Compatibility Equation and Revision on the Expression of Cord-line Component of Dynamical Cable Tension

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Abstract

The problem in deriving the relation of the chord-line component of the dynamical cable tension and the deflections in the conventional cable theory is investigated. The expression for the relation of the chord-line component of the dynamical cable tension and the deflections is reasonably derived and improved by introducing a compatibility condition for three-dimensional vibrations of the cable. The difference between the chord-line component of the dynamical cable tension based on the conventional cable theory and the proposed compatibility equation are investigated for different sag-to-span ratios and inclined angles. It is found that the error caused by the conventional cable theory can be too large to be acceptable when the sag-span ratio or the inclined angle is large to some extent.

Consider the inclined suspended steel cable as shown in Figure 1 with mass per unit length being $4539.6kg/m$, length between two supports being 1000m.

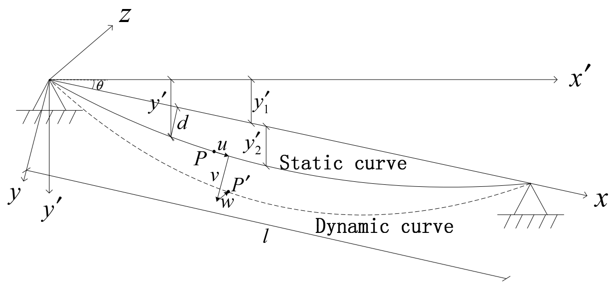

Figure 1: Inclined cable and its coordinate system.

If the cable is subjected to the external dynamic force $f_y(x,t)$ and $f_z(x,t)$ per unit length in y and z directions, respectively, three-dimensional equations of motion of the cable are easily obtained with h unknown, which is the chord-line component of additional cable tension induced by dynamical displacement. However, the conventional derivation of h is shown as follows, which is not theoretically based, because it made use of the average dynamical strain of the cable.

$$h = \frac{EA}{L_e}\int_0^l \left(y_x v_x + \frac{1}{2}v_x^2 + \frac{1}{2}w_x^2 \right) dx, \tag{1}$$

where

$$L_e = \int_0^l \left(\frac{ds}{dx} \right)^3 dx = \int_0^l (1+y_x^2)^{\frac{3}{2}} dx.$$

After the compatibility equation of the change of the length of cable under dynamical deformation is formulated, the revised expression of h is given as,

$$h = \frac{EA}{L_e'}\int_0^l \left(y_x v_x + \frac{1}{2}v_x^2 + \frac{1}{2}w_x^2 \right) dx, \tag{2}$$

where

$$L_e' = \int_0^l \left(\frac{ds}{dx} \right)^2 dx = \int_0^l (1+y_x^2) dx.$$

The results of L_e and L_e' for different sag-span ratio are calculated numerically and compared as shown in Table 1 to Table 5. It is observed that if the inclined angle remains the same, the difference between L_e and L_e' becomes larger as the sag-to-span ratio increases. If the sag-to-span ratio remains the same, the difference between L_e and L_e' becomes larger as the inclined angle increases.
It can be seen from Table 1 that when the cable is suspended horizontally and the sag-to-span ratio is $1/4$, the difference between L_e and L_e' reaches 18.45%, which is too large to be acceptable. From Table 5, it can be seen that when the inclined angle is 60° and the sag-to-span ratio is $1/10$, the difference between L_e and L_e' could reaches about 5%. So the difference could significantly influence the responses of three-dimensional vibrations of the cable. The error caused by the conventional cable theory can be too large to be acceptable when the sag-to-span ratio or the inclined angle is large to some extent.

Table 1: Difference between L_e and L_e' for $\theta = 0°$.

sag-span ratio	L_e	L_e'	$(L_e - L_e')/L_e'(\%)$
1/10	1085.34	1053.61	2.73
1/8	1130.72	1084.01	4.31
1/6	1240.27	1150.22	7.83
1/4	1590.94	1343.18	18.45

Table 2: Difference between L_e and L_e' for $\theta = 15°$.

sag-span ratio	L_e	L_e'	$(L_e - L_e')/L_e'(\%)$
1/10	1046.6	1018.41	2.77
1/8	1095.07	1048.76	4.42
1/6	1208.42	1116.64	8.22
1/4	1613.65	1330.88	21.25

Table 3: Difference between L_e and L_e' for $\theta = 30°$.

sag-span ratio	L_e	L_e'	$(L_e - L_e')/L_e'(\%)$
1/10	941.89	915.25	2.91
1/8	991.42	945.91	4.81
1/6	111.23	1021.08	9.81
1/4	1819.06	1338.03	35.95

Table 4: Difference between L_e and L_e' for $\theta = 45°$.

sag-span ratio	L_e	L_e'	$(L_e - L_e')/L_e'(\%)$
1/10	777.17	752.22	3.32
1/8	832.91	785.71	6.01
1/6	1027.74	888.64	15.65
1/4	5981.87	2067.58	189.32

Table 5: Difference between L_e and L_e' for $\theta = 60°$.

sag-span ratio	L_e	L_e'	$(L_e - L_e')/L_e'(\%)$
1/10	572.12	545.19	4.94
1/8	672.36	599.19	12.21
1/6	2024.96	1017.46	99.02

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