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Structure preserving optimal control of a 3d-dimensional upright gait

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Abstract

The straight posture of the homo sapiens' gait is a characteristical attribute of the human species, whereby the evolution of the human gait results from an anthropological optimisation process. Initiated by climbing down from trees and leaving forests, the survival in velds necessitates a physical adoption and the results are reflected at the kind of human locomotion respectively at the physique: an upright gait shows benefits like a distinct all-round visibility and allows the possibility to use tools and weapons at the struggle of survival.

The goal of this work is to optimally control the human upright gait using a structure preserving variational integrator, whereby different physiologically motivated cost functions are tested. Based on Lagrangian mechanics, the action is discretised and with respect to the discrete variational principle the resulting discrete Euler-Lagrange equations are equipped with symplectic and momentum preserving properties. The discussed multibody system of an bipedal walker represents the consistent further development of the monopedal jumper models discussed in [1] and [2]. In the first citation, the simple footless monopedal jumper consists of three rigid bodies – upper part of the body, thigh and calf – consequently, a single point is used to model the contact. In [2], a more realistic behaviour is targeted, because the inclusion of the jumper's foot is a basic detail and it has enormous influence of the jumping movement before the contact between the foot and the ground is established or released. The properties of this monopedal jumper's leg are adopted to the multibody system of the bipedal walker. Herein, the human upright gait is analysed using a simple model consisting of seven rigid bodies representing the human's torso plus the thigh, calf and foot of both legs, which are symmetrical to each other. The hips are modelled as spherical joints and the thigh and calf are chained with a revolute joint, where the unit vector $\mathbf{n}_1 \in \mathbb{R}^3$ specifies the axis of rotation of the knee. In reality, the human ankle joint has two rotational degrees of freedom, whose axis of rotation are diagonally located to each other – the implemented ankle joints are simplified using a spherical joint with anatomically adapted restrictions. The constrained multibody system is described by the configuration variable $\mathbf{q} \in \mathbb{R}^{84}$ and due to the rigid body formulation in use, $m_{int} = 42$ internal constraints are present. The joint interconnections cause $m_{ext} = 22$ external constraints and therefore the $k = 84$-dimensional system is restricted by $m = 64$ holonomic constraints. Corresponding to the $k - m = 20$ degrees of freedom, the generalised coordinates read $\mathbf{u} = \left[\mathbf{u}^{UP}; \boldsymbol{\theta}^{UP}; \boldsymbol{\theta}_H^R; \theta_K^R; \boldsymbol{\theta}_A^R; \boldsymbol{\theta}_H^L; \theta_K^L; \boldsymbol{\theta}_A^L\right]^T \in \mathbb{R}^{20}$ and $\boldsymbol{\tau} = \left[\boldsymbol{\tau}_H^R; \tau_K^R; \boldsymbol{\tau}_A^R; \boldsymbol{\tau}_H^L; \tau_K^L; \boldsymbol{\tau}_A^L\right]^T \in \mathbb{R}^{14}$ represents the actuation in the hip, knee and ankle joints of both legs (see Figure 1). The herein modelled feet have

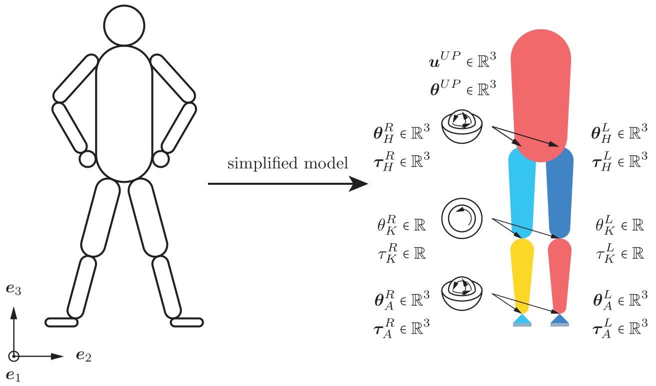

Figure 1: Seven link model of the simplified bipedal walker.

