Reduction of a high-fidelity vehicle dynamic model using the proper orthogonal decomposition

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Abstract

The demand for high-fidelity automotive simulation models has been increasing in recent decades. State-of-the-art automotive design and analysis paradigms benefit from more realistic simulation models in many respects, including the reduction of experimental costs. Sophisticated models can also incorporate more complex physical phenomena in the simulation of mechanical systems, which can improve the reliability, identifiability, and controllability of the models. The computational capabilities of modern processors, coupled with the performance of modern software packages, provide the essential tools to create complex dynamic simulations of automotive systems.

For a dynamic system to be *proper* for a particular application, it should produce accurate results with a minimal degree of complexity [1]. A proper model preserves the fundamental characteristics of the dynamic system behavior, and the computational effort involved in simulating the model is minimized for the application of interest (e.g., system identification or advanced control design). Formal model order reduction methods are popular for designing dynamical systems, where computation time and storage requirements are prominent considerations in many applications. Reducing the complexity of a model while preserving the input–output behavior of the system is an essential part of the design process, and plays a key role in control design and real-time applications.

In this work, we reduce a high-fidelity model of a vehicle with double-wishbone front suspensions and trailing-arm rear suspensions, as shown in Figure 1. We use the Fiala tire model [2] to simulate the interaction between the tire contact patch and the road. The vehicle model is developed in MapleSim [3], which generates a system of fully symbolic differential-algebraic equations (DAEs). The model is represented by 26 generalized coordinates that are coupled by 12 algebraic constraint equations, leaving 14 degrees of freedom: 3 translations and 3 rotations of the chassis, the jounce/rebound motion of each

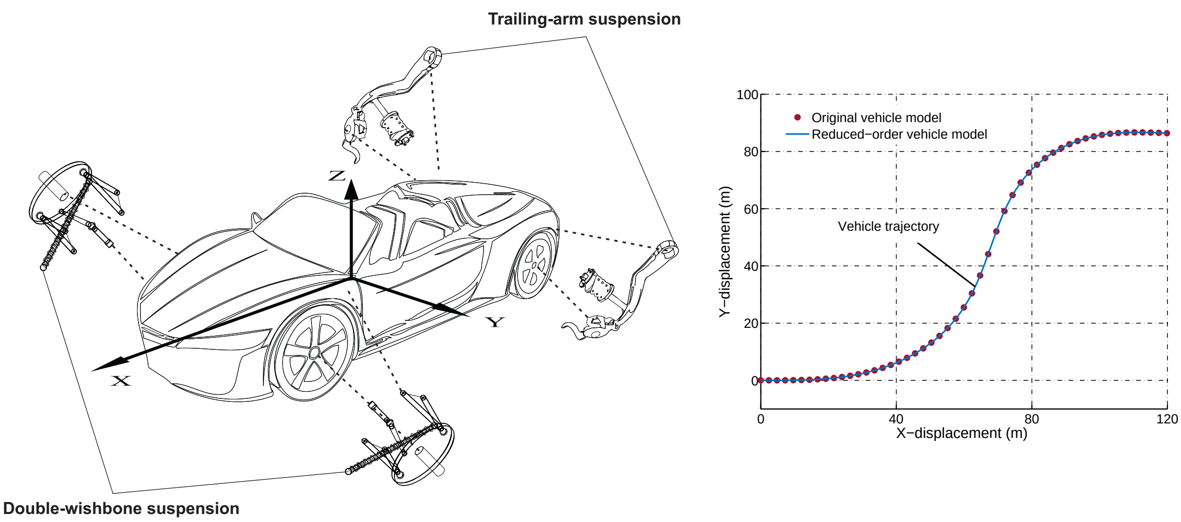

Figure 1: Schematic of vehicle model with double-wishbone front suspensions and trailing-arm rear suspensions (left) and simulated trajectory of the high-fidelity vehicle model using original and reduced-order models (right).

suspension, and the spin of each wheel; the steering angle is an input to the model. The nonlinear DAEs governing the dynamic behavior of the vehicle can be expressed in the following general form:

$$\boldsymbol{G}\dot{\boldsymbol{x}}(t) = \boldsymbol{F}(t, \boldsymbol{x}(t), \boldsymbol{u}(t)) \tag{1}$$

where $\boldsymbol{G}$ is the coefficient matrix, $\boldsymbol{x}$ is the state vector, $\boldsymbol{F}$ is the vector of generalized forces, and $\boldsymbol{u}$ is the vector of inputs to the model. Note that $\boldsymbol{G}$ is singular due to the algebraic constraints. Baumgarte constraint stabilization [4] is employed to convert the governing DAEs to pure ordinary differential equations (ODEs). A single-lane-change maneuver is simulated by specifying an initial forward speed and providing a sinusoidal steering input. We first simulate the system using the original ODEs, then repeat the simulation using a reduced model. All numeric parameters for the vehicle body, suspension systems, and tires are obtained from an example application in the MapleSim model gallery [3].

The proper orthogonal decomposition is used to construct a reduced-order model from the detailed dynamic system. This projection-based model reduction scheme has been widely used for reducing nonlinear systems [5, 6]. We use the proper orthogonal decomposition to form lower-dimensional approximations of general dynamic systems, reducing the system size while preserving the most informative dynamic behavior [7]. Snapshots from the state trajectories of the high-fidelity vehicle model are used to construct a basis onto which the original system states are projected. Our reduced-order model simulates 23% faster than the original model, produces results that are in very close agreement to those obtained with the original model (see Figure 1), and is robust to perturbations of the input signal. The proper orthogonal decomposition is a reliable model reduction tool for small-scale, nonlinear multibody systems.

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