

# Dynamic Fracture Simulation of Flexible Multibody Systems via Absolute Nodal Coordinate Formulation and SPH Method

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## Abstract

A new computation method is proposed to study the dynamic fracture of the flexible multibody system, where the potential fracture part with large deformation is modeled by using the Smoothed Particle Hydrodynamics (SPH) method [1, 2] and the other parts are described by using the Absolute Nodal Coordinate Formulation (ANCF) [3, 4]. Further research about SPH and ANCF can be referred to the works by Schörrghumer et al [5] and Hu et al [6]. The dynamic equations for the SPH particles are expressed as

$$\frac{d\mathbf{v}_a}{dt} = \sum_{b=1}^N m_b \left( \frac{\boldsymbol{\sigma}_a}{\rho_a^2} + \frac{\boldsymbol{\sigma}_b}{\rho_b^2} - \Pi_{ab} \right) \nabla_a W_{ab} + \mathbf{g} , \quad (1)$$

where  $N$  is the total number of particles inside the support domain of particle  $a$  which is shown as Figure 1,  $\boldsymbol{\sigma}_a$  and  $\boldsymbol{\sigma}_b$  are the Cauchy stress tensor at the particle  $a$  and  $b$ ,  $\rho_a$  and  $\rho_b$  are the density of particle  $a$  and  $b$ ,  $\mathbf{g}$  is the gravitational acceleration vector,  $\Pi_{ab}$  is the artificially introduced viscosity term and  $W_{ab}$  is the smoothing kernel function between particle  $a$  and  $b$ .

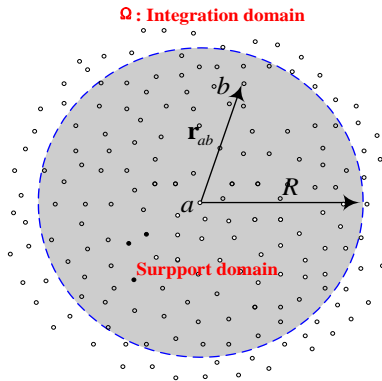


Figure 1: Particles within the support domain of particle  $a$

The dynamic equations for the ANCF elements are expressed as

$$\begin{cases} \mathbf{M}\ddot{\mathbf{q}} + \Phi_q^T \boldsymbol{\lambda} + \mathbf{F}(\mathbf{q}) = \mathbf{Q}(\mathbf{q}, \dot{\mathbf{q}}) \\ \Phi(\mathbf{q}, t) = \mathbf{0} \end{cases}, \quad (2)$$

where  $\mathbf{M}$  is the constant mass matrix of the ANCF elements,  $\mathbf{F}(\mathbf{q})$  is the elastic force vector,  $\Phi(\mathbf{q}, t)$  represents the vector that contains the system constraint equations,  $\Phi_q$  is the derivative matrix of constraint equations with respect to the generalized coordinate vector  $\mathbf{q}$ ,  $\boldsymbol{\lambda}$  is the Lagrange multiplier vector,  $\mathbf{Q}(\mathbf{q}, \dot{\mathbf{q}})$  is the external force vector including the forces applied by the SPH particles.

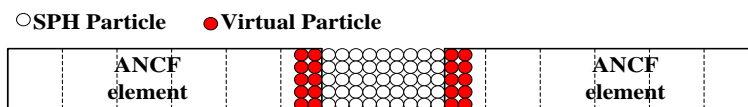


Figure 2: The connection configuration of particles and elements

In order to maintain the deformation field continuation, extra virtual particles are introduced and embedded in the boundary of ANCF elements, as shown in Figure 2, which are connected to the physical SPH particles. The interaction forces between the SPH particles and the ANCF elements are transmitted by the virtual particles. The domain decomposition and GPU based parallel algorithms are used to improve the efficiency of interaction detection in SPH computation. A predictor-corrector scheme is used to solve the governing equations of potential fracture part discretized by SPH particles. The generalized-alpha method based on sparse matrix storage skill is used to solve the huge set of multibody system equations. Finally, the dynamic fracture simulation of a crank slider model is given to validate the proposed computation method, Figure 3 shows the dynamic configurations of the system .

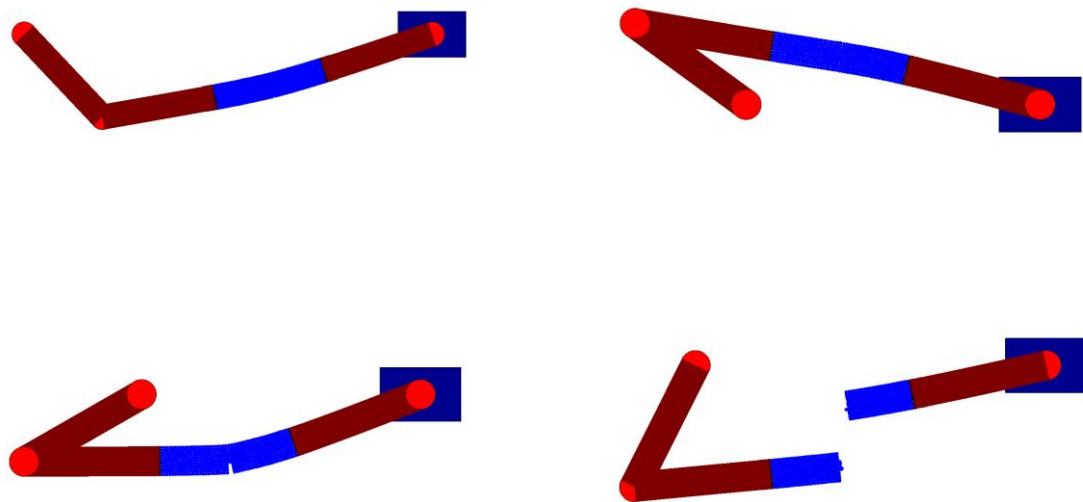


Figure 3: Dynamic configurations of the crank slider

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