## Computing Frictional Contact Forces in Rigid Multibody Dynamics Using a Primal-Dual Interior Point Method

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## Abstract

The dynamical simulation of systems involving unilateral contacts between bodies is complicated by the nonsmooth nature of the frictional constraints. When the number of contacts between bodies increases to thousands or millions, as in the case of granular flows in silos or in soil dynamics, the computational efficiency of traditional methods can become an issue even on supercomputers. To the best of our knowledge, no numerical method has been demonstrated to reliably solve a sizeable problem of this type; i.e., the direct shear test shown in Fig. (1). The direct shear test is used to measure the shear strength properties of a soil, specifically the cohesion, angle of friction, and shear modulus. A sample of the soil is contained in a shear box which is aligned under a load cell that applies a normal force to the soil. The top of the shear box is clamped so that the lower half can be translated horizontally by a specified displacement. The horizontal force required to displace the soil is measured to produce a plot of the shear stress as a function of shear displacement.



Figure 1: One million spheres interacting through contact and friction.

We present a primal-dual interior point (PDIP) method for solving a cone complementarity problem (CCP) associated with multibody dynamics frictional contact problems approached within a differential variational inequality (DVI) framework using the parallel sparse solver, SPIKE::GPU, to accelerate the solution. The CCP can be reformulated as a cone-constrained optimization problem as described in Eq. (1), where the cost function  $\mathbf{q}(\gamma)$  is obtained by combining the discretized equations of motion, a complementarity condition between the gap function and normal contact forces  $\gamma$ , and the Coulomb dry friction model.

$$\min \mathbf{q}(\gamma) = \frac{1}{2} \gamma^T \mathbf{N} \gamma + \mathbf{r}^T \gamma$$
  
s.t.  $\gamma_i \in \Upsilon_i$ , for  $i = 1, 2, ..., n_c$  (1)

The PDIP method can be applied to large-scale convex optimization problems with inequality constraints  $c(\gamma) < 0$  [?]. As a barrier method, the constraints are included into the cost function, converting the original constrained problem in Eq. (1) into an unconstrained one Eq. (2). The Karush-Kuhn-Tucker (KKT) conditions are derived in Eq. (3), where  $\gamma$  and  $\lambda$  are the primal and dual variables, respectively. When the barrier parameter  $\mu$  is set to zero, Eq. (3) is the same as the KKT condition of Eq. (2).

$$\min \phi(\gamma) := \mathbf{q}(\gamma) - \mu \sum_{i=1}^{m} ln(c_i(\gamma))$$
(2)

The perturbed KKT conditions Eq. (3) are solved using Newton's method. Only an approximate solution of the barrier problem with a fixed value of  $\mu$  is necessary to guarantee global convergence. The barrier parameter  $\mu$  is updated at a superlinear rate to ensure fast convergence [?]. Once the search direction is identified, a step size is determined that satisfies the feasibility of the primal and dual variables. Solving a sequence of barrier problems eventually drives the perturbation parameter  $\mu$  to zero, and the solution sequence is proven to converge to the solution of the original constrained problem.

$$c_{i}(\lambda) > 0, \ i = 1, ..., m$$
$$\lambda_{i} > 0, \ i = 1, ..., m$$
$$\nabla \mathbf{q}(\lambda) - \sum_{i=1}^{m} \lambda_{i} \nabla c_{i}(\lambda) = 0$$
$$\lambda_{i}c_{i} = \mu, \ i = 1, ..., m$$
(3)

Our study focuses on large-scale frictional contact problems, see Fig. (1). The associated CCP was solved using the PDIP and the Accelerated Projected Gradient Descent (APGD) methods [?] at one time step. The residual and convergence information, shown in Table 1, indicate that the PDIP method converges faster than the APGD algorithm. However, the computational cost of PDIP remains high since it calls for the solution of a sequence of primal-dual linear systems.

	APGD			PDIP		
Solver	# of	Residual	Total Shear	# of	Residual	Total Shear
Tolerance	iterations		Force	iterations		Force
0.1	261	0.098779	-16108.1	28	0.067072	-63536.7
0.01	2308	0.009998	-67744.6	62	0.00959	-123132
0.001	8561	0.001	-201568	101	0.00088	-166467
0.0001	57882	1.00E-04	-170950	108	9.04E-05	-168788
0.00001	70562	9.99E-06	-169141	115	9.44E-06	-168986
0.000001	3000000	3.46E-06	-168956	120	2.40E-07	-169005

Table 1: Performance statistics for the APGD and PDIP solvers during a single time step of the direct shear test.

## References

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