Multiple-point elastoplastic smooth collisions in multibody systems

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Abstract

Impact problems are often encountered in different branches of mechanics. They appear in robotics (robot-environment interaction), in physics of granular media, in biomechanics (heel-ground collisions in human gait), or in musical acoustics (hammer-string collisions), among others.

Different approaches have been proposed to study and simulate impacts in rigid-body systems [1, 2]. They can be roughly classified into variable-configuration (VC) approaches and constant-configuration (CC) ones. VC methods rely on compliant models to explore the interactions at the colliding points. In CC methods, conversely, the unavoidable deformation associated with impact is localized at the colliding points. As the time interval elapsed between the beginning and the end of the impact is very small (as compared to the time scale of non-impact dynamics), the system configuration is assumed to be constant throughout the collision interval.

Single-point collisions are usually treated through Routh and Darboux methods. The most relevant feature of these methods is the use of the normal impulse as the integration variable (instead of time) to determine the velocity changes. This allows the determination of the velocity changes without using any particular compliant model but does not give any information about the value of the force at the collision point. Consequently, the detection of the collision end (which corresponds to a zero force value) has to be established through a plausible hypothesis concerning energy losses, usually formulated through a coefficient of restitution (COR).

The aforementioned methods may not be applicable to multiple-point collisions [3]. In that case, the use of a compliant model is unavoidable. The constitutive laws allow the time integration of the equations of motion (simplified by the CC hypothesis) thus yielding the evolution of both velocities and normal forces at the collision points. The advantage of that integration is the easy detection of collisions end as a zero force state, thus avoiding the use of CORs (which may be energetically inconsistent).

Two important features appear in multiple-point collisions, which are not possible in single-point ones. On one hand, redundancy may appear if the normal velocities of some colliding points are linearly related. On the other hand, in elastoplastic and inelastic impacts, unilateral constraints may appear, thus reducing the system's number of degrees of freedom.

The present study proposes an efficient method to simulate smooth 3D multiple-point collisions in multibody systems to overcome those difficulties. The model is an extension of a previous version, restricted to the perfectly elastic case, able to account for the high sensitivity to initial conditions and for redundancy without assuming any particular collision sequence [4]. The main idea consisted of assuming a finite linear normal stiffness (high enough to assume constant configuration throughout the process) at each impact point and solving a vibrational problem. Two different time and space scales were used. At the macro scale, the overall system configuration was assumed to be constant.

In this work, we present an extension that includes energy dissipation (with or without permanent indentation) associated with material deformation. It is introduced through a linear-by-part elastoplastic model consisting on two series bistiffness springs (Fig.1). The first one accounts for inelastic behavior (energy loss without permanent indentation, $0 \leq \mu < 1$), while the second one introduces plasticity (that is, permanent indentation, $\mu' = 1$).

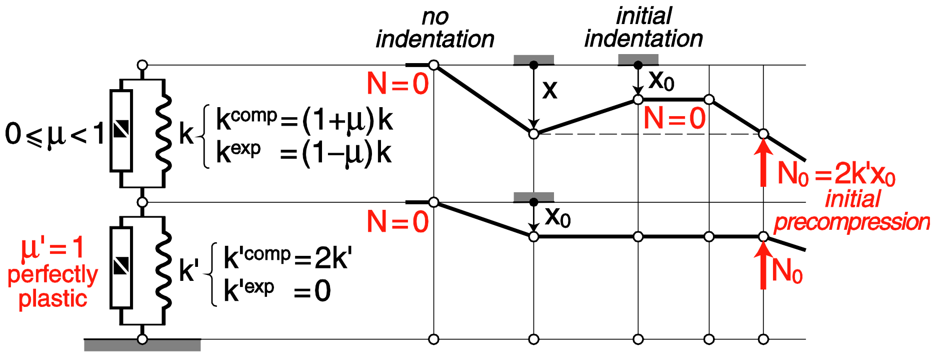

Figure 1: Viscoelastic model consisting of two series bistiffness springs.

When introducing a deformation x (or a normal displacement $\delta = -x$), if the initial value of x_0 is zero, both systems undergo a compression (and thus $x_0 \neq 0$) . If having attained a certain value $x_0 \neq 0$ the upper system begins an expansion phase, the lower one will be retained by the damper. That lower damper, combined with the parallel stiffness k', will be able to generate a maximum force of $2k'x_0$. Only if a new compression of the upper set generates a force higher than $2k'x_0$, will the lower set compress further.

Figure 2 shows an example of a two-point impact of a rod on a fixed ground. Different phases can be observed: compression of both springs, compression of just the upper one, unilateral constraint and single-point contact.

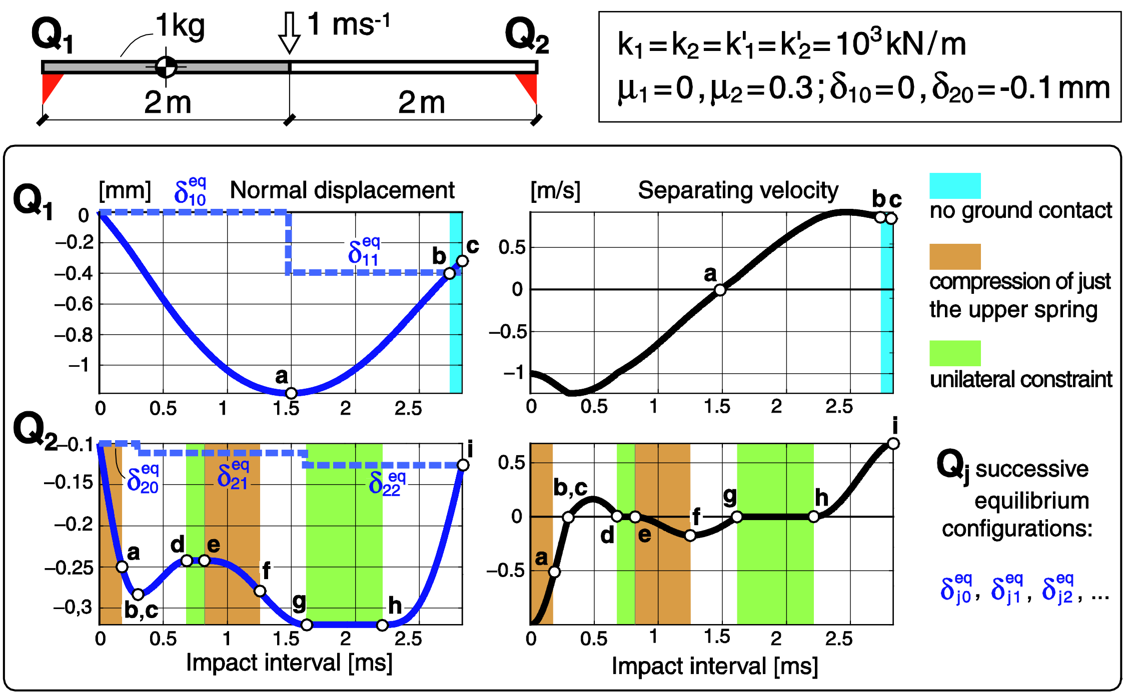

Figure 2: Normal displacement δ and separating velocity $\dot{\delta}$ in a two-point impact of a rod with initial pure downwards translation. The mass is concentrated on half of the rod length.

References

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