Nonlinear Vibration of Cantilever under the Action of Lateral Harmonic Excitations and Axial Load

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Abstract
In this paper, an uniform cantilever beam under the action of the lateral harmonic load on the tip and axial self weight has been studied. In order to find the effects of high order nonlinear terms on the response, such as the third and fifth order nonlinear terms, a variational approach based on the extended Hamilton principle is employed to derive the equations of motion and boundary conditions governing the planar nonlinear vibrations of the isotropic and inextensible Euler-Bernoulli beams. Three second-order partial differential equations containing up to third and fifth order nonlinearities are obtained. With Galerkin method, the nonlinear dynamical system in time domain is formulated. Then the Runge-Kutta method is adopted to analyse the responses of the system. The nonlinear behaviour of the cantilever beam has been examined. The nonlinearity of the cantilever beam under the lateral harmonic load on the tip and the axial self weight is investigated with different frequencies of harmonic excitations and axial forces. Some hardening and softening behaviours been observed through the numerical analysis.

Introduction
In order to model the systems accurately, one needs to incorporate nonlinearity in the model. Especially when the system undergo a large motion, the geometric nonlinearity and inertial nonlinearity are activated and their effect cannot be ignore. Under such circumstance, the effects of higher-order nonlinear terms need being taken into account. Generally, there are two categories in the analytical investigations of the responses of cantilever beams. In the first category, the influence of nonlinearity was either neglected or partially considered. Silva and Glynn used the equations they derived to investigate the flexural-flexural responses of near-square cantilever beams subject to primary resonances (1978) [1, 2]. They found that the response amplitude-frequency curves shows much difference behavior at the first natural frequency from those at higher natural frequencies. Furthermore, the influence of the nonlinear curvature terms on the response diminishes for higher modes. Nayfeh and Pai (1989) [3] used the equations of Silva and Glynn (1978a, b) [1, 2] to investigate the nonlinear nonplanar responses of cantilever beams subject to parametric excitations in the presence of a one-to-one internal resonance involving two flexural modes. Pai and Nayfeh (1990a) [4] also used the equations of Silva and Glynn (1978b) [2] to investigate the nonplanar oscillations of square and rectangular cantilever beams subject to lateral base excitations. They found that the geometric nonlinearity dominates the inertia nonlinearity for the low-frequency modes, whereas the inertia nonlinearity dominates the geometric nonlinearity for the high-frequency modes. The geometric nonlinearity produces a hardening spring effect and the inertia nonlinearity produces a softening spring effect. Eftekhari, Mahzoon and Ziaei-rad (2012) [5] used multiple scales method to carry out a comparative study between a beam with and without a tip mass and found out that when the excitation frequency or amplitude of excitation force is slowly changed, the presence of tip mass altering the stability of solutions.

Numerical Results
A planar forced vibration of an isotropic, inextensible, Euler-Bernoulli cantilever beam carrying nonlinearity up to fifth order is investigated numerically. The force-response and frequency-response curves (FRC) are obtained. Some of the numerical results are shown in the following figures. From the

FRC we can see that both hardening and softening effects can be observed around the first natural frequency.

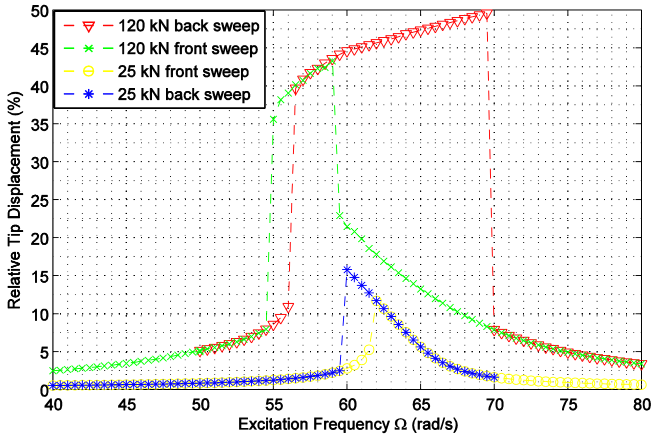

Figure 1: (a):FRC of the cantilever beam under different excitation amplitude

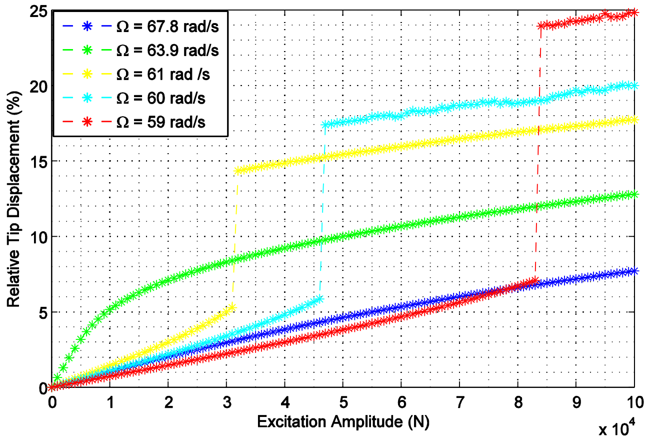

Figure 1: (b): Force-Response curve of the cantilever beam around the first natural frequency.

References

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