Partially-linearized multibody equations of railroad vehicles on arbitrary tracks for on-board applications

Javier F. Aceituno, José L. Escalona, Daniel García-Vallejo

Department of Mechanical and Manufacturing Engineering
University of Seville
Camino de los Descubrimientos s/n, 41092, Sevilla, Spain
[aceituno, escalona, dgvallejo]@us.es

Abstract

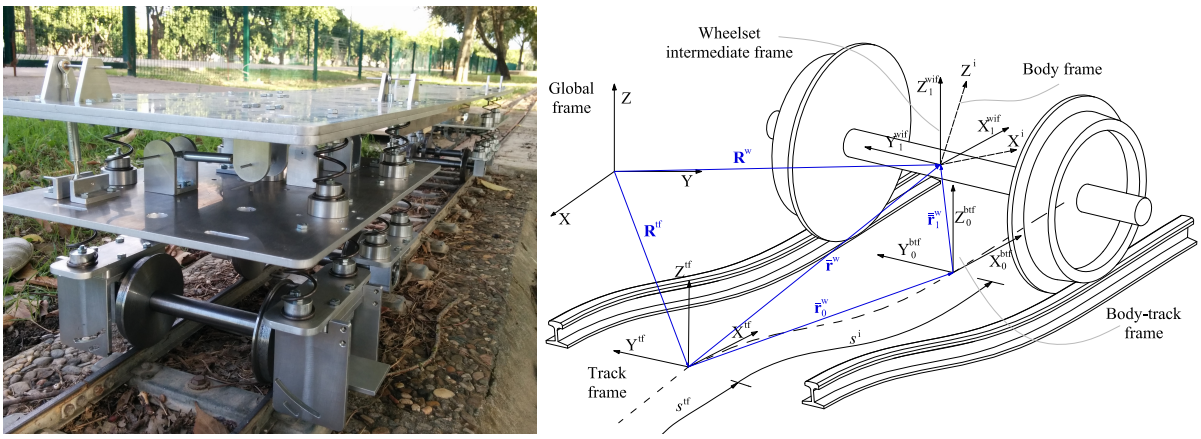
In railroad dynamics, an accurate prediction of the wheel-rail contact geometry is one of the most essential issues for evaluating vehicle stability, curve negotiations or ride comfort. In addition, if this prediction were an on-board one, a great reduction of railroad maintenance costs could be achieved allowing every vehicle body the possibility to check *online* track irregularities. To this end, this paper shows a partial linearization of multibody equations of motion applied to a multibody model of the scaled railroad vehicle of Fig. 1a for on-board simulations on arbitrary tracks. The equations of motion of Eq. (1) are first symbolically obtained as a transformation of the Newton-Euler equations of the vehicle bodies which are referred to a track frame (TF) that accompanies the vehicle [1]. These equations take the form

$$\begin{aligned} &\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}_{\mathbf{q}}^{\mathrm{T}}\lambda = \mathbf{Q} \\ &\mathbf{C}(\mathbf{q},t) = \mathbf{0} \end{aligned} \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{C}_{\mathbf{q}}$ is the jacobian matrix of the constraints vector $\mathbf{C}$, λ is the vector of Lagrange multipliers, $\mathbf{Q}$ is the vector of generalized forces that includes gravity, suspension, and quadratic in system velocities forces, and $\ddot{\mathbf{q}}$ is the accelerations of the system generalized coordinates $\mathbf{q}$. Equation (1) can be expressed in terms of independent coordinates $\mathbf{q}_{ind}$ [2]. When these contraints are not explicit function of time (*scleronomic* constraints), matrix $\mathbf{B}$ relates the velocities of the generalized coordinates to the independent ones as $\dot{\mathbf{q}} = \mathbf{B}\dot{\mathbf{q}}_{\mathrm{ind}}$. Thus, Eq. (1) can be written as $\mathbf{f}(\ddot{\mathbf{q}}_{\mathbf{ind}}, \dot{\mathbf{q}}_{\mathbf{ind}}, \mathbf{q}_{\mathbf{ind}}) = \mathbf{0}$ to then, symbolically be linearized respect to an equilibrium position $\mathbf{q}_{std}$ such as

$$\mathbf{M}_{lin}(s^{\mathrm{tf}})\ddot{\mathbf{q}}_{ind} + \mathbf{C}_{lin}(s^{\mathrm{tf}})\dot{\mathbf{q}}_{ind} + \mathbf{K}_{lin}(s^{\mathrm{tf}})\mathbf{q}_{ind} = \boldsymbol{\gamma}_{std} \tag{2}$$

In Eq. (2), $\mathbf{M}_{lin}$, $\mathbf{C}_{lin}$ and $\mathbf{K}_{lin}$ are the linearized mass, damping and stiffness matrices as functions of the TF arc length s^{tf}, and $\boldsymbol{\gamma}_{std}$ is the vector that contains the terms related to the equilibrium position.



