A consistent and efficient framework for the time integration of multibody systems with impacts and friction

Thorsten Schindler

Institute of Applied Mechanics
Technische Universität München
Boltzmannstraße 15, 85748 Garching, Germany
thorsten.schindler@mytum.de

Abstract

In this work, we discuss time integration schemes for the dynamic simulation of nonsmooth flexible multibody systems. One the one hand, we analyze mechanical systems which undergo impulsive, i.e., impacting, and non-impulsive motion. Thereby, we consider dry friction, i.e., arbitrary stick-slip transitions. On the other hand, even large deformations due to flexibility of specific machine parts are covered. For such applications, standard time integration schemes known from computational mechanics [3] usually suffer from oscillations in the relative contact velocities [2]. That is why, we take another path.

We develop a framework for the consistent treatment of velocity jumps, e.g. due to impacts. A non-impulsive trajectory of state-variables is improved by an impulsive correction after each time-step if necessary. This correction is automatically chosen starting from a non-impulsive base integration scheme, which discretizes the propagation within the time-step. As indicated in Figure 1, we calculate approx-

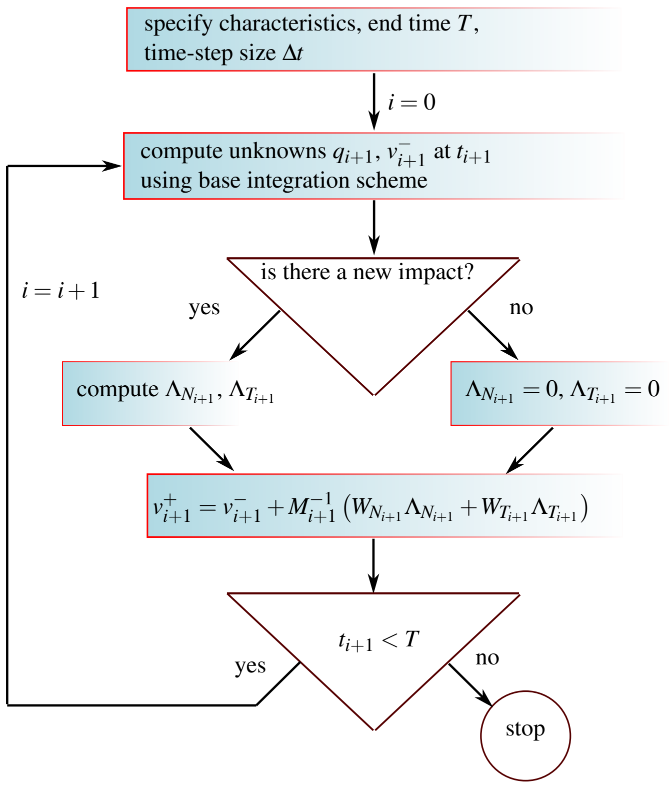

Figure 1: Flowchart of computational algorithm [4].

imations for the position q_{i+1} and the left-sided limit of the velocity v^-_{i+1} of the mechanical system at the end of a time interval $[t_i, t_{i+1}]$ given the initial values, i.e., q_i and the right-sided limit of the velocity v^+_i. This computation assumes that the propagation $(q_i, v^+_i) \to (q_{i+1}, v^-_{i+1})$ is non-impulsive. However, active contacts like bilateral and unilateral frictional constraints are considered on the level of the tuple (gap velocities, contact forces) in a differential-algebraic equations (DAE) sense with reduced drift-off. Knowing in particular the new position q_{i+1}, we can compare criteria at the beginning and at the end of the time-step for each contact k: e.g. the gaps $g_k(q_i)$ and $g_k(q_{i+1})$. If there is at least a contact which is active at the end of the time-step $g_{k_0}(q_{i+1}) \leq 0$ but has not been active at the beginning of the time-step $g_{k_0}(q_i) > 0$, we start an impulsive correction of the velocities with the tuple $(\dot{g}(q_{i+1}, v^+_{i+1}), \Lambda_{i+1})$. Therefore for simplicity, we consider the whole mechanical system with mass matrix M_{i+1}, normal and tangential force directions $W_{N_{i+1}}$, $W_{T_{i+1}}$ as well as force parameters $\Lambda_{N_{i+1}}$, $\Lambda_{T_{i+1}}$.

Within the preceding framework, consistency is achieved due to the impulsive corrections on the same kinematic level as the treatment of non-impulsive constraints. This idea stems from a time-discontinuous Galerkin setting [5], but is generalized concerning the splitting of non-impulsive and impulsive force propagation in [4, 6]. Efficiency and further attributes are gained from the base integration schemes. In this work, we compare the behaviour of four different base integration schemes in the newly developed framework as well as a classical Moreau-Jean timestepping scheme [1] concerning selected criteria and examples from academics and industry. In particular, we discuss a half-explicit trapezoidal rule satisfying the time-discontinuous Galerkin setting, called HETS, the generalized-α method, the Bathe-method and the ED (energy decaying)-α method as base integration schemes. For examples, we consider a bouncing ball, a flexible slider-crank mechanism within the floating-frame approach with impacts and friction, an impacting elastic bar and a rubbing rotor. We analyze the local order of the schemes, the damping ability of spurious oscillations also in the nonlinear case, the computing time per time-step and the necessary computing time to achieve a prescribed tolerance. In addition, we discuss the convergence of finite element discretizations of flexible machine parts concerning modal references.

It turns out that the half-explicit timestepping is a very robust and the most efficient method as far as we deal with non-stiff problems. The timestepping schemes based on the generalized-α method, the Bathe method and the ED-α method become most efficient for stiff problems with spurious oscillations. In our test cases the generalized-α method is the most efficient base integration scheme concerning computing time, however it may get unstable in the nonlinear regime. The ED-α method satisfies exactly the opposed characteristics. It is the Bathe-method, which seems to be the best compromise concerning stability and efficiency in the nonlinear regime. We propose it as a base integration scheme for timestepping methods whenever stiff problems with impacts and friction have to be solved.

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