Dynamic Modeling Approach for a Continuous Moving Belt

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Abstract

Belt drives are used in numerous applications, such as automotive engines, household applications or industrial engines, to transmit power between different machine elements. Because of their simple installation and low maintenance together with their ability to absorb shocks, they are frequently used instead of chain or geared transmission systems. However, they can exhibit complex dynamic behaviors, such as the transverse vibrations of the belt spans, sliding of the belt over the pulley, etc. All this phenomena impact the belt life and also the acoustic comfort. It is therefore of interest to predict the dynamic response of such systems using numerical models. Examples of related simulation models and references to this class of problems can be found in [1] and [2].

In this work a mathematical model for the non-synchronous belt drive is presented, which is able to describe the entire belt (free spans as well as wrapped arc sections), and which makes the need of dissections unnecessary. In order to simulate the dynamic behavior of the belt, the longitudinal movement and the transversal vibrations are modeled. Furthermore, the belt-pulley interaction is incorporated and modeled by two functions, which define the belt-pulley repulsion force, modeled by elastic contact conditions, and the transferred torque, modeled by a slip approach.

Model

The movement of the belt drive can be separated into longitudinal motion and lateral vibration, where the lateral displacement describes the belt configuration as offset from a reference state. This reference configuration agrees to the unloaded ideal geometric (static) shape of the drive. Hence, the reference configuration of the belt can be described by ideal straight segments and arcs wrapped around the pulleys, and which as a whole describe a closed curve in two dimensions:

$$\vec{x}_0 : [0, L_0] \to \Re^2 \quad \text{with } \vec{x}_0(0) = \vec{x}_0(L_0) \text{ and } \|\vec{x}_0'\| = 1 \tag{1}$$

The parameter L_0 is the length of the belt in the reference configuration. The position of a moving belt is distorted with respect to its reference state and can be described by the lateral displacement

$$w = w(s,t) \tag{2}$$

which is a function of arc length (of the reference state) s and time t. The curve of the belt at time t is given as:

$$\vec{x}(s,t) = \vec{x}_0(s) + w(s,t) R \vec{x}_0'(s) \tag{3}$$

where R denotes the 90° counter clockwise rotation matrix: $R := \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

The lateral displacement of the belt w is represented as a linear combination of N time invariant basis functions w_i:

$$w(s,t) = \sum_{0 \le i < N} q_i(t) w_i(s) = \underline{q}(t)^T \, \underline{w}(s) \tag{4}$$

The local longitudinal dynamics of the belt is neglected, therefore only one global uniform longitudinal displacement q_{long} and global transport velocity v of the belt is taken into consideration.

$$q_{\mathrm{long}} = \int_{t_0}^{t} v dt \quad \Rightarrow \quad v = \dot{q}_{\mathrm{long}} \tag{5}$$

Equation of Motion

The Lagrange-d'Alembert principle is used in order to derive the equation of motion. The state variables for this problem are the lateral displacements q_i and the longitudinal displacement q_{long}. The contributing forces for the modeling of the equations are the pulley-belt contact forces, the strain forces due to longitudinal elongation, the shear forces between pulley and belt and the bending forces. Additionally, a slip approach is incorporated to model the variable friction coefficient at the pulley belt contact and for simplification and model reduction the following assumptions are made:

- The variations of the mass density m due to elongation can be ignored.
- The magnitude of the lateral displacement is small in comparison the radii of the pulleys.

Finally the state of the belt drive can be expressed by a system of equations for the transversal motion:

$$\begin{aligned} & m\left(\underline{\underline{C}}_0\underline{\ddot{q}} + \underline{\underline{C}}_1\underline{q}\dot{v} + 2\underline{\underline{C}}_1\underline{\dot{q}}v + \left(\underline{K}_0 - \underline{\underline{C}}_2\underline{q}\right)v^2\right) - \left(F_0 + EA\frac{\Delta L}{L_0} + DA\frac{\Delta \dot{L}}{L_0}\right)\left(\underline{K}_0 - \underline{\underline{C}}_2\underline{q}\right) \\ & + \left(EI\left(\underline{K}_2 + \underline{\underline{C}}_4\underline{q}\right) + DI\underline{\underline{C}}_4\underline{\dot{q}} + DIv\left(\underline{\underline{C}}_5\underline{q} - \underline{K}_3\right)\right) = \sum_{0 \le i < N_p} \underbrace{M_p^{(i)}(t)\left(\underline{\underline{T}}_i\underline{q} + \underline{R}_i\right) + \int_0^{L_0} \underline{w} F_{rep}^{(i)}(d_i.\dot{d}_i)\frac{\left(\vec{x} - \vec{c}_i\right)\cdot R\vec{x}_0'}{\left\|\vec{x} - \vec{c}_i\right\|}ds}_{\text{joint force contribution of pulley } i} \end{aligned} \tag{6}$$

and a single equation for the belts global longitudinal motion:

$$m\left(L_0\dot{v} - 2v\underline{\dot{q}}^T\left(\underline{K}_0 - \underline{\underline{C}}_2\underline{q}\right) + \underline{\ddot{q}}^T\underline{\underline{C}}_1\underline{q}\right) = \sum_{0 \le i < N_p} \underbrace{\kappa_i M_p^{(i)}(t) + \int_0^{L_0} F_{rep}^{(i)}(d_i.\dot{d}_i)\frac{\left(\vec{x} - \vec{c}_i\right)\cdot \vec{x}_0'}{\left\|\vec{x} - \vec{c}_i\right\|}ds}_{\text{joint force contribution of pulley } i} \cdot \tag{7}$$

m is the mass density, F_0 the initial longitudinal preload and ΔL is the actual total elongation of the belt. EI and DI are stiffness and damping coefficients due to bending and EA and DA the stiffness and damping coefficient due to its longitudinal elongation. N_p is the number of pulleys and $F_{rep}^{(i)}$ is the contact force between a pulley and the belt depending on the depth d_i and the rate $\dot{d}_i$ of indentation into the pulley. $M_p^{(i)}$ is the transferred moment and $\vec{c}_i$ and κ_i are center and curvature of pulley i. The Matrices and Vectors $\underline{\underline{C}}_0 - \underline{\underline{C}}_5$, $\underline{\underline{T}}_i$, $\underline{K}_0 - \underline{K}_3$, $\underline{R}_i$ are model parameters depending on the reference geometry and the used ansatz functions.

This belt drive model is embedded into the flexible multi-body software AVL-EXCITE (cf. [3]).

The verification of this developed model is done for a two-pulley belt-drive. The numerically obtained results of the contact (normal-) forces are compared with an analytical solution of the equations of motion, which describe the pulley-belt contact zone (developed and presented in [4]). Finally, the applicability of the belt-drive model is demonstrated by simulation of the typical engineering task of a preloaded three-pulley belt drive investigation, considering non-steady operation conditions.

References

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