Underactuated approach for the control-based forward dynamic analysis of acquired gait motions

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Abstract
The analysis of acquired gait motion through forward dynamics instead of traditional inverse dynamics offers certain advantages, such as superior dynamic consistency, ability to consider muscle activation and contraction dynamics when descending at muscular level, and feasible computation of contact forces between the subject and the ground or assistive devices. Furthermore, since the forward dynamic analysis implies the integration in time of the model equations of motion, it must face the inherent challenges of gait dynamics (intermittent contact, stability, etc.), and can be perceived as an intermediate step to motion prediction, having less uncertainty as the resulting motion is known.

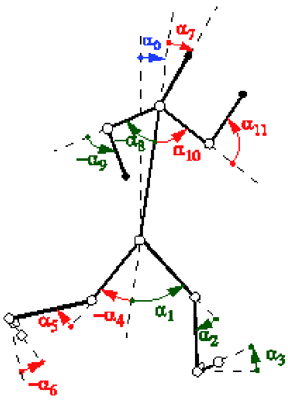

Figure 1: Planar human model.

In a previous work [1], the authors addressed the forward dynamic analysis of an acquired gait motion by means of trajectory tracking controllers associated to all the degrees of freedom of the model. It was shown that the computed torque control (CTC) method provided good accuracy and was extremely robust with respect to the selected gain values. In fact, the computed muscle control (CMC) proposed in [2] at muscular level, uses the CTC method at joint level (introducing then an optimization loop to compute the muscular excitations that generate the obtained joint torques).
However, the human body is not a fully-actuated system, but an underactuated one. Therefore, if an approach closer to reality is sought, it is not admissible to control the six degrees of freedom of the base body (usually, the pelvis or the foot in contact with the ground). Instead, actuators can only be associated to human joints, while the external reactions coming through the feet can be represented by foot-ground contact models. An example of this approach can be found in [3] for a jumping exercise.
The foot-ground contact model plays a key role in the proposed problem. If a force model is chosen, the system is certainly underactuated, and control methods for such types of systems must be used, with the additional difficulty of the unstable nature of gait. If a constraint method is selected seeking to have a fully-actuated system at all times, constraints must be alternatively imposed to the feet (thus perturbing the continuous motion they experience during gait, even at stance phase), and the impact at landing must be dealt with in some way. Consequently, a force model has been used in this work.
Moreover, there is a problem that must be faced when using force contact models in acquired gait motions: the selection of the contact model parameters and, more importantly, of the foot boundary. A not sufficiently good location of feet boundaries can yield huge contact forces that make the

simulation fail. Therefore, an optimization method to select the mentioned characteristics of the contact model, as those proposed in [4,5], is required to be applied, prior to the forward dynamic analysis, to ensure reasonable contact forces during the simulation.

The general formulation of the underactuated system dynamics can be stated as follows. First, the equations of motion of the system are written in minimum number of coordinates as,

$$\mathbf{M}\ddot{\mathbf{z}} = \mathbf{Q} + \mathbf{B}\mathbf{u} \tag{1}$$

where **M** is the system mass matrix, $\ddot{\mathbf{z}}$ is the vector of second time-derivatives of the coordinates (accelerations), **u** is the vector of actuations, projected to the coordinates space through matrix **B**, and **Q** is the vector of velocity-dependent and remaining applied forces (gravitational, ground reactions, etc.). The required outputs are considered to be either functions of the coordinates (e.g. joint trajectories), or of the coordinates and their first derivatives (e.g. ground reactions as a force model).

$$\mathbf{y} = \begin{Bmatrix} \mathbf{y}_1(\mathbf{z}) \\ \mathbf{y}_2(\mathbf{z},\dot{\mathbf{z}}) \end{Bmatrix} \tag{2}$$

