

# Planar oscillations of a dumb-bell of a variable length in a central field of Newtonian attraction. Exact approach

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## Abstract

Investigation of orbital systems with a variable mass distribution arises to early 1960s. In particular, a rule for the mass redistribution allowing to keep fixed a direction to the attracting center in the axes fixed in the body, was proposed by V. A. Sarychev and W. Schiehlen within a so-called “satellite approximation” (see also [1, 2, 3, 4]). For the dumbbell-like body of variable length necessary conditions of stability of the radial configuration were studied in [5]. These results were rediscovered and partially completed in [6, 7]. Another aspect of the orbital dynamics of bodies with a variable mass distribution relates to possibility of using of this redistribution for variation of orbital parameters. Its investigation arising to [8] (see also [9]), is a subject of some modern publications (cf. [10, 11]). The third aspect relates to problems of deployment – retrieval of orbital tethered systems with tethered elements of a finite mass [12, 13]. Finally there are investigations devoted to using of dumbbell-like bodies of a variable length for verification of relativistic hypothesis [14, 15, 16].

Here we investigate dynamics of a dumb-bell of a variable length in a central field of Newtonian attraction. It is assumed that the body moves in a plane fixed in the absolute space and passing through an attracting center. The law of length’s variation providing an existence of stationary configurations is pointed out. For these configurations the dumb-bell forms a constant angle with a local vertical passing through the center of mass of the dumb-bell, which moves in an elliptic orbit similar to the Keplerian. In particular, the mentioned constant angle may be equal to zero. In contrast to previous investigations [6, 7, 17] the problem is solved within the exact formulation, without supplementary simplifying assumptions concerning smallness of the dumb-bell in comparison to its distance from the attracting center. Under assumptions mentioned we find the steady solutions corresponding to traditional relative equilibria. After that we investigate their stability using equations in variations with help of exact estimations for parameters of the problem.

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