Nonlinear random vibration of the cable modeled as MDOF system and excited by filtered Gaussian white noise

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Abstract

We investigate the nonlinear random vibration of the cables with small sag and excited by colored or filtered Gaussian white noise uniformly distributed on the cable. The cable and many other systems in science and engineering can be modeled as nonlinear stochastic dynamical (NSD) systems with multiple degrees of freedom (MDOF). It is known that the analysis on the probabilistic solutions of MDOF-NSD systems has been a challenge for almost a century, especially for the systems with strong nonlinearity or large number of nonlinear terms. There are three methods that were used to analyze the MDOF-NSD systems. The first one is the Monte Carlo simulation (MCS) method that was proposed by Metropolis and Ulam in 1949 in their research about atomic physics [1, 2, 3]. There are some challenges in using MCS method for analyzing the strongly nonlinear stochastic dynamic systems with multiple degree of freedoms, such as the problems of round-off error, numerical stability, convergence, and requirement for huge number of samples for strongly nonlinear system. The second one is the equivalent linearization (EQL) method which was proposed by Booton in 1954 in the research about nonlinear random dynamics of electronic circuit [4, 5]. It is well known that the EQL method is suitable for analyzing the weakly nonlinear systems excited by Gaussian excitation for obtaining the probabilistic solutions of the system responses. The third method named state-space-split and exponential polynomial closure (SSS-EPC) method that was proposed in 2010 for the probabilistic solutions of large MDOF-NSD systems with polynomial type of nonlinearity or solving the Fokker-Planck-Kolmogorov (FPK) equations in high dimensionality [6, 7]. It was extended for analyzing the systems excited by colored or filtered Gaussian white noise [8]. The SSS method can make the problem of solving the FPK equation in high dimensionality become the problem of solving some FPK equations in low dimensionality or make the large NSD system decoupled into some small NSD systems. Therefore, the FPK equations in low dimensionality can be solved with the exponential polynomial closure method [9, 10]. In this paper, the SSS-EPC method is further used to analyze the probabilistic solutions of the in-plane vibration of the cable with small sag and excited by filtered Gaussian white noise uniformly distributed on the cable. The equation of motion of the cable is a nonlinear partial differential equation in time and space [11, 12]. With Galerkin method, the nonlinear partial differential equation is reduced to MDOF-NSD system. The results obtained with the SSS-EPC method are compared with those obtained with EQL and MCS to show the effectiveness of the SSS-EPC method in this case and the advantage of SSS-EPC method over EQL and MCS in analyzing the formulated MDOF system for the cable with small sag, even and strong nonlinearity, and large number of nonlinear terms.

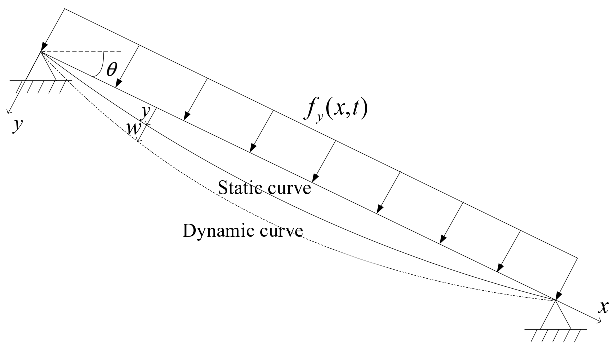

Figure 1: Inclined cable.

For the horizontal steel cable with Young's modulus being 2.1×10^{11}, damping ratio for each mode being 0.01, length being $120m$, diameter of cross section being $0.1m$, material density being $7,850kg/m^3$, small sag-to-span ratio being given. The cable is excited by uniformly distributed force with density $100f(t)$ in which $f(t)$ is a filtered Gaussian white noise. The PDFs and the logarithms of the PDF of the deflection in the middle of the cable are obtained with SSS-EPC, MCS, and EQL and compared when the cable is modeled as MDOF-NSD system. The dimension-reduction procedure of the SSS method is adopted to make the FPK equation governing the PDF solution of the nonlinear random vibration of cable reduced to the FPK equation in four-dimensional space. Then the EPC method is used to solve the FPK equation in four-dimensional space. From numerical analysis it is observed that the SSS-EPC method works well for obtaining the PDF of the deflection of the cable excited by filtered Gaussian white noise. It is also observed that the results obtained by modeling the cable as MDOF-NSD system is improved in comparison with those obtained by modeling the cable as SDOF system. We also found that the computational time needed by MCS for the MDOF-NSD system is huge compared to that needed by the SSS-EPC method. The solution from EQL is not acceptable for this nonlinear random vibration problem of cable.

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