A transcription method for optimal control problems in multibody dynamics

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Abstract
The present work deals with the optimal control of multibody systems in terms of a transcription scheme, see also [1, 3, 4], i.e. both the formulation of the necessary conditions of optimality as well as the structure of the embedded equations of motion have to be considered on equal terms. In this regard, we will focus on multibody systems in terms of natural coordinates, see also [2]. This approach provides a basic framework for the object-oriented assembly of multibody systems, the systematic implementation of open-loop and closed-loop systems and the design of energy-momentum integration schemes.

Introduction

In a wide range of applications, the preservation of resources is becoming increasingly important. Hence, the main focus will be set on the minimization of the control effort which is necessary for moving a multibody system from a specific initial to a specific terminal state. The multibody system's propagation in time herein is captured by the equations of motion, also denoted as state equations which take up the role of path constraints within the optimal control framework. In this regard the focus of the present work is set on rigid multibody systems in terms of redundant coordinates. This approach ultimately leads to differential-algebraic equations with a well-suited structure for the assembly of multibody systems due to the fact that additional rigid bodies, joints and other specific objects solely affect localized parts of the state equations.

Optimal control

The optimal control task at hand is stated as follows: Find the trajectory, comprising both the state variables $\boldsymbol{x}(t)$ and control inputs $\boldsymbol{u}(t)$, that minimizes the criteria $\mathcal{J}(\boldsymbol{u})$ and fulfills the boundary conditions on the time domain Ω = [0,T]. Hence, we deal with a multi-point boundary value problem in time. At this point, we introduce the general augmented cost functional of form

$$J = \boldsymbol{\eta} \cdot \boldsymbol{\Psi}\Big|_T + \int_\Omega \mathcal{J}(\boldsymbol{u}) + \boldsymbol{\mu} \cdot \boldsymbol{g}(\boldsymbol{x}, \dot{\boldsymbol{x}}, \boldsymbol{u})\,\mathrm{d}t \qquad \text{with} \qquad \mathcal{J}(\boldsymbol{u}) = \frac{1}{2}\boldsymbol{u} \cdot \boldsymbol{u} \tag{1}$$

where $\boldsymbol{\Psi}, \boldsymbol{\eta}$ and $\boldsymbol{g}, \boldsymbol{\mu}$ account for the terminal conditions and state equations together with their respective Lagrangian multipliers and costate variables. Note that the subscript notation $(\cdot)|_T$ refers to the fact, that the terminal constraints $\boldsymbol{\Psi}$ apply to the final time node $t_N = T$. The running cost $\mathcal{J}$ is solely dependent on the control inputs and accounts for the control effort over the respective time domain.

Dynamics

Ensuing from the choice of redundant coordinates, we define the feasible configuration manifold Q and the conjugated tangent space T_qQ as follows

$$\{\mathsf{Q}, T_q\mathsf{Q}\} = \{\boldsymbol{q}, \boldsymbol{v} \in \mathbb{R}^n \mid \phi_b(\boldsymbol{q}) = 0,\ \nabla\phi_b(\boldsymbol{q}) \cdot \boldsymbol{v} = 0,\ 1 \le b \le m\} \tag{2}$$

In this context, we solely ensure the confinement of the state variables on configuration level, i.e. $\boldsymbol{q} \in \mathbb{R}^n$, to the aforementioned configuration manifold Q via holonomic constraints $\boldsymbol{\phi} = \phi_i \boldsymbol{e}_i \in \mathbb{R}^m$. Consequently the continuous set of differential-algebraic state equations yields

$$\boldsymbol{g}(\boldsymbol{x}, \dot{\boldsymbol{x}}, \boldsymbol{u}) = \begin{bmatrix} \dot{\boldsymbol{q}} - \boldsymbol{v} \\ \boldsymbol{M}\dot{\boldsymbol{v}} + \sum_b \nabla\phi_b \lambda_b - \boldsymbol{f}(\boldsymbol{q}, \boldsymbol{u}) \\ \boldsymbol{\phi} \end{bmatrix} \tag{3}$$

where the state variables on configuration and velocity level as well as the dynamical lagrangian multipliers $\boldsymbol{\lambda} \in \mathbb{R}^m$ are condensed in the vector $\boldsymbol{x} \in \mathbb{R}^{2n+m}$.

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