Efficient embedding of a geometrically exact Cosserat rod in multibody dynamics using kinematic coupling constraints

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Abstract

In vehicle design and production, one typically has to deal with flexible components. Simulation of these flexible components as part of a multibody system is especially challenging, if fast simulations up to realtime are demanded. This is the case, e.g., in interactive assembly simulations of cables and hoses.
In our work, we use the discrete Cosserat rod model as presented in [1], which is based on finite differences. It uses the special structure of one-dimensional components and, thus, is very efficient, but accurate at the same time.

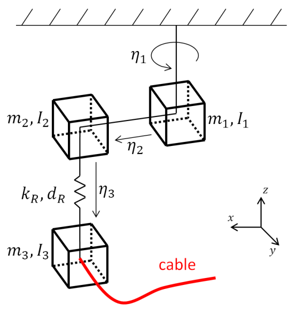

Figure 1: Sketch of the coupling example.

To couple this flexible rod model (F) with the surrounding rigid multibody system (R), we propose a coupling approach that uses a kinematic coupling constraint $0 = g_{co}(q_R^{co}, q_F^{co})$, where only the coupling states q_R^{co}, q_F^{co} appear, and no inner states q_R^{in}, q_F^{in}. Starting formally with a monolithic description of the coupled problem, a force-displacement co-simulation is developed. Here, the multibody system is driven by constraint-forces

$$F^{coupl} = -\frac{\partial g_{co}}{\partial q_R}^T \Lambda, \tag{1}$$

while the coupling states of the flexible structure, i.e. the end-points of the cable, are prescribed. The prescribed states $q_F^{pre}, \dot{q}_F^{pre}$ can be computed via the kinematic coupling from the multibody states $q_R^{co}, \dot{q}_R^{co}$, whereas the constraint-forces, i.e. the Lagrange multiplier Λ, could be derived by solving the monolithic index-1-system. However, for efficiency reasons the forces are approximated with a size-reduced monolithic system, where we exploit the zero-blocks $\frac{\partial g_{co}}{\partial q_R^{in}} = 0$ and $\frac{\partial g_{co}}{\partial q_F^{in}} = 0$ in the constraint gradient. Only equations of motion that correspond to coupling states appear in this smaller system, whose size is related to the number of coupling states (details will be in [2]). Depending on the mass ratio of the subsystems, the approximation can cause an instability. This observation is similar to the one in [3, 4].
Our coupling approach does not introduce a bushing-element for the kinematic coupling and, thus, no artificial stiffness. Moreover, it is easy to formulate more complex coupling joints, which is non-trivial with bushing-elements, cf. [5].
For time integration we use a parallel co-simulation, since it is more efficient than a sequential one. Therefore, both coupling terms, forces and displacements, have to be extrapolated for the integration up to the next macro-point, cf. [6]. On the one hand, the multibody system can be integrated with any standard MBS-solver. On the other hand, for realtime-capability, a linear-implicit method is used to simulate the flexible component, cf. [7].

In a first coupling example, the cable model is embedded in a small multibody system, as sketched in figure 1. One end of the cable is fixed at the wall, the other one is coupled to the body with mass m_3. In this example, the cable is fully clamped to the rigid body, but also changes of the coupling joint are possible with little effort.
A challenging aspect is the staggered discretization of the rod model, as depicted in figure 2. Rotational degrees of freedom – parameterized with quaternions $p_{\frac{1}{2}}, \ldots, p_{N-\frac{1}{2}}$ – are located on the edges and not in the nodes, such that there are no discrete rotational variables at the cable end points. To handle clamped boundary rotations p_0, a virtual ghost quaternion $p_{-\frac{1}{2}}$ is introduced beyond the cable end point, and is defined as the spherical linear extrapolation of $p_{\frac{1}{2}}$ via p_0. Thus, the virtual ghost quaternion appears in the rotational coupling constraint

$$0 = g_{co}^{rot}\left(\phi,\ p_0(p_{-\frac{1}{2}}, p_{\frac{1}{2}})\right), \tag{2}$$

where ϕ represents the rotation of the coupling body of the multibody system. To derive the coupling forces (exactly or approximately), constraint gradients and corresponding equations of motion are necessary. Therefore, we have to set up virtual equations of motion for the ghost quaternion $p_{-\frac{1}{2}}$, which is, however, computationally cheap, since all involved terms are evaluated already.

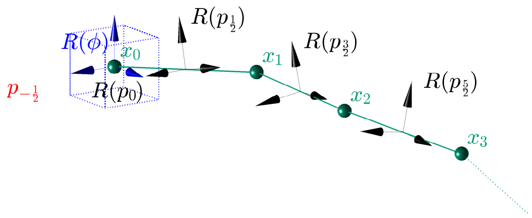

Figure 2: Staggered discretization of the cable model.

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