Constrained State and Input Estimation for a MacPherson Suspension Using the Unscented Kalman Filter and a 3D Multibody Model

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A novel approach to estimate states and inputs of constrained multibody systems is presented. It is based on a constrained Unscented Kalman Filter (UKF) with unknown inputs. A 3D MacPherson suspension is used as a numerical example to show the effectiveness of the estimation using a minimum number of sensors.

Introduction

Research on control strategies for semi-active and active suspensions of vehicles has been carried out extensively in the last two decades. 1D and 2D analytical models, like the quarter-car model, are commonly employed for suspension control synthesis and analysis. 3D analytical models of both the suspension and the full vehicle have also been employed more recently. The skyhook, groundhook or hybrid control algorithms are usual model–free control algorithms for semi-active suspensions (SAS) that are tuned using the aforementioned models [1]. More advanced control techniques such as optimal control, adaptive control, pole location control, fuzzy control and sliding mode have also been applied to the control of semi- and active suspensions [2]. All these control strategies assume that velocities (for example the one of the sprung mass or the one of the damper) can be measured. However because such sensors are not widely available in practice, instead the velocities have to be computed both from position measurements by derivation or from acceleration measurements by integration, which is a challenging task. For example position sensors are not suitable for measurements above ~20 Hz while acceleration sensors are not suitable for measurements under ~5 HZ [3]. The aforementioned reasons justify the use of model-based observers to compute these quantities and make them available to the controller as virtual sensors [4].

3D rigid-body model of the MacPherson suspension

The 3D MacPherson suspension presented in [5] has been modeled in natural coordinates by means of 8 points and 3 unit vectors. An index-3 augmented Lagrangian formulation with mass-orthogonal projections in velocities and accelerations has been chosen to simulate the multibody system due to its computational efficiency. The corresponding equations of motion are presented in Equation (1).

$$\mathbf{M}\ddot{\mathbf{q}} + \boldsymbol{\Phi}_{\mathbf{q}}^T \boldsymbol{\alpha} \boldsymbol{\Phi} + \boldsymbol{\Phi}_{\mathbf{q}}^T \boldsymbol{\lambda}^* = \mathbf{Q} \tag{1a}$$

$$\boldsymbol{\lambda}_{i+1}^* = \boldsymbol{\lambda}_i^* + \boldsymbol{\alpha} \boldsymbol{\Phi}_{i+1} \qquad i = 0, 1, 2... \tag{1b}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{q}$ the vector of