Damping amplification caused by a mechanism that trans-pass through its singular position

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Abstract

In the paper, vibrations of a hybrid multibody-continuous system are investigated. Despite of their presence in the each day experiences, vibrations of mechanical elements are undesired in most of the technical cases. Thus, for all the mechanical devices, investigations of effective damping methods are pointed as crucial topics of the design process of mechanisms. Presently, methods based on viscous dampers and elasto-viscous elements look dominant. In the paper, an alternative based on modal disparity is investigated [3, 4].

When zooming on the consequences of the structural damping present in continuous systems, high frequency vibrations are well damped in general (excluding the cases of resonances or the self-excited vibrations). Harmonic vibrations of low frequency are more challenging, as intensity of the structural damping depends on the velocity of element deformation, and the last is proportional with the frequency. To accelerate the damping, external velocity amplification can be useful, or energy transfer between different modes can be helpful. The last is impossible in the linear systems, where behaviour of different modes can be treated independently (fundamental property of the modal analysis). However, the energy transfer become possible, when nonlinear elements are present. Such combinations are challenging, however. Parametric vibrations or even chaotic motions can be obtained [1]. The requested nonlinearity can be obtained with different methods. At present, a mechanical system is introduced and set in a neighbourhood of its kinematic singular position [4, 5].

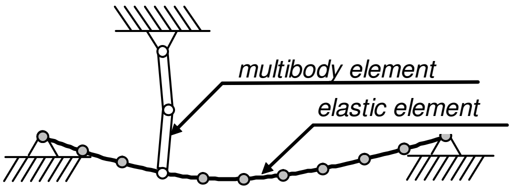

Figure 1: Main elements of the considered system.

To test the hypothesis of damping effectiveness, a numerical model is proposed and tested. The system is planar (fig. 1). Its model consists of two parts: a multibody part and an elastic beam part. The last is considered as a structural element composed of finite elements.

The mechanism is modelled as a multibody system composed of two rigid bodies Massless joints are used to connect the bodies. Joint displacements are taken as system generalized coordinates. The Newton/Euler's equations are developed for free body diagrams of the all the bodies of the system. The dynamic equations are combined with the kinematics equations of the considered kinematical chain. Projections of the dynamics equations are taken on joint axes and the final matrix form of the dynamics equations is [5, 6]

$$\boldsymbol{M}^b(\boldsymbol{q}^b)\cdot\ddot{\boldsymbol{q}}^b+\boldsymbol{F}^b(\dot{\boldsymbol{q}}^b,\boldsymbol{q}^b)-\boldsymbol{Q}^b(\dot{\boldsymbol{q}}^b,\boldsymbol{q}^b,\boldsymbol{f}^e,\boldsymbol{t}^e,t)=0\,. \tag{1}$$

To express dynamics of the continuous beam, finite elements are used (Fig. 1). Deformations of a set of selected points (nodes) are considered as the coordinates. Dynamics equation of the system is [7]

$$\boldsymbol{M}^c\cdot\ddot{\boldsymbol{q}}^c+\boldsymbol{D}^c\cdot\dot{\boldsymbol{q}}^c+\boldsymbol{K}^c\cdot\boldsymbol{q}^c=\boldsymbol{P}^c\,. \tag{2}$$

To joint the subsystems, constraint equations are introduced. The end point of the last arm of the multibody system is joined with the beam by use of a spherical joint constraint. The spherical joint coincides with a node of the elastic structure. The constraint equations are expressed at the position level, and then the time derivatives of the constraint equations are developed. With the introduced constraint equations, the dynamics equations Eqs. (1) and (2) have to be accompanied with Lagrange's multipliers (reaction forces at the constrained connecting point). Resulting equations are [4, 5]:

$$\boldsymbol{M}^b \cdot \ddot{\boldsymbol{q}}^b + \boldsymbol{F}^b - \boldsymbol{Q}^b + \boldsymbol{J}^{b^T} \cdot \boldsymbol{\lambda} = 0\,; \tag{3}$$

$$\boldsymbol{M}^c \cdot \ddot{\boldsymbol{q}}^c + \boldsymbol{D}^c \cdot \dot{\boldsymbol{q}}^c + \boldsymbol{K}^c \cdot \boldsymbol{q}^c + \boldsymbol{J}^{c^T} \cdot \boldsymbol{\lambda} = \boldsymbol{P}^c \ . \tag{4}$$

Next, the Lagrange's multipliers and the dependent coordinates are eliminated. It is made with a modified version of the classical coordinate partitioning method [2]. Presently, to avoid numerical problems sourced in numerical singularity of the considered matrices, one of the multibody system coordinates is considered as independent. After the announced elimination the dynamic equations are:

$$\boldsymbol{M}^* \cdot \ddot{\boldsymbol{q}}^* + \boldsymbol{F}^* - \boldsymbol{Q}^* = 0 \ . \tag{5}$$

These equations are composed of accelerations of the independent coordinated, only. Basing it on the announced equations, accelerations of the independent coordinates are evaluated.

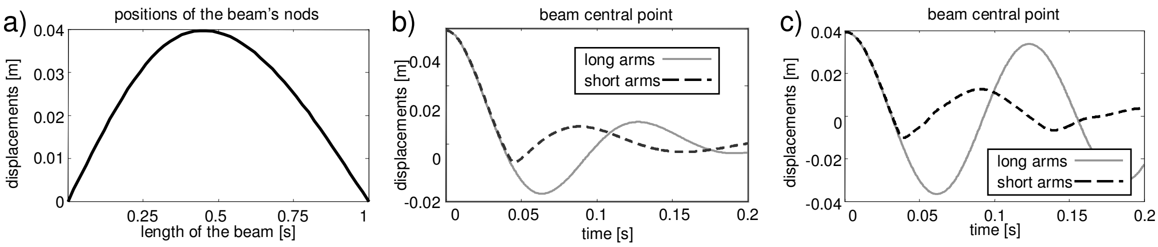

Figure 2: The beam selected deformations: initial position of the beam (a); time evolution of the position of the beam's central point – higher structural damping (b); time evolution of the position of the beam's central point – lower structural damping (c)

The introduced model is tested numerically. The statically loaded beam drops its load and responses of the system are observed (Fig. 2). Except of some beam structural damping, other damping elements are not present in the mechanism. Tests are performed with different values of the structural damping, as well as with different lengths of the mechanism's arms. The shorter arms lead to the mechanism singular configuration with the end effecter position slightly below of the beam equilibrium position.

References

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