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Multibody model reduction by parameter elimination

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Abstract

Model reduction is a more and more interesting topic in Multibody Dynamics as the number of applications which use real-time models increases. Multibody models are usually computationally expensive and reaching the real-time computation usually needs for model reduction or simplification.

When writing the Lagrange equations of a Multibody System,

$$\boldsymbol{M}(\mathbf{q},\dot{\mathbf{q}})\ddot{\mathbf{q}} - \boldsymbol{\delta}(\mathbf{q},\dot{\mathbf{q}}) = \boldsymbol{\tau}$$

if the Inverse Dynamics Model (IDM) is desired, it is customary to write the equations as:

$$\mathbf{K}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\boldsymbol{\phi} = \boldsymbol{\tau} \tag{1}$$

where $\mathbf{K}$ is the one instant Observation Matrix that depends on the generalized coordinates $\mathbf{q}$ and their first and second derivatives $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$, vector $\boldsymbol{\phi}$ represents the model parameters and $\boldsymbol{\tau}$ are the external forces. The IDM can always be written (as in Eq.(1)) linearly with respect to the inertial parameters, and linear-in-the-parameters models should be used for friction and other phenomena. A customary IDM reduction method is based on Model Selection which determines the elements of $\boldsymbol{\phi}$ (and the corresponding columns of $\mathbf{K}$) that could be deleted from the model so that:

$$\mathbf{K}_R(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\boldsymbol{\phi}_R \approx \mathbf{K}(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\boldsymbol{\phi} = \boldsymbol{\tau}.$$

When one needs to identify the dynamic parameters of a robotic system, it is always necessary to re-parameterize the model in order to assure the identifiability of the model parameters. This procedure leads to a model, in terms of a set of *base parameters*, which depends on a smaller number of parameters so that the re-parameterization can be interpreted as a model reduction technique. This reduction can be performed numerically [1] and symbolically [4].

When writing the inverse dynamics equations of a multibody system for a set of n instants, collecting all of them, the Observation Matrix ($\mathbf{W}$) is obtained as:

$$\mathbf{W} = \begin{bmatrix} \mathbf{K}(\mathbf{q}_1,\dot{\mathbf{q}}_1,\ddot{\mathbf{q}}_1) \\ \dots \\ \mathbf{K}(\mathbf{q}_n,\dot{\mathbf{q}}_n,\ddot{\mathbf{q}}_n) \end{bmatrix} \tag{2}$$

It turns out that for a general set of instants, matrix $\mathbf{W}$ tends to be rank deficient meaning that linear dependencies between its columns exist. Eliminating the columns of $\mathbf{W}$ that are linear combinations of others, matrix $\mathbf{W}_b$ is obtained, and writing the new parameters expressions as a linear combination of the original parameters $\boldsymbol{\phi}$, the base parameters $\boldsymbol{\phi}_b$ are obtained. The *base parameters model*, as written in Equation (3), would represent a reduction of Equation (1) without any approximation at all.

$$\mathbf{W}_b\boldsymbol{\phi}_b = \boldsymbol{\tau} \tag{3}$$

Once the *base parameter* reduction has been performed, it is still possible to further reduce the model by eliminating from the model the base parameters (and therefore the corresponding columns of $\mathbf{W}_b$) that have a small significance in the model. In this case, further model reductions will imply an approximation. In this paper, the *base parameter* determination procedure will be extended in order to use it as a general model reduction method.

A customary criterion for model reduction is focused on checking explicitly the influence of each parameter ϕ_i on the external forces $\boldsymbol{\tau}$. The influence of a certain parameter (ϕ_i) on $\boldsymbol{\tau}$ can be measured, for example, in terms of the cross-correlation between the column vector $\mathbf{W}_i$ and $\boldsymbol{\tau}$. Statistical tests can be applied [6] to neglect or not the parameter ϕ_i checking if the correlation between $\mathbf{W}_i$ and $\boldsymbol{\tau}$ $(E(\mathbf{W}_i'\boldsymbol{\tau}))$ lays into a certain interval or not. The techniques based on the errors in $\boldsymbol{\tau}$ are fit for reducing models when the interest lays on estimating the external forces with the highest possible precision.
The model reduction by parameter elimination is a problem very close to that of Model Selection for which it exist an extensive bibliography (see for instance [2, 3, 6]). The main objective of the Model Selection procedures is to obtain a reduced model that depends on the minimum possible number of parameters which is still able to make accurate predictions. The reduction (parameter elimination) procedures relay on a variety of criteria for finding a compromise between the number of parameters and the accuracy of the model. Some of the classical criteria are the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC), the Minimum Description Length (MDL) and the Final Prediction Error (FPE). However, these and other criteria suppose a certain *order of relevance* for the parameters so that the criteria can be applied adding (or eliminating) one parameter at a time and evaluating the criterion function until a minimum is found. Determining a certain emphorder of relevance between the parameters of a multibody system is a key and cumbersome task in any Model Selection procedure.
In the present paper, a new model reduction technique will be presented. It will be supposed that the purpose of the model is to estimate the forces $\boldsymbol{\tau}$, and therefore, the model reduction will be based on the estimate error. The algorithm also proposes how to order the parameters in terms of their relevance to calculate $\boldsymbol{\tau}$. Moreover, the reduction method is able to calculate a set of *generalized base parameters* that hold part of the information of the parameters that have been eliminated. These *generalized base parameters* could be indeed used in forward dynamic simulations which would take profit of the model reduction.
The algorithm is based on the fact that some columns of $\mathbf{W}_b$ are collinear or nearly collinear. Let us call $\mathbf{w}_i$ to the i^{th} column of $\mathbf{W}_b$ and $\mathbf{W}_{\hat{i}}$ to the matrix obtained eliminating column vector $\mathbf{w}_i$ from $\mathbf{W}_b$. In the model reduction algorithm, a column vector $\mathbf{w}_i$ is eliminated from $\mathbf{W}_b$ (and the corresponding parameter ϕ_i from $\boldsymbol{\phi}$) if the norm of the part of $\mathbf{w}_i\phi_i$ that can not be written as a linear combination of the columns of $\mathbf{W}_{\hat{i}}$ is smaller than a tolerance. Or mathematically, if

$$\frac{\|\boldsymbol{\tau} - \mathbf{W}_{\hat{i}}(\mathbf{W}_{\hat{i}}^{+}\boldsymbol{\tau})\|}{\|\boldsymbol{\tau}\|} < TOL,$$

where $\mathbf{W}_{\hat{i}}^{+}$ represents the pseudo-inverse matrix of $\mathbf{W}_{\hat{i}}$. A one-by-one elimination of columns leads to the reduced model with the minimum number of parameters and with a prediction error smaller than a desired tolerance. An equivalent algorithm can also be implemented adding one column vector (one parameter) at a time until the prediction error is smaller than the desired tolerance.
Once the to-be-eliminated parameters have been selected, the *generalized base parameters* of the reduced model can be determined using the customary algorithm of Gautier [1].

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