

From Line Impact to Rolling Resistance¹

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Abstract

Interaction between rough surfaces displays complex coupling among multiple spatiotemporal scales, which is at the root of their use in many applications. Rolling resistance basically characterizes this feature, and attracts interests from lots of authors[1, 2, 3, 4, 5]. Here, we aim at discovering the energy dissipation of the rolling motion of a real cylinder by a variably-scaling method. We first investigate the pure rolling motion of prisms, where the energy dissipation is induced by a sequence of line impacts. We theoretically and experimentally learned the rupture process of the interaction comes to an end when no detachment occurs after a line impact. The energetic dissipation of line impacts is depicted by the non-detachment condition[6]. Supposing the dissipation of a rolling cylinder be the limit of that of an equivalent prism whose edge width is determined by Hertz contact theory, we deduced the rolling resistance of the cylinder from sequence of line impacts of prisms. We successfully express the resistance torque as the function of the square of angular velocity plus an exponent term about the normal load. In contrast to the classical model of the rolling resistance, where the energetic dissipation in rolling motion is attributed to either the minute slip[1, 2], or the viscosity of materials[3, 4, 5], our model attributes the rolling resistance to the rupture process of interface stresses that generates the elastic hysteresis losses. We further validate our model by comparing it with that in [7] for the low rotary speed and by investigating the 3D motion of an Euler disk rolling on a rough horizontal surface for high rotary speed. The new model for rolling resistance proposed in this paper can well explain the finite-time singularity of the disk discovered in [8].

Rolling resistance is very small and can be hardly distinguished it from disturbance experimentally. But if it is investigated by a prism, the edge number of the prism can be decreased and the intensity of its line impacts is enhanced. Thus, experimental investigation can be carried out. After the line impact is understood fully, it is then applied to the equivalent prism with large edge number. At last, the rolling resistance of the cylinder is revealed from the equivalent prism. Therefore, the investigating approach in this paper is called as a variably-scaling method.

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