An Analysis of Several Methods for Handling Hard-Sphere Frictional Contact in Rigid Multibody Dynamics

Hammad Mazhar*, **Daniel Melanz***, **Michael C. Ferris**#, **Dan Negrut***

* Department of Mechanical Engineering
University of Wisconsin-Madison
1513 University Ave., Madison, WI, 53706, USA
[hmazhar, melanz, negrut]@wisc.edu

Department of Computer Sciences
University of Wisconsin-Madison
1210 W Dayton St., Madison, WI, 53706, USA
[ferris]@cs.wisc.edu

Abstract

We analyze several methods to solve the equations of motion (EOM) associated with the dynamics of rigid bodies that interact through contact and friction. We rely on a Differential Variational Inequality (DVI) approach to solve the EOM using three different formulations of the DVI problem: the primal [1, 2], dual [3], and generalized. Several benchmark tests are used to provide insights into the pitfalls and artifacts associated with these formulations and their numerical solution. The goal is to understand how several solvers: e.g., Mosek [4], PATH [5], Accelerated Projected Gradient Descent (APGD) [6], Jacobi and Gauss Seidel, perform in conjunction with these three formulations. At a high level, the primal formulation first solves for body velocities and subsequently recovers the remaining unknown quantities such as the frictional contact force. The dual formulation goes the opposite way: it solves for the frictional contact forces first and proceeds to recover the rest of the solution unknowns. The generalized formulation solves for all unknowns in one fell swoop.

The testing infrastructure utilized relies on Matlab, the General Algebraic Modeling System (GAMS) [7], and a C++ library called Chrono [8]. The GAMS modeling language supports modeling Second Order Cone Programs (SOCP), which are used by Mosek in conjunction with the primal formulation. APGD solves for the rigid body dynamics with frictional contact by concentrating on the dual formulation. Additionally the EOM can be directly modeled using the GAMS Extended Mathematical Programming (EMP) framework, which automatically forms the KKT conditions associated with the DVI problem and then uses the PATH solver to find a solution. In this "generalized" formulation the reduction of the discretized EOM to a smaller problem, which is the strategy adopted for the primal and dual formulations, is completely bypassed. While convenient to pose, the generalized formulation leads to large and highly nonlinear problems that are challenging to solve and significantly less efficient.

Preliminary Results

Several benchmark tests that we will report on indicate that both Mosek and APGD, which implement the primal and dual formulations, respectively, converge to similar objective function values in similar times. The simulation results proved rather sensitive for the dual formulation – for instance, small changes in the number of spheres in a "pile of bodies at rest" simulation resulted in large changes in the objective value. In such cases the number of iterations required to converge for APGD varied greatly while Mosek, which is a primal dual interior point method, consistently converged within the same number of iterations and wall clock time. The PATH solver, used herein to implement the generalized formulation, provided results similar to APGD and Mosek in terms of velocities and reaction impulses for contacts yet its performance scaled poorly, see Figure 1.

The relaxation of the nonlinear complementarity problem (NCP) to a cone complementarity problem (CCP) embraced in the dual formulation, implemented here using the AGPD solver, introduces numerical artifacts at high friction and/or sliding velocity. This is demonstrated using a simple test with a 3D rigid ball sliding on the ground with an initial velocity of $-2\,\mathrm{m/s}$ in the x direction. It has a radius of $1\,\mathrm{m}$ and the contact has a friction value of $\mu = .2$. The ball, which is initially sliding, slowly begins to roll due to friction and eventually gets into a steady state rolling motion. The time it takes to get to this state is $t_{rolling} = \frac{2v_0}{7\mu g}$ [9]. For an initial velocity of $2\,\mathrm{m/s}$ and $g = 9.81 m/s^2$, the ball will be fully rolling at $t_{rolling} = .291s$. A numerical integration step size of $h = 0.0025\,\mathrm{s}$ was used to capture these dynamics. Figure 2 shows results obtained with both the relaxed and non-relaxed methodologies and displays an artifact in the high sliding regime for the dual formulation. As expected, these artifacts are due to the relaxation that transforms the NCP into a numerically less challenging CCP.