(a) (b)

Figure 1: (a) Scaled railroad vehicle. (b) Kinematics description of a wheelset

Wheel-rail contact constraints are treated with precalculated lookup tables which can take into account the track irregularities as in [3] and consequently, five different types of reference frames are used for the kinematic description of railroad vehicle bodies as it can be seen in Fig. 1b. As generalized coordinates are referred to the TF, a body track frame (BTF) must be identified for every wheelset in which the lookup

table is used. In addition, to avoid the wheelset *pitch* rotation when obtaining the location of the contact points, a wheelset intermediate frame (WIF) is defined that shares the position of the body frame (BF) but does not show *pitch* rotation.
The procedure followed when applying this formulation in a full railroad vehicle model in an arbitrary track is:

- In a preprocessing stage, the different mass, damping and stiffness matrices are evaluated for every identified strecth of the track such as tangent and constant radius curve ones.
- When integrating the linearised dynamic equation of motion of Eq. (2), if the vehicle runs in a transition strecth, precalculated mass, damping and stiffness matrices are interpolated.
- In every time step of the dynamic simulation, the corresponding BTF related to each wheelset body is obtained by solving the following non-linear algebraic equation

$$\bar{\mathbf{x}}_0^{\mathrm{btf}}\left(s^{\mathrm{tf}}+s^i\right)^{\mathrm{T}}\left(\bar{\mathbf{r}}^w-\bar{\mathbf{r}}_0^w\left(s^{\mathrm{tf}}+s^i\right)\right)=0 \tag{3}$$

  where as shown in Fig. 1a, $\bar{\mathbf{x}}_0^{\mathrm{btf}}$ is the longitudinal tangent vector to the track centerline as a function of the arc lenght parameters s^{tf} and s^i, $\bar{\mathbf{r}}^w$ is the position vector of the wheelset respect to the TF, and $\bar{\mathbf{r}}_0^w$ the position vector of the BTF respect to the TF. All of them expressed in the TF.

In this paper the numerical results of the simulation with the developed linearised formulation as in Fig. 2 are compared with the results of the simulation with a fully nonlinear formulation.

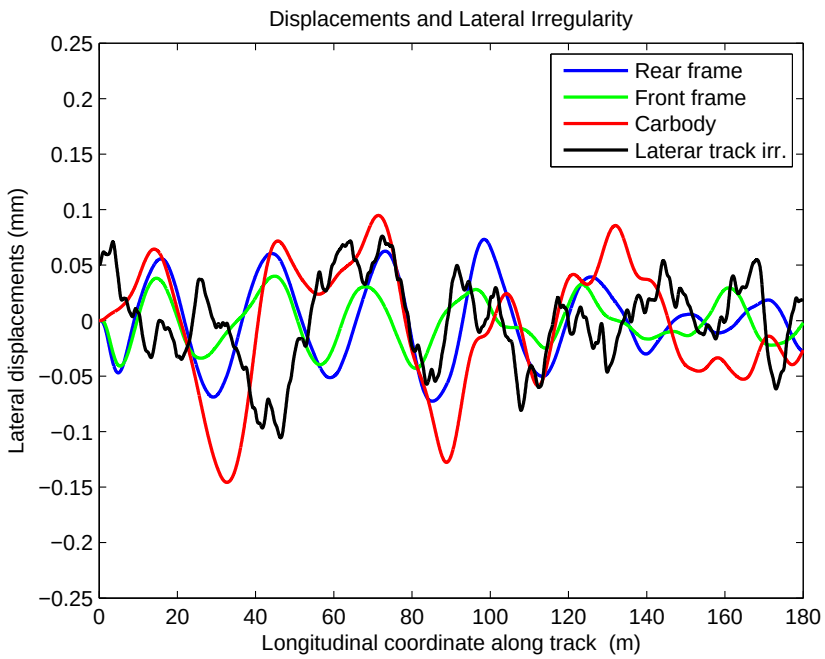


Figure 2: Lateral displacement of a vehicle on an irregular track.

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