Differentiating Eq. (2) with respect to time (twice for $\mathbf{y}_1$ and once for $\mathbf{y}_2$), and substituting then $\ddot{\mathbf{z}}$ from Eq. (1) yields,

$$\hat{\mathbf{y}} = \begin{Bmatrix} \ddot{\mathbf{y}}_1(\mathbf{z}) \\ \dot{\mathbf{y}}_2(\mathbf{z},\dot{\mathbf{z}}) \end{Bmatrix} = \begin{bmatrix} \dot{\mathbf{H}}_{1\mathbf{z}} & \mathbf{H}_{1\mathbf{z}} \\ \mathbf{H}_{2\mathbf{z}} & \mathbf{H}_{2\dot{\mathbf{z}}} \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{z}} \\ \ddot{\mathbf{z}} \end{Bmatrix} = \begin{bmatrix} \dot{\mathbf{H}}_{1\mathbf{z}} \\ \mathbf{H}_{2\mathbf{z}} \end{bmatrix} \dot{\mathbf{z}} + \begin{bmatrix} \mathbf{H}_{1\mathbf{z}} \\ \mathbf{H}_{2\dot{\mathbf{z}}} \end{bmatrix} \ddot{\mathbf{z}} = \mathbf{A}\dot{\mathbf{z}} + \mathbf{D}\ddot{\mathbf{z}} = \mathbf{A}\dot{\mathbf{z}} + \mathbf{D}\mathbf{M}^{-1}\left(\mathbf{Q} + \mathbf{B}\mathbf{u}\right) \tag{3}$$

so that the vector of actuations **u** can be worked out from Eq. (3) as,

$$\mathbf{u} = \left(\mathbf{D}\mathbf{M}^{-1}\mathbf{B}\right)^{-1}\left(\hat{\mathbf{y}} - \mathbf{A}\dot{\mathbf{z}} - \mathbf{D}\mathbf{M}^{-1}\mathbf{Q}\right) \tag{4}$$

Now, calling $\mathbf{P} = \mathbf{D}\mathbf{M}^{-1}\mathbf{B}$, and considering that feedback is introduced for the outputs, it results,

$$\mathbf{u} = \mathbf{P}^{-1}\left(\begin{Bmatrix} \ddot{\mathbf{y}}_1^* + \mathbf{C}_{\mathrm{v}}\left(\dot{\mathbf{y}}_1^* - \dot{\mathbf{y}}_1\right) + \mathbf{C}_{\mathrm{p}}\left(\mathbf{y}_1^* - \mathbf{y}_1\right) \\ \dot{\mathbf{y}}_2^* + \mathbf{K}_{\mathrm{p}}\left(\mathbf{y}_2^* - \mathbf{y}_2\right) \end{Bmatrix} - \mathbf{A}\dot{\mathbf{z}} - \mathbf{D}\mathbf{M}^{-1}\mathbf{Q}\right) \tag{5}$$

where the asterisk indicates the desired values of the outputs, different from the current ones (without asterisk), and $\mathbf{C}_{\mathrm{v}}$, $\mathbf{C}_{\mathrm{p}}$ and $\mathbf{K}_{\mathrm{p}}$ are diagonal matrices containing the gains associated to each output.

If the number of outputs is equal to that of actuators, matrix **P** is square and the required inputs can be determined from Eq. (5). If the number of outputs is greater than that of actuators, the required outputs can be satisfied in a minimum squares sense only, the system of equations to be solved being,

$$\mathbf{u} = \left(\mathbf{P}^{\mathrm{T}}\mathbf{W}\mathbf{P}\right)^{-1}\mathbf{P}^{\mathrm{T}}\mathbf{W}\left(\begin{Bmatrix} \ddot{\mathbf{y}}_1^* + \mathbf{C}_{\mathrm{v}}\left(\dot{\mathbf{y}}_1^* - \dot{\mathbf{y}}_1\right) + \mathbf{C}_{\mathrm{p}}\left(\mathbf{y}_1^* - \mathbf{y}_1\right) \\ \dot{\mathbf{y}}_2^* + \mathbf{K}_{\mathrm{p}}\left(\mathbf{y}_2^* - \mathbf{y}_2\right) \end{Bmatrix} - \mathbf{A}\dot{\mathbf{z}} - \mathbf{D}\mathbf{M}^{-1}\mathbf{Q}\right) \tag{6}$$

with **W** the weight diagonal matrix which assigns more weight to more relevant outputs.

In this work, different alternatives in the outputs choice are investigated, along with the conditions for a stable solution. The human body model considered is a 12-segment planar model with 14 degrees of freedom (Figure 1).

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