Real-time Vehicle Dynamics Analysis Using Generalized Maxwell Model for Modeling of Rubber Bush

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Abstract

Multibody dynamics is an effective method to reduce the cost and the period in the development of mechanical products. Recently, a multibody vehicle model is used for some applications of real-time simulation such as a driving simulator. In this simulation, rubber bushes in a suspension system should be contained in the vehicle model, because it is an important element for the ride comfort and the driving stability. However, a rubber bush is a high stiffness element, and a real-time simulation with such a stiff element can be unstable because there is a limitation to shrink the step time for a real-time simulation. In this research, the generalized scheme was used for this problem. The generalized- α scheme is a

powerful approach because it allows an optimal combination of accuracy at low-frequency and numerical damping at high-frequency [1]. The algorithm of the generalized- α scheme is as follows. Let us consider the constrained mechanical system

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} = \mathbf{f}(\mathbf{q},\dot{\mathbf{q}},t) - \mathbf{\Phi}_{\mathbf{q}}^{T}\boldsymbol{\lambda}$$
(1)

$$\mathbf{0} = \mathbf{\Phi}(\mathbf{q}, t) \tag{2}$$

where equation (1) represents the dynamics of the mechanical system and equation (2) represents the kinematic constraints. The vectors \mathbf{q} and $\boldsymbol{\lambda}$ denote the generalized coordinates and the Lagrange multipliers, \mathbf{M} is the symmetric mass matrix, the vector of apparent forces \mathbf{f} collects external forces, internal forces and complementary inertia forces, and $\boldsymbol{\Phi}_{\mathbf{q}}$ is the matrix of constraint gradients [1]. In each timestep, the generalized- α scheme requires re-evaluation of the iteration matrix

$$\mathbf{S}_{t_0} = \begin{bmatrix} (\beta' \hat{\mathbf{M}} + \gamma' \hat{\mathbf{C}} + \hat{\mathbf{K}}) & \hat{\mathbf{\Phi}}_{\hat{\mathbf{q}}}^T \\ \hat{\mathbf{\Phi}}_{\hat{\mathbf{q}}} & \mathbf{0} \end{bmatrix}$$
(3)

By describing the equation of motion (1) and the constrained equation (2) with the updated reference frame to a body of the vehicle model, the variation of the iteration matrix remains small. In this study, the iteration matrix \mathbf{S}_{t_0} was fixed at the initial state during the numerical simulation in order to reduce the amount of calculation.

The rubber bushes which are contained in the vehicle model were modeled by the generalized Maxwell model. The generalized Maxwell model is represented by damper elements and spring elements connected in series [2] to express viscoelastic properties such as a stress relaxation and a frequency dependence. A total external force u of the generalized Maxwell model is written as follows

$$u = k_0 x + \sum_{i=1}^{n} k_i x_{k_i}$$
(4)

and a state x_{k_i} is required to satisfy the following equation

$$\dot{x}_{k_i} = \dot{x} - \frac{k_i}{c_i} x_{k_i} \tag{5}$$

where x is the total displacement of the model, x_{k_i} is the displacement of a spring in the Maxwell element, k_i is the stiffness of spring elements and c_i is the damping coefficient of the damper element. Velocities \dot{x} and \dot{x}_{k_i} can be obtained by taking differential of x and x_{k_i} with respect to time.

The bush model shown in Figure 1 includes a number of generalized maxwell models, each of which has different parameters to represent the variation of the characteristics according to the direction. The



Figure 1: Bush model.

Figure 2: Identification result.

parameters for the bush model were identified based on the measurement result of a rubber bush for a passenger car. Figure 2 shows the comparison between measurement result and the calculation result of the bush model in radial direction.

The performance of the generalized- α scheme with fixed iteration matrix was examined with a multibody vehicle model shown in Figure 3, in which the bush model is included in the rear suspension. A real-time simulation with the J-turn steering maneuver prescribed in ISO 7401 was carried out, and the elapsed time of the calculation in each time step was evaluated. In this evaluation, the step size was set to 1 ms. The results are compared with the conventional method in which the iteration matrix was evaluated in each time step. From Figure 4, the generalized- α scheme with fixed iteration matrix realizes smaller elapsed time of the calculation compared to the conventional method.



Figure 3: Multibody vehicle model.

Figure 4: Simulation results.

References

Body 7

Body 2

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