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A validation and comparison study of two discrete element methods for granular dynamics

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Abstract

Two alternative approaches have emerged as viable solutions for large frictional contact problems in granular flow dynamics. Widely adopted and more mature, the so-called Penalty Method (PM) is a regularization (or smoothing) approach which relies on a relaxation of the rigid-body assumption [1]. Normal and tangential forces are calculated using various laws [2, 3], based on the local body deformation which is defined as the penetration (overlap) of the two rigid bodies. Once contact forces are known, the time evolution of each body in the system is obtained by integrating the Newton-Euler equations of motion (EOM). Commonly known as the Discrete Element Method (DEM), this is the approach adopted by all leading multibody dynamics and DEM commercial packages. However, due to large contact stiffness, it is limited to very small integration step-sizes. This leads to very long simulation times and/or the requirement of expensive hardware (e.g. distributed computing on supercomputers).

A second approach, relatively more recent, is based on a Lagrangian approach to the contact problem. Here, the non-penetration constraints are written as complementarity conditions which, in conjunction with a Coulomb friction law, lead to a Differential Variational Inequality (DVI) form of the EOM [4]. Not limited by stability considerations, DVI allows for much larger integration steps than PM. However, this involves a more complex and costly solution sequence as, upon discretization, the DVI approach leads to a mathematical program with complementarity and equality constraints. Various relaxations result in tractable linear complementarity or cone complementarity problems [5].

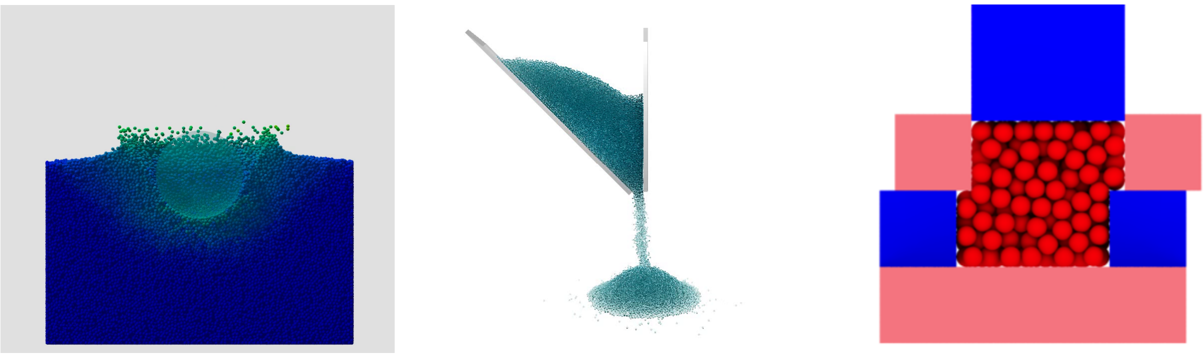

Figure 1: Snapshots from Chrono simulations of the three benchmark problems: projectile impact (left), mass flow rate (middle), direct shear test (right).

In this study we introduce three problems (see Fig. 1), coming from different application areas, which can be used to (i) validate modeling methods and verify simulation software for granular dynamics and (ii) compare different approaches to the solution of large-scale frictional contact problems. Because of the inherent difficulties in experimentally observing and validating the motion of individual particles, these validation problems focus on bulk properties of the granular material.

The first problem models an experiment set up to study the low-speed impact of spherical projectiles into granular material and the resulting crater formation. Used as a simplified model of meteorite crater formation [6], this experiment involved dropping, from different heights, spherical projectiles with different densities into a bed of granular material. An experimentally inferred relationship between the drop height and the maximum penetration depth offers a simple global measure for validation and verification studies. This empirical formula indicates that the coefficient of friction and the bulk density of the granular material, but not the granule size, dictate the resulting crater depth.

The second experiment was designed to quantify the mass flow rate of granular material. The experimental setup [7] is composed of two machined components, a translational stage, a linear actuator, and a scale/collector. After filling the reservoir with a fixed amount of granular material with known geometry, the translating wall is moved to a predefined gap size and the granular material flows onto the scale/collector where the accumulated mass is measured over time. Experimental results are available for different, precisely controlled, values of the opening gap size.

The direct shear test [8] is used to measure the shear strength properties of a soil, specifically the cohesion, angle of friction, and shear modulus. A sample of the soil is contained in a shear box which is aligned under a load cell that applies a normal force to the soil. The top of the shear box is clamped so that the lower half can be translated horizontally by a specified displacement. The horizontal force required to displace the soil is measured to produce a plot of the shear stress as a function of shear displacement.

Using the Chrono [9] parallel multi-physics simulation package, we present results obtained using both the PM and DVI methods on the above three validation problems. In addition to verifying the extent to which simulation results are able to reproduce the experimental results, we also compare the two methods in terms of accuracy, robustness, efficiency, and scalability. While DVI methods are capable of taking large steps, the numerical solution at each step is much more laborious than for PM; in an attempt to gain insights relevant to problem sizes of engineering relevance, we investigate the point where solution cost of DVI offsets the advantage of larger integration steps.

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