Forward Dynamics of Variable Topology Mechanisms – The Case of Constraint Activation –

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Variable topology mechanisms (VTM) form a class of mechanisms that can switch between different kinematic topologies, and thus change their kinematic mobility and possibly their DOF. This property is shared with kinematotropic mechanisms. The latter transit between motion modes via kinematic singularities while keeping their kinematic topology. VTM on the other hand change their mobility due to switching constraints, respectively (bilateral) contacts. In this paper VTM with switching constraints are considered that are referred to as *quasi-scleronomic VTM*.

The configuration space (c-space) of a holonomic quasi-scleronomic VTM is the time dependent variety $V := h^{-1}(\mathbf{q},t)$ defined by a system of *quasi-scleronomic* geometric *constraints* $h(\mathbf{q},t) = \mathbf{0}$ of the form

$$h(\mathbf{q},t) = \begin{cases} h_1(\mathbf{q}), & t \in [t_0, t_1) \\ h_2(\mathbf{q}), & t \in [t_1, t_2) \\ \cdots & \\ h_i(\mathbf{q}), & t \in [t_{i-1}, t_i) \end{cases} \tag{1}$$

where the constraint switching occurs at t_i. Each individual constraint $h_i(\mathbf{q}) = \mathbf{0}$ corresponds to a kinematic topology, and defines a variety $V_i := h_i^{-1}(\mathbf{q},t)$, where $V_i \cap V_j \neq \emptyset$, which is the *c-space of the VTM at topology i*. A special class, with practical relevance, are VTM with *regular topology changes due to the activation of additional constraints*. The quasi-scleronomic constraints can be expressed as

$$h(\mathbf{q},t) = \begin{cases} h_-(\mathbf{q}) := h_1(\mathbf{q}), & t < 0 \\ h_+(\mathbf{q}) := \begin{pmatrix} h_1(\mathbf{q}) \\ h_2(\mathbf{q}) \end{pmatrix}, & t \geq 0 \end{cases}, \quad \mathbf{J}(\mathbf{q},t) = \begin{cases} \mathbf{J}_-(\mathbf{q}) := \mathbf{J}_1(\mathbf{q}), & t < 0 \\ \mathbf{J}_+(\mathbf{q}) := \begin{pmatrix} \mathbf{J}_1(\mathbf{q}) \\ \mathbf{J}_2(\mathbf{q}) \end{pmatrix}, & t \geq 0 \end{cases}. \tag{2}$$

Here $h_1(\mathbf{q}) = \mathbf{0}$ is a set of persistent constraints to which another set of constraints $h_2(\mathbf{q}) = \mathbf{0}$ is added at $t = 0$.

A topology change at time t_0 with configuration $\mathbf{q}_0(t_0) \in V_1 \cap V_2$ is *regular* iff $\mathbf{q}_0$ is a regular point of V_1 and of V_2, i.e. the VTM does not encounter a singularity during the topology change. Topology variations are accompanied by non-smooth transitions between different motion modes, i.e. discontinuous system trajectories. This is a challenge for the numerical simulation as well as for model-based control that has been addressed in [1],[2] for instance.

In this paper a momentum consistent formulation for the forward dynamics of VTM exhibiting regular topology changes, and a momentum consistent time stepping scheme is presented.

Suitable Form of Motion Equations: Starting from Gauß' principle, with the acceleration constraints $\mathbf{J}_i\ddot{\mathbf{q}} + \dot{\mathbf{J}}_i\dot{\mathbf{q}} = \mathbf{0}$ corresponding to $h_i(\mathbf{q}) = \mathbf{0}$, the equations of motion (EOM) governing the VTM dynamics on V_i are

$$\ddot{\mathbf{q}} = \mathbf{M}^{-1}(\mathbf{q})\,\mathbf{N}^T_{\mathbf{J}_i,\mathbf{M}}\big(\mathbf{u}(\mathbf{q},t) - \mathbf{C}(\dot{\mathbf{q}},\mathbf{q})\,\dot{\mathbf{q}} - \mathbf{Q}(\dot{\mathbf{q}},\mathbf{q},t)\big) - \mathbf{J}^+_{i,\mathbf{M}}(\mathbf{q})\,\dot{\mathbf{J}}_i\dot{\mathbf{q}} \tag{3}$$

with $\mathbf{J}^+_{i,\mathbf{M}}$ being the $\mathbf{M}$-weighted right pseudoinverse of $\mathbf{J}_i$, and $\mathbf{N}_{\mathbf{J}_i,\mathbf{M}} = \mathbf{I} - \mathbf{J}^+_{i,\mathbf{M}}\mathbf{J}_i$ is the corresponding projector to the null-space of $\mathbf{J}_i$. This formulation is free of Lagrange multipliers while it does not require selection of independent (minimal) coordinates, as the common minimal coordinate formulations for constrained MBS do. The system (3) can be rewritten as [3]

$$\mathbf{N}^T_{\mathbf{J}_i,\mathbf{M}}\big(\mathbf{M}(\mathbf{q})\,\ddot{\mathbf{q}} + \mathbf{C}(\dot{\mathbf{q}},\mathbf{q})\,\dot{\mathbf{q}} + \mathbf{Q}(\dot{\mathbf{q}},\mathbf{q},t) - \mathbf{u}(\mathbf{q},t)\big) = \mathbf{0}. \tag{4}$$

This is not directly applicable to forward dynamics simulation, as it is a system of n equations of which only $n - m$ are independent, since rank $\mathbf{N}_{\mathbf{J},\mathbf{M}} = n - m$, but it gives rise to a tailored momentum balance condition.

