Dynamics and Control of a Novel Two-degree-of-freedom Drive

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Abstract

Actuators capable of producing cylindrical motion, i.e., independent rotations about an axis and translations in its direction, are of interest in fields such as robotics, mechatronics and industrial automation. For instance, they are being developed for use as drives of parallel manipulators, where it is advantageous to fix all motors to the base, thereby obtaining a load-carrying capacity higher than if the motors were floating [1]. Recently, a new cylindrical actuator was proposed, termed cylindrical-drive, or C-drive for brevity, intended for driving a four degree-of-freedom (dof) parallel pick-and-place robot [2]. It consists of a two-dof RHHR single-loop closed kinematic chain, where R and H denote revolute and helical joints, respectively, as shown in Figure 1. The collar is driven *cylindrically* along its axis by the differential rotation of the two coaxial screws, the latter having the same pitch but opposite hands. This design is notable for its symmetries, which allow for the use of two identical motors. The kinematics of the C-drive is succinctly reproduced below for quick reference:

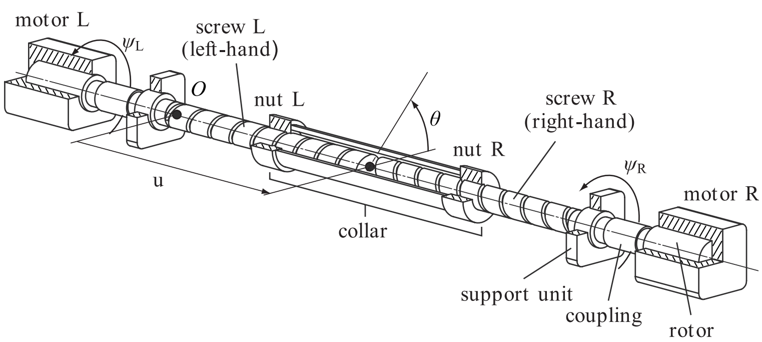

Figure 1: Diagram of C-drive

$$\mathbf{w} = \mathbf{J}\boldsymbol{\psi}, \quad \mathbf{J} \equiv \frac{p}{4\pi}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \quad \boldsymbol{\psi} \equiv \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix}, \quad \mathbf{w} \equiv \begin{bmatrix} u \\ v \end{bmatrix}, \quad v \equiv \frac{p}{2\pi}\theta \quad \Rightarrow \quad \dot{\mathbf{w}} = \mathbf{J}\dot{\boldsymbol{\psi}} \tag{1}$$

where $\boldsymbol{\psi}$ and $\mathbf{w}$ are, respectively, the vectors of *motor* and *collar coordinates*, as defined in the figure, p is the screw pitch, while $\mathbf{J}$ is the Jacobian matrix that maps motor into collar coordinates. Moreover, this Jacobian turns out to be not only constant, but also isotropic: its singular values are identical, namely $\sqrt{2}p/(4\pi)$. The dynamics of the C-drive is governed by a damped linear time-invariant system:

$$\mathbf{M}\ddot{\boldsymbol{\psi}} + \mathbf{D}\dot{\boldsymbol{\psi}} = \boldsymbol{\tau}(t) \tag{2a}$$

$$\mathbf{M} \equiv \begin{bmatrix} I_h + \frac{I_c}{4} + \frac{p^2 m}{16\pi^2} & \frac{I_c}{4} - \frac{p^2 m}{16\pi^2} \\ \frac{I_c}{4} - \frac{p^2 m}{16\pi^2} & I_h + \frac{I_c}{4} + \frac{p^2 m}{16\pi^2} \end{bmatrix}, \qquad \mathbf{D} \equiv \begin{bmatrix} \beta + \frac{1}{2}\gamma & -\frac{1}{2}\gamma \\ -\frac{1}{2}\gamma & \beta + \frac{1}{2}\gamma \end{bmatrix} \tag{2b}$$

where the generalized inertia matrix (GIM) and the viscous damping matrix, denoted $\mathbf{M}$ and $\mathbf{D}$, respectively, are 2×2 symmetric positive-definite matrices, and $\boldsymbol{\tau}$ is the two-dimensional vector of motor torques, while m is this mass of the collar, I_c and I_h are the moments of inertia about the C-drive axis of the collar and rotational parts (screws and motor rotors), respectively, while β and γ are viscous friction terms [2]. No stiffness term is present, as the elasticity of the components is neglected. We now introduce a change of generalized coordinates by means of similarity transformations, which simplifies the analysis and control of the C-drive while preserving the invariant properties of the system matrices, namely their eigenvalues and eigenvectors, the latter being multiplied by the similarity-transformation matrix. As the collar, rather than the motor positions, are the variables of interest, $\boldsymbol{\psi}$ is substituted with its expression in terms of $\mathbf{w}$ in Equation (2a). Premultiplication of both sides of this equation by the Jacobian $\mathbf{J}$ results in similarity transformations of $\mathbf{M}$ and $\mathbf{D}$, namely,