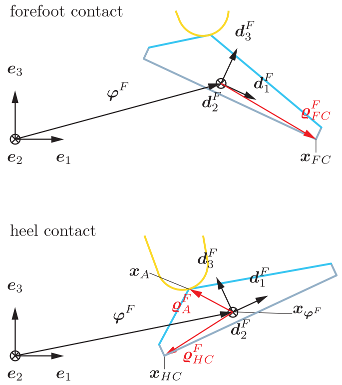

Figure 2: Visualisation of the forefoot and heel contact of the walker's right leg.

two contact points and thus three different contact scenarios are possible, i.e. two single contact phases (forefoot contact (FC), respectively, heel contact (HC)) as illustrated in Figure 2 and the complete contact support phase, at which the forefoot and the heel are in contact with the ground. Each contact point between the foot and the ground is fixed at the ground in $\mathbf{x}_i \in \mathbb{R}^3$ for $i = FC, HC$ by a spherical joint, consequently the contact function reads

$$\mathbf{g}_i^{(S)}(\mathbf{q}) = \boldsymbol{\varphi}^F + \boldsymbol{\rho}_i^F - \mathbf{x}_i \in \mathbb{R}^3, \tag{1}$$

where $\boldsymbol{\rho}_i^F$ points from the centre of mass to the respective contact point, while $\boldsymbol{\rho}_A^F$ points to the ankle. The condition for double contact (FC and HC) is different from the contact formulation for one contact point in (1), because the combination of two spherical joints does not completely restrict the foot's degrees of freedom. The constructive approach used here is to fix one of the contact points at the ground by a spherical joint and for the other contact point, only the vertical component of the remaining spherical joint formulation is fixed. To prevent the residual rotational motions around the line connecting the FC and HC point, two further contact constraints are necessary, in this work the $\boldsymbol{e}_2$-component of the center of mass and of the ankle are picked; for example, the constraint formulation of the double contact phase (FHC) reads

$$\boldsymbol{g}_{FHC}(\boldsymbol{q}) = \begin{bmatrix} \boldsymbol{g}_{FC}^{(S)}(\boldsymbol{q}) \\ (\boldsymbol{e}_2)^T \cdot (\boldsymbol{\varphi}^F - \boldsymbol{x}_{\boldsymbol{\varphi}^F}) \\ (\boldsymbol{e}_2)^T \cdot ((\boldsymbol{\varphi}^F + \boldsymbol{\rho}_A^F) - \boldsymbol{x}_A) \\ (\boldsymbol{e}_3)^T \cdot \boldsymbol{g}_{HC}^{(S)}(\boldsymbol{q}) \end{bmatrix} = \mathbf{0} \in \mathbb{R}^6,$$

whereby the spherical joint formulation is exemplarily used at the forefoot and $\boldsymbol{x}_{\boldsymbol{\varphi}^F}, \boldsymbol{x}_A \in \mathbb{R}^3$ represent the location vectors where the center of mass and of the ankle are fixed in space.

The direct transcription method DMOCC is used to transform the optimal control problem into a constrained optimisation problem. The human gait can be considered as an examples of a hybrid system, where the dynamics is subject to an inherent switch due to the closing or opening of contacts at the feet. As specified in Figure 3, the motion sequence – not the switching time – in which the contact phases follow each other are considered as known. To perpetuate the structure preservation of the variational integrator and the geometrical correctness during the releasing and establishing of the contact at the feet, the non-smooth problem is solved including the computation of the contact or contact release configuration as well as the contact time and force, instead of relying on a smooth approximation of the contact problem via a penalty potential. Due to the cycle structure of human gait, only one half step is optimised, such that periodical boundary conditions are considered as equality conditions in the optimisation problem. The optimised course of motion with minimal control effort is shown in Figure 4.

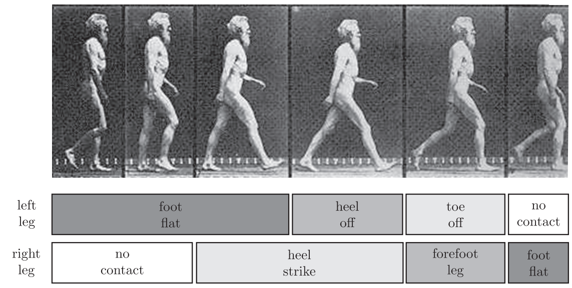

Figure 3: Characteristical configurations during the human bipedal walk.

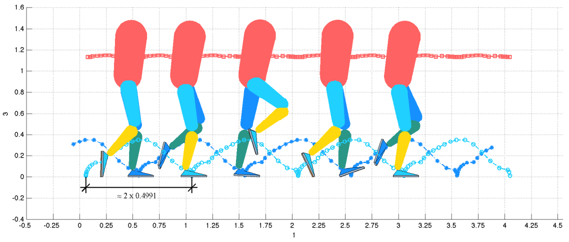

Figure 4: Snapshots of an optimised bipedal gait minimising the control effort.

References

[1] M. W. Koch, S. Leyendecker. Structure Preserving Simulation of Monopedal Jumping. The Archive of Mechanical Engineering, Vol. LX, No. 1, pp. 127–146, 2013.

[2] M. W. Koch, S. Leyendecker. A structure preserving approach to the simulation of non-smooth dynamics. PAMM, 2014, accepted.