# **Energy-Efficient Point-to-Point Trajectory Generation**

# for Industrial Robotic Machines

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### Abstract

Robots and mechatronic applications are widely used for process automation in plants and factories. Trajectory planning is a fundamental issue for operating these industrial machines [1].

In the past, most of the research works covered the solution of minimum time problems and, later, attention has been given to the jerk minimization problem to reduce the vibration phenomena and the mechanical failure risks [2, 3]. Recently, in order to develop energy efficient robotic and tool machines, thanks to the spread of variable frequency devices and energy recovery systems, new trajectory planning techniques aiming at reducing the energy expenditure can be developed.

In [3] the simultaneous evaluation of both the energy efficiency and the smoothness in the most significant off-line non-model based methods and algorithms currently adopted in industrial robotic applications is presented. In [4] the minimum-energy trajectory optimization problem is treated considering the electrical energy exchange via the shared inverter DC link, thus allowing to find a different energy minimum with respect to the available approaches.

In this work, a point-to-point (PTP) trajectory based on a S-curve has been designed to reduce the consumed energy of a typical mechatronic system, i.e. a robotic linear axis made of an electric-motor that moves a payload on a plane by means of a transmission system and a toothed belt.

The S-curve trajectory has been chosen with a cycloid motion during the acceleration and deceleration periods to have a limited acceleration value at the starting and ending points.

Basically, it can be described by four parameters: acceleration time, constant velocity time, deceleration time and constant velocity magnitude, Equations (1) and (2).

$$v(t) = \begin{cases} \frac{v_0}{2} (1 - \cos(\omega_1 t)) & \text{if } t \in [0, t_1) \\ v_0 & \text{if } t \in [t_1, t_1 + t_2) \\ \frac{v_0}{2} (1 + \cos(\omega_3 t')) & \text{if } t \in [t_1 + t_2, t_1 + t_2 + t_3) \end{cases}$$
(1)  
$$\left(\frac{v_0}{2} (\omega_1 \sin(\omega_1 t)) & \text{if } t \in [0, t_1) \right)$$

$$a(t) = \begin{cases} \frac{1}{2} (\omega_1 \sin(\omega_1 t)) & t \neq [0, t_1) \\ 0 & if \ t \in [t_1, t_1 + t_2) \\ \frac{v_0}{2} (\omega_3 \sin(\omega_3 t')) & if \ t \in [t_1 + t_2, t_1 + t_2 + t_3) \end{cases}$$
(2)

with  $t_1$ ,  $t_2$ ,  $t_3$  and  $v_o$  the acceleration time, the constant velocity time, the deceleration time and the maximum velocity, respectively, and  $t' = t - t_1 - t_2$ ,  $\omega_1 = \pi/t_1$  and  $\omega_3 = \pi/t_3$ .

The electro-mechanical model of the robotic axis has been defined taking also into account the load (l) mass *m* and inertia  $j_m$ , the viscous *D* and Coulomb friction  $T_a$ , as well as the resistive losses in the motor windings; then, both the motor torque  $\tau_m$  and the instantaneous motor current *i* and voltage *e* in the motor phase have been found. The instantaneous power P(t) can then be expressed as in Eq. (3):

$$P(t) = R i(t)^{2} + K_{e}/K_{r}v_{l}(t)i(t)$$
(3)

where with  $K_t$  the torque constant and  $K_e$  the back-EMF constant. The first term is the power lost in the motor winding and the second is the power used to move the payload; if the latter is positive, the

system is in direct motion, otherwise retrograde motion occurs and the drive system recovers energy through a regenerative system (i.e. it is providing a negative work). Finally, following and extending the approach in [5], the total energy *E* formulation for the robotic axis in the generic case of  $t_1 \neq t_3$  has been found. Thus, the main issue in reducing the wasted energy is to reduce the one used and lost in the armature resistance.

Given the energy formulation and once fixed the motion displacement, the S-curve trajectory is described by three parameters, i.e. acceleration time, deceleration time and total motion time, and the minimum of the energy consumption can be found either in a closed-form or numerically using a genetic algorithm.

The energy optimization has been solved in the cases:

(a)  $t_1$  and  $t_3$  free and total time fixed (see Fig. 1);

(b)  $t_1 = t_3$  free and total time free by means of the optimization of an equation in two variables through the computation of the matrix Hessian;

(c)  $t_1$ ,  $t_3$  e total time free by means of a genetic algorithm procedure.



Mass	m	10 kg
Track length	L	2 m
Transmission ratio	Kr	0.016 m/rad
Torque constant	Kt	0.14 Nm/A
Back-EMF constant	Ke	0.08 Vs/rad
Motor inertia	j <sub>m</sub>	0.00035 kgm <sup>2</sup>
Motor resistance	R	0.22 Ω
Gravity acc.	G	$9.81 \text{ m/s}^2$
Dynamic friction	Ta	30 N

Figure 1. Total energy consumption with respect to the variation of  $t_1$  and  $t_3$  and total time free for the test case parameters reported in the table.

In this manner, the parameters that allow to achieve the minimum energy consumption for the robotic axis under study, given the possibility to recover energy when braking, can be found in different cases either in a closed form or through a numerical solution.

#### References

- [1] A. Gasparetto, P. Boscariol, A. Lanzutti, R. Vidoni, (2012) Trajectory Planning in Robotics, Mathematics in Computer Science, Volume 6, Issue 3, Pages 269-279.
- [2] P.J. Barre, R. Bearee, P. Borne, E. Dumetz, (2005) Influence of a jerk controlled movement law on the vibratory behaviour of high-dynamics systems, Journal of Intelligent and Robotic Systems, Volume 42, Issue 3, Pages 275-93
- [3] R. Vidoni, N. Lever, D. Richiedei, A. Gasparella, A. Trevisani, A. Gasparetto (2013), Energy Efficiency and smoothness in robotics trajectory planning: numerical simulation and comparison, Proc. of the Austrian Robotics Workshop (ARW 2013), Wien (Austria), May 23-24, 2013.
- [4] C. Hansen, J. Kotlarski, T. Ortmaier, Experimental validation of advanced minimum energy robot trajectory optimization, (2013) Proc. of the IEEE 16th International Conference on Advanced Robotics (ICAR13), Montevideo, Uruguay, 25-29 Nov. 2013.
- [5] K.D. Nguyen, I-M. Chen, T-C. Ng, Planning Algorithms for S-curve Trajectories, Proc. of the 2007 IEEE/ASME international conference on Advanced intelligent mechatronics, Zurich, Switzerland, 4-7 Sept. 2007.