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On the Use of the Wrench Exertion Capability as a Performance Index for Cable Driven Robot

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Abstract

The evaluation of the performances of cable driven robots must explicitly take into account the considerable limitations introduced by cable tension bilateral bounds, translating the physical need that neither negative nor limitless cable forces can be exerted. As a result, though cable driven robots are basically parallel robots, the performance indices typically suggested for parallel robots cannot be employed straightforwardly. In [1] a novel approach to cable robot performance evaluation has been presented and applied to solely redundant cable robots (i.e. robots with more cables than degrees of freedom of the moving platform). Such an approach is based on the computation of the maximum force which can be exerted by the cables on the moving platform along a specific direction. By extending the reasoning behind the approach in [1], this paper introduces a novel performance index called Wrench Exertion Capability (WEC) which can be applied to any cable robot topology.

The computation of the WEC is based on the solution of a linear programming problem involving cable tensions, cable tension limits, and a suitable representation of the wrench matrix. The wrench matrix (or structure matrix) $\boldsymbol{S}$ of a cable robot usually defines the relation between the wrench $\boldsymbol{w_c}$ exerted by the cable forces on the moving platform and the tension vector $\boldsymbol{\tau}$ containing the cable forces: $\boldsymbol{w_c} = \boldsymbol{S\tau}$. The structure matrix $\boldsymbol{S}$ only allows computing the cable wrench $\boldsymbol{w_c}$ exerted by the cables on the moving platform. In general, this is not the sole wrench applied to the moving platform. In order to compute the total wrench $\boldsymbol{w} := [\boldsymbol{f}^T \quad \boldsymbol{t}^T]^T$ applied to the moving platform, external loading, including, for example, gravity force, should be taken into account. In the previous definition of $\boldsymbol{w}$ vectors $\boldsymbol{f}$ and $\boldsymbol{t}$ are respectively the overall forces and torques exerted on the moving platform by the cables and the external forces. In order to account explicitly for external forces, a novel definition for the wrench matrix (denoted by $\boldsymbol{W}$) is introduced, which is obtained by simply aggregating the structure matrix $\boldsymbol{S}$ and the external wrench $\boldsymbol{w}_e$:

$$\boldsymbol{w} = \boldsymbol{w}_c + \boldsymbol{w}_e = \boldsymbol{S\tau} + \boldsymbol{w}_e = [\boldsymbol{S} \quad \boldsymbol{w}_e]\begin{Bmatrix}\boldsymbol{\tau}\\ 1\end{Bmatrix} := \boldsymbol{W}\begin{Bmatrix}\boldsymbol{\tau}\\ 1\end{Bmatrix} \tag{1}$$

Once the matrix definition of $\boldsymbol{w}$ in Equation (1) is introduced, it is possible to develop cable robot performance analysis following a well established approach. In particular, in the performance analysis of parallel manipulators, it has been proved convenient to split Jacobian matrices into their "translational" parts and "rotational" ones [2]. By applying the same idea to the novel definition of wrench matrix $\boldsymbol{W}$ of a cable robot, it is here suggested to split it into two parts, namely $\boldsymbol{W}_f$ and $\boldsymbol{W}_t$ (where $\boldsymbol{W} := [\boldsymbol{W}_f^T \quad \boldsymbol{W}_t^T]^T$) to analyze separately force and torque exertion capabilities. Moreover, in order to refer the evaluation to a specific direction d, a rotation matrix $\boldsymbol{R}$ can be introduced to define such a direction of interest univocally in an absolute reference frame. The reason for referring the evaluation to a given direction d comes from a typical practical need when designing a cable robot, which is predicting the maximum force that can be exerted along such a direction of interest, usually keeping null wrench components, both in terms of forces and torques, along the other directions. This is basically what we mean by evaluation of the WEC of a cable robot. Clearly, WEC is not limited to maximum force evaluations.

Once the direction d is defined, symbols *o1* and *o2* can be adopted to denote orthogonal Cartesian directions, and the following expression can be adopted to rotate matrices $\boldsymbol{W}_f$ and $\boldsymbol{W}_t$:

$$\boldsymbol{R}^T\boldsymbol{W}_f := \begin{bmatrix}\boldsymbol{W}_{f_d}\\ \boldsymbol{W}_{f_{o1}}\\ \boldsymbol{W}_{f_{o2}}\end{bmatrix}; \qquad \boldsymbol{R}^T\boldsymbol{W}_t := \begin{bmatrix}\boldsymbol{W}_{t_d}\\ \boldsymbol{W}_{t_{o1}}\\ \boldsymbol{W}_{t_{o2}}\end{bmatrix} \tag{2}$$