Momentum Balance: Assume that the switching occurs at $t = 0$, and denote the corresponding configuration with $\mathbf{q}_0 := \mathbf{q}(0)$. The balance of generalized momentum of a VTM subjected to the quasi-scleronomic constraints (2) can be derived as

$$\mathbf{M}(\mathbf{q}_0)\Delta\dot{\mathbf{q}} + \mathbf{N}^T_{\mathbf{J}_1,\mathbf{M}}(\mathbf{q}_0)\mathbf{J}^T_2(\mathbf{q}_0)\Lambda_+ dt = \mathbf{U}(\mathbf{q}_0) \tag{5}$$

where $\Delta\dot{\mathbf{q}} := \dot{\mathbf{q}}_+ - \dot{\mathbf{q}}_-$ is the velocity jump, $\Lambda_+ := \int_0^\varepsilon \lambda dt$ is the impulsive constraint force after the event, and $\mathbf{U}(\mathbf{q}_0) = \int_{-\varepsilon}^\varepsilon \mathbf{u}(\mathbf{q}_0, t)\, dt$ is an impulsive applied force during the event.

Kinematic Compatibility: The persistent constraints can be written as $\mathbf{J}_1(\mathbf{q}_0)\dot{\mathbf{q}}_+ - \mathbf{J}_1(\mathbf{q}_0)\Delta\dot{\mathbf{q}} = \mathbf{0}$, and the additional constraints for $t \geq 0$ as $\mathbf{J}_2(\mathbf{q}_0)\Delta\dot{\mathbf{q}} + \mathbf{J}_2(\mathbf{q}_0)\dot{\mathbf{q}}_- = \mathbf{0}$. These can be combined to

$$\mathbf{J}_2(\mathbf{q}_0)\mathbf{N}_{\mathbf{J}_1,\mathbf{M}}(\mathbf{q}_0)\Delta\dot{\mathbf{q}} = -\mathbf{J}_2(\mathbf{q}_0)\dot{\mathbf{q}}_-. \tag{6}$$

Overall Transition Condition: The momentum balance together with the kinematic compatibility conditions can be summarized as

$$\begin{pmatrix} \mathbf{M}(\mathbf{q}_0) & \mathbf{N}^T_{\mathbf{J}_1,\mathbf{M}}(\mathbf{q}_0)\mathbf{J}^T_2(\mathbf{q}_0) \\ \mathbf{J}_2(\mathbf{q}_0)\mathbf{N}_{\mathbf{J}_1,\mathbf{M}}(\mathbf{q}_0) & \mathbf{0} \end{pmatrix} \begin{pmatrix} \Delta\dot{\mathbf{q}} \\ \Lambda_+ \end{pmatrix} = \begin{pmatrix} \mathbf{U}(\mathbf{q}_0) \\ -\mathbf{J}_2(\mathbf{q}_0)\dot{\mathbf{q}}_- \end{pmatrix}. \tag{7}$$

The special case of MBS only subjected to a set of constraints that is activated at $t = 0$ is included with $\mathbf{N}_{\mathbf{J}_1,\mathbf{M}} = \mathbf{I}$. The so determined $\Delta\dot{\mathbf{q}}$ is the admissible velocity jump such that the momentum balance and the constraints before and after the event are satisfied. As a by product, the impulsive constraint force Λ_+ due to the event is determined.

The system (7) is invoked within numerical time stepping schemes to determine the velocity jump at the switching point. To this end the system trajectory is numerically determined until the switching event, providing the velocity $\dot{\mathbf{q}}_-$ prior to the event and the configuration $\mathbf{q}_0 \in V$.

It is only necessary to solve (7) at the switching event, which does not intrude the numerical integration. Moreover, any dynamics formulation (absolute or relative coordinates) and integration scheme can used. The compatibility condition (7) yields the full (redundant) state of the system, from which any desired set of coordinates can be selected, e.g. minimal coordinates. The impulsive force $\mathbf{U}(\mathbf{q}_0)$ can possible substituted by a contact model. The method is easily extended to account for non-holonomic velocities $\mathbf{v}$ according to $\mathbf{v} = \mathbf{A}(\mathbf{q})\dot{\mathbf{q}}$.

The method has been applied to several VTM examples that will be presented. Such an example is the metamorphic hand in Fig. 1. that has been reported in [4]. The particular feature of this hand is its palm. The palm consists of a spherical 5-bar linkage imitating the inherent mobility of the human hand. The latter is actuated by two motors. As humans do, the palm can be folded so to essentially collapse. This is achieved by locking the drive 1, thus reducing its DOF and so changing the kinematic topology.

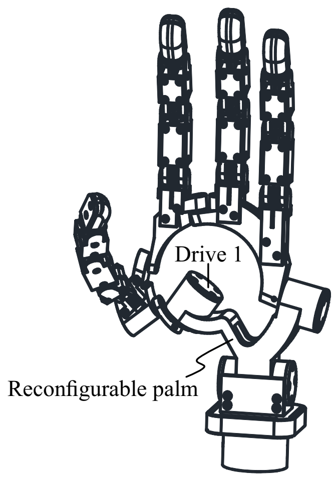

Figure 1: Metamorphic Hand with reconfigurable palm. [Courtesy of Prof. Jian Dai]

References

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