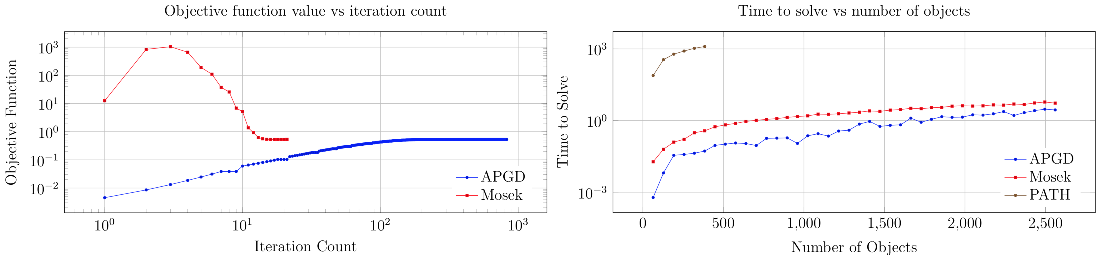

Figure 1: Left: Convergence behavior of APGD and Mosek. The latter initially increases the objective function value before converging to the correct solution. Note that the absolute value of the objective function is used and shown on a log scale. Right: Scaling analysis of APGD, Mosek and PATH for a problem with spheres filling a container (stacking problem). The number of spheres was increased to lead to an increasingly larger number of contacts.

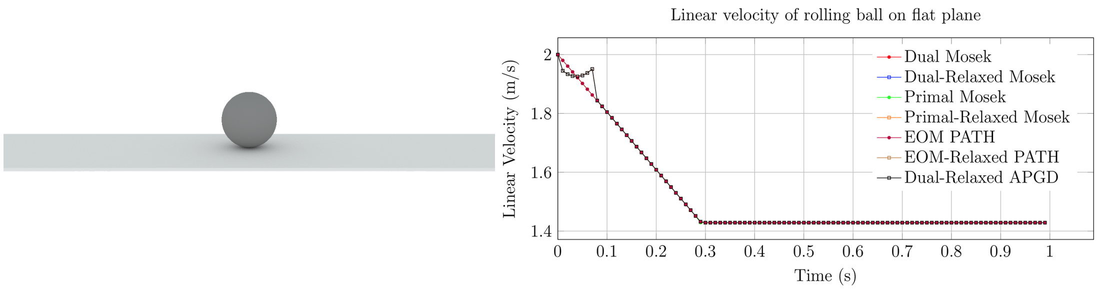

Figure 2: Comparison of different formulations showing how the relaxation that transforms the NCP into a CCP creates artifacts when two bodies in contact are sliding with friction.

References

[1] F. Cadoux. An optimization-based algorithm for coulomb's frictional contact. *ESAIM: Proc.*, 27:54–69, 2009.

[2] V. Acary, F. Cadoux, C. Lemarechal, and J. Malick. A formulation of the linear discrete coulomb friction problem via convex optimization. *ZAMM-Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik*, 91(2):155–175, 2011.

[3] M. Anitescu and G. D. Hart. A constraint-stabilized time-stepping approach for rigid multibody dynamics with joints, contact and friction. *International Journal for Numerical Methods in Engineering*, 60(14):2335–2371, 2004.

[4] E.D. Andersen, C. Roos, and T. Terlaky. On implementing a primal-dual interior-point method for conic quadratic optimization. *Mathematical Programming*, 95(2):249–277, 2003.

[5] S. Dirkse and M. Ferris. The path solver: A non-monotone stabilization scheme for mixed complementarity problems. 1993.

[6] H. Mazhar, A. Heyn, Tasora, and D. Negrut. Using Nesterov's method to accelerate Multibody Dynamics with friction and contact. *ACM Transactions on Graphics (TOG)–under review*, 2014.

[7] GAMS Development Corporation. General Algebraic Modeling System (GAMS) Release 24.2.1. Washington, DC, USA, 2013.

[8] Project Chrono. An Open Source Framework for the Physics-Based Simulation of Dynamic Systems, 2014.

[9] J. C. Trinkle. Formulation of multibody dynamics as complementarity problems. In *Volume 5: 19th Biennial Conference on Mechanical Vibration and Noise, Parts A, B, and C*. ASME.