Then, for example, the WEC of a fully constrained cable robot, expressed in terms of maximum force w_{f_d} that can be exerted along the direction d (and hence referred to as WEC_d^f) while keeping bounded

cable tensions and given values $\tilde{\boldsymbol{w}}_R$ of the other wrench components along the directions d, $o1$, and o2, can be computed by solving the following linear programming problem (the symbol $\preccurlyeq$ stands for the componentwise inequality):

$$WEC_d^f := max\left(w_{f_d} = \boldsymbol{W}_{f_d} \begin{Bmatrix} \boldsymbol{\tau} \\ 1 \end{Bmatrix} \right) \quad s.t.: \begin{cases} \begin{bmatrix} \boldsymbol{W}_{f_{o1}} \\ \boldsymbol{W}_{f_{o2}} \\ \boldsymbol{W}_{t_d} \\ \boldsymbol{W}_{t_{o1}} \\ \boldsymbol{W}_{t_{o2}} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\tau} \\ 1 \end{Bmatrix} := \boldsymbol{A} \begin{Bmatrix} \boldsymbol{\tau} \\ 1 \end{Bmatrix} := \tilde{\boldsymbol{w}}_R \\ \boldsymbol{\tau_{min}} \preccurlyeq \boldsymbol{\tau} \preccurlyeq \boldsymbol{\tau_{max}} \end{cases} \tag{3}$$

As previously mentioned it can be interesting setting $\tilde{\boldsymbol{w}}_R$ equal to $\boldsymbol{0}$. Such an approach can be extended to other cable robot topologies, including underactuated cable robots. In underactuated cable robots, however, it is impossible to apply the proposed optimization unless a sufficient number of equations in the linear problem $\boldsymbol{A\tau} = \tilde{\boldsymbol{w}}_R$ is removed: in practice, it is impossible to assign finite values to all the $\tilde{\boldsymbol{w}}_R$ components, but only to some of them. These aspects are discussed in detail in the paper.
In the representative example shown in Figure 1, a three-dof planar, underconstrained, and redundant robot is considered. The robot is assumed to move in a vertical plane. In the subplot on the left the robot is schematically depicted at a point P: the red vector provides a scale representation of the WEC_x^f (i.e the maximum force that can be exerted on the moving platform in the rightward horizontal direction) whose value (in *N*) is written above the vector. The green vector represents the gravity vector while blue segments represent the cables and the tensions within them (bold scaled arrows). The subplot on the right represents the isolines of the WEC_x^f values computed within the Statically Feasible Workspace delimited by the green solid lines shown in the subplot on the left. The region where the best performance is achieved can be immediately recognized: it is the one where the isolines take the highest values (i.e. those in red). The WEC_x^f is here computed considering a moving platform mass by 5 kg and cable tensions limited in the range 5-100 *N*.
Other significant examples of WEC computation are discussed in the paper and an approach is suggested to introduce in the linear programming problem a suitable number of constraints in the form of inequality whenever it is impossible (e.g. with underactuated robots), or it is not necessary for the given application, to provide all the constraints in equality form.

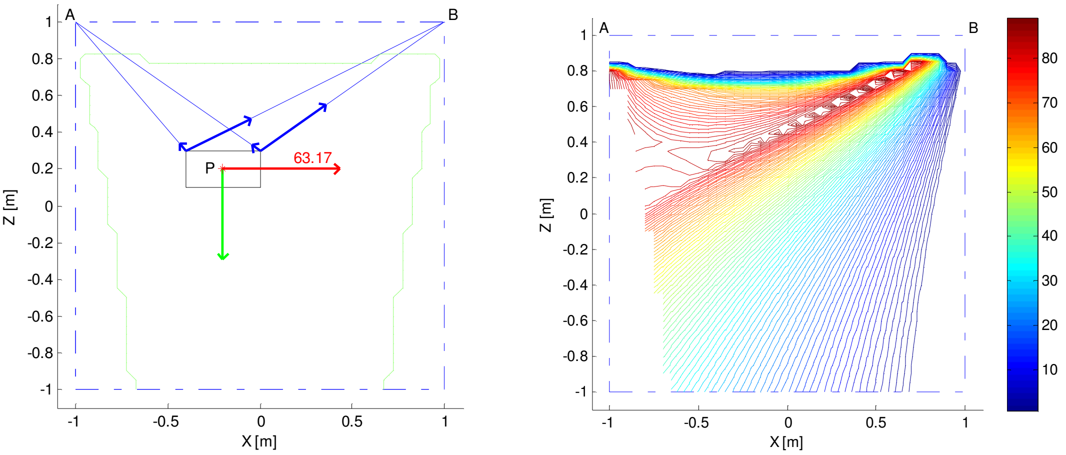

Figure 1: WEC_x^f of a redundant and underconstrained robot computed at a single point P (plot on the left) and throughout the SFW (plot on the right).

References

[1] G. Boschetti, A. Trevisani. Performance Evaluation for Cable Direct Driven Robot. In Proceedings 12th Biennial Conference on Engineering Systems Design and Analysis "ESDA 2014", Copenhagen, Denmark, 2014.

[2] JP. Merlet. Jacobian, manipulability, condition number, and accuracy of parallel robots. ASME Journal of Mechanical Design, Vol. 128, pp. 199–206, 2006.