$$\mathbf{M}\mathbf{J}^{-1}\ddot{\mathbf{w}}+\mathbf{D}\mathbf{J}^{-1}\dot{\mathbf{w}}=\boldsymbol{\tau}\Rightarrow\underbrace{\mathbf{J}\mathbf{M}\mathbf{J}^{-1}}_{\mathbf{E}}\ddot{\mathbf{w}}+\underbrace{\mathbf{J}\mathbf{D}\mathbf{J}^{-1}}_{\mathbf{G}}\dot{\mathbf{w}}=\mathbf{J}\boldsymbol{\tau},\quad \mathbf{E}=\mathrm{diag}\left(\varepsilon_1^2,\varepsilon_2^2\right),\mathbf{G}=\mathrm{diag}\left(\gamma_1,\gamma_2\right) \tag{3}$$

where $\mathbf{E}$ and $\mathbf{G}$ are diagonal matrices whose entries ε_i^2 and γ_i, $i=1,\ 2$, are the eigenvalues of their respective matrices. That is, the model thus resulting is *decoupled*, by virtue of the columns of $\mathbf{J}$, and hence of $\mathbf{J}^{-1}$, being proportional to the eigenvectors of $\mathbf{M}$ and $\mathbf{D}$, respectively. In other words, $\mathbf{J}$ is a scalar multiple of the "modal matrix" [3], where $\mathbf{D}$ plays the role of the stiffness matrix $\mathbf{K}$ in the undamped case. The central portion of Equation (3) is rendered monic, i.e., the second-derivative term becomes multiplied by the identity matrix, via the substitutions

$$\mathbf{F}\equiv\sqrt{\mathbf{E}},\quad \boldsymbol{\sigma}\equiv\mathbf{F}\mathbf{w},\quad \mathbf{u}\equiv\mathbf{F}^{-1}\mathbf{J}\boldsymbol{\tau},\quad \boldsymbol{\Theta}\equiv\mathbf{F}^{-1}\mathbf{G}\mathbf{F}^{-1}=\mathrm{diag}\left(\gamma_1/\varepsilon_1^2,\ \gamma_2/\varepsilon_2^2\right)\equiv\mathrm{diag}\left(\vartheta_1,\ \vartheta_2\right) \tag{4}$$

where $\boldsymbol{\Theta}$ is termed the dissipation matrix, its diagonal entries ϑ_1 and ϑ_2 being its eigenvalues, while $\mathbf{u}$ is the input vector, thereby leading to the dynamics in terms of $\boldsymbol{\sigma}$, the transformed collar coordinates:

$$\ddot{\boldsymbol{\sigma}}+\boldsymbol{\Theta}\dot{\boldsymbol{\sigma}}=\mathbf{u} \tag{5}$$

In the absence of potential-energy sources, the model lacks the zeroth-order terms; a trivial change of variable thus renders the system formally identical to a first-order system:

$$\dot{\mathbf{x}}+\boldsymbol{\Theta}\mathbf{x}=\mathbf{u},\qquad \mathbf{x}\equiv\dot{\boldsymbol{\sigma}} \tag{6}$$

which is readily rearranged into state space form, the state matrix being $-\boldsymbol{\Theta}$, the input matrix the 2×2 identity matrix, the output matrix $\mathbf{J}^{-1}\mathbf{F}^{-1}$ and the output vector $\dot{\boldsymbol{\psi}}$. The latter being the motor velocities, it cannot be measured directly by a control system equipped only with motor shaft encoders, which measure displacements $\boldsymbol{\psi}$. A reduced-order observer [4] is therefore synthesized based on a four-dimensional state-space realization obtained from Equation (5), thereby yielding an estimate of $\dot{\mathbf{w}}$ in real-time. The details are omitted here for brevity. The C-drive prototype is computer-controlled at 1 kHz using either independent joint proportional-derivative (PD) controllers or a model-based computed torque controller (CTC) [5] that incorporates the observer. The foregoing control scheme decouples the error dynamics using a feedforward acceleration term, but is sensitive to modelling errors. The coupling is avoided in the first place using task-space rather than joint-space control, that is, calculating the PD error in terms of collar coordinates rather than motor coordinates. Therefore, a PD task-space controller and a resolved-acceleration controller—the task-space counterpart of CTC—are also tested [6]. The simulated and experimental performances of these four controllers are compared based on application requirements.

References

[1] D. Kohli, et al. Manipulator configurations based on rotary-linear (RL) actuators and their direct and inverse kinematics. ASME Journal of Mechanical Design 110.4, pp. 397–404, 1988.

[2] T. Harada, T. Friedlaender, J. Angeles. The development of an innovative two-dof cylindrical drive: design, analysis and preliminary tests. Proceedings IEEE International Conference on Robotics and Automation, pp. 6612–6617, Hong Kong, China, 2014

[3] F. Tisseur, K. Meerbergen. The quadratic eigenvalue problem. SIAM Review 43.2, pp. 235–286, 2001

[4] D. Luenberger. Observers for multivariable systems. Automatic Control, IEEE Transactions on 11.2, pp. 190–197, 1966.

[5] P. Khosla, K. Pradeep, T. Kanade. Real-time implementation and evaluation of the computed-torque scheme. Robotics and Automation, IEEE Transactions on 5.2, pp. 245–253, 1989.

[6] F. Caccavale et al. Resolved-acceleration control of robot manipulators: A critical review with experiments. Robotica 16.05, pp. 565–573, 1998.