An aggregation-based multigrid method for solving large multibody problems

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Abstract

Most multibody formulations share the common problem of solving at least one linear problem at each time step; often it is a saddle-point problem with the following structure:

$$\begin{bmatrix} \mathbf{H} & \mathbf{C_q}^T \\ \mathbf{C_q} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{x} \\ \mathbf{y} \end{Bmatrix} = \begin{Bmatrix} \mathbf{a} \\ \mathbf{b} \end{Bmatrix} \tag{1}$$

where $\mathbf{H}$ is a sparse matrix, often a mass matrix, $\mathbf{C_q}$ is a sparse jacobian of constraints, $\mathbf{a}$ and $\mathbf{b}$ are known terms, and $\mathbf{x}$ and $\mathbf{y}$ are unknowns (often $\mathbf{x}$ contains the unknown accelerations and $\mathbf{y}$ contains the unknown reactions in constraints).

The solution of such system is trivial in the case of a small number of coordinates, where direct methods can be efficiently used; for larger numbers of coordinates the usual approach is based on iterative methods of Krylov type, such as GMRES or MINRES [1]. The problem is that Krylov methods perform poorly when the matrix has a high conditioning number, that is the case, for instance, of mechanical systems with odd mass ratios or odd stiffness ratios. Preconditioners for Krylov iterations can alleviate the problem but, to our knowledge, classic preconditioners fall short in terms either in terms of scalability or in terms of efficiency.

This motivated our research on multigrid methods for the solution of this class of problems. We adopted a K-cycle aggregation-based multigrid method that uses multigrid as a preconditioner for a Krylov outer loop. When compared to classic multigrid, this aggregation-based multigrid uses simpler coarsening and interpolating operators: this fits better in our sparsity-oriented multibody implementation; on the other hand the performance penalty caused by the rough interpolation is compensated by the special K-cycle that replaces the usual V or W cycles of traditional multigrid schemes.

This idea of K-cycle aggregation-based multigrid was presented in [2], and we adapted it to the saddle-point nature of our multibody problems. The original scheme can operate on diagonally-dominant matrices, but this is not the case discussed here. In case $\mathbf{H}$ is easily invertible, for instance if $\mathbf{H} = \mathbf{M}$, a diagonal mass matrix, we tested that the method can be directly operated on the Schur complement $\mathbf{N} = \mathbf{C_q}\mathbf{H}^{-1}\mathbf{C_q}^T$; yet our customized coarsening strategy takes into account the fact that flipping the direction of some constraints leads to different $\mathbf{N}$ and different quality of coarsening. In case $\mathbf{H}$ is not easily invertible, such as during the iteration of a Newton-Raphson loop for an implicit solver, where $\mathbf{H}$ contains terms proportional to a tangent stiffness matrix, the Schur complement cannot be computed, so a special coarsening method has been developed, that operates separately on dual and primal variables in a monolithic K-cycle.

We compared the resulting multigrid method to state-of-the-art numerical methods for large linear systems, see for instance Figure 1. In cases where classical Krylov methods exhibit slow convergence because of a critical conditioning of the system, the aggregation-based multigrid always converges better.

The availability of an efficient multigrid solver allows the simulation of large problems, such as multibody systems interacting with granular dynamics, whose solution proved to be difficult with usual iterative solvers with slow convergence, especially in case of odd mass ratios [3]. An interesting outlook of this research is that, for problems with unilateral contacts, the complementarity problem can still benefit from the presence of a linear solver of this type, because complementarity problems can be solved with interior point methods as a sequence of linear problems [4].

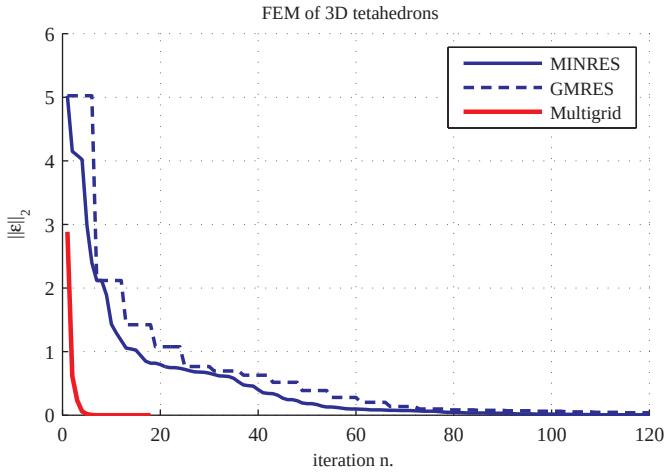

Figure 1: Benchmark with a 3D FEM model (232 tetahedrons and 36 constraints).

References

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