Generalization of the Divide-And-Conquer Algorithm for the Uncertainty Quantification of Multibody Systems

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Abstract

Reliability assessment plays the key role in the design and analysis of complex multibody dynamics systems in various applications including robotics, MEMS, aerospace and automotive industry.
Polynomial chaos expansion (PCE) provides a powerful intrusive tool to simulate stochastic models of dynamical systems. PCEs are essentially a way to compactly represent random variables. In this method, each stochastic response output and random input $R(\zeta,t)$ is projected onto the space of appropriate independent orthogonal polynomial base functions as

$$R(\zeta,t) = \sum_{i=0}^{\infty} r_i(t)\Psi_i(\zeta) \cong \sum_{i=0}^{N_t} r_i(t)\Psi_i(\zeta). \tag{1}$$

Herein, r_i are time-dependent modal coefficients, while $\Psi_i(\zeta_{i1},\zeta_{i2},...,\zeta_{in})$ are time-invariant generalized polynomial chaos of order of $n(i)$, in terms of multidimensional random variables $\zeta(\zeta_{i1},\zeta_{i2},...,\zeta_{in})$. These polynomials form a complete orthogonal basis for the Hilbert space of square integrable random variables [1]. The series presented in Equation (1) converges to any random process in L^2 sense. In practice, one truncates the infinite expansion such that the polynomial chaos expansion includes a complete basis of polynomials up to a fixed total order specification P. In this case, for a system with N_u finite number of uncertain variables $\zeta = (\zeta_1,\zeta_2,...,\zeta_{N_u})$, the total number of modal terms in the summation, N_t+1, is computed as [1]

$$N_t + 1 = \frac{(N_u+P)!}{N_u!P!}. \tag{2}$$

Using the PCE method, stochastic parameters of the system such as the expected value and variance of R are calculated as

$$E[R] = r_0, \tag{3}$$

$$Var(R) = E[R^2]-(E[R])^2 = \sum_{i=1}^{\infty} r_i^2 < \Psi_i^2 >, \tag{4}$$

where $< \Psi_i^2 >= \int \Psi_i^2(\zeta)w(\zeta)d\zeta$ [2].
In general, the equations governing the dynamics of a deterministic constrained multibody system may be expressed as follows:

$$u = C(q,t)\dot{q} + D(q), \tag{5}$$

$$M(q,t;p)\dot{u} = K(q,\dot{q},t;p) + \Phi,q^T(q,t;p)\lambda, \tag{6}$$

$$\Phi(q,t;p) = 0. \tag{7}$$

In these equations, q is a column matrix of n generalized coordinates of the system. Time is denoted by t, and p is the set of all system design parameters. Equation (5) represents the mapping between $\dot{q}$ and the generalized speeds of the system u. The term M in Equation (6) is the mass matrix of the entire system while Φ,q represents the constraint Jacobian. K is the column matrix of centripetal and Coriolis terms, as well as applied and body loads. Finally, λ is a column matrix of n_c Lagrange multipliers.
Proceeding in the PCE scheme, one should express all state variables and Lagrange multipliers in terms of modal coefficients and orthogonal polynomials with N_t+1 terms as indicated in Equation (1). Replacing these expressions in Equations (5)-(7), and using traditional methods of forming the equations of motion

of constrained multibody systems [1], one can express the PCE-based DAEs for a multibody system as [2]

$$\begin{bmatrix} \mathscr{M}_{n(N_t+1)\times n(N_t+1)} & \mathscr{B}^T_{n(N_t+1)\times n_c(N_t+1)} \\ \mathscr{B}_{n_c(N_t+1)\times n(N_t+1)} & 0_{n_c(N_t+1)\times n_c(N_t+1)} \end{bmatrix} \begin{bmatrix} \dot{u}_{10} \\ \vdots \\ \dot{u}_{nN_t} \\ \lambda_{10} \\ \vdots \\ \lambda_{n_cN_t} \end{bmatrix} = \begin{bmatrix} \mathscr{K}_1 \\ \mathscr{K}_2 \end{bmatrix}. \quad (8)$$

In the above relations, the subscript i is used to show the components along the deterministic dimension, while j denotes the component along the stochastic dimension. The DAE formulation of the stochastic constrained multibody system contains $n \times (N_t+1)$ differential and $n_c \times (N_t+1)$ independent algebraic constraint equations. The overall computational cost associated with forming and solving stochastic DAEs of Equation (8) at each evaluation is on the order of $(n(N_t+1))^3 + (n(N_t+1))^2 n_c(N_t+1) + n(N_t+1)(n_c(N_t+1))^2 + (n_c(N_t+1))^3$ operations per integration time step using direct (non-iterative) methods. Moreover, according to Equation (2), the value of N_t increases rapidly with the increase in the number of the uncertain parameters N_u and the order of the polynomial chaos P as shown in Table 1. For instance, for a small non-constrained system with 20 generalized coordinates and 10 uncertain parameters, if one uses polynomial chaos up to the order 3, the number of the states of the stochastic system rapidly increases to 20×286. As such, traditional methods of forming and solving PCE-based DAEs impose undesirable computational burden on the simulation tool as the system becomes more complex in terms of the number of generalized coordinates, constraints, and uncertain parameters.

Table 1: Number of unknown polynomial chaos coefficients $N_t + 1$ associated with each generalized coordinate

	$N_u = 5$	$N_u = 10$	$N_u = 15$
P = 2	21	66	136
P = 3	56	286	816
P = 4	126	1001	3876

This paper extends the method of Divide-and-Conquer Algorithms (DCA) [4] for the uncertainty analysis of multibody systems in the PCE framework [2]. In the PCE-based DCA approach, the stochastic mass and Jacobian matrices of the entire system are not explicitly formed. In this paper, stochastic handle equations of motion of each individual body in the DCA framework as well as kinematic constraints at connecting joints are developed in terms of base functions and modal terms. Then the mathematical formulation to form the stochastic handle equations of the assemblies are presented. Using the Divide-and-Conquer scheme, the entire system is then swept in the assembly and disassembly passes to recursively form and solve non-deterministic equations of motion for modal values of spatial accelerations and constraint loads. The structure of the method is in such a fashion that it easily lends itself to the parallel implementation framework. As such, using this approach, computational complexity of forming and solving the system's equations of motion will increase as a linear and logarithmic function of $n(N_t+1)$ in the serial and parallel implementations, respectively. This is a significant improvement over the traditional methods of forming and solving stochastic DAEs in which the cost increases as a cubic function of $n(N_t+1)$. The developed equations are then used to study the uncertainty propagation in the double pendulum problems with the uncertainty in the location of the mass center of one of the constituent links. The results will then be compared with other traditional techniques such as Monte Carlo simulation.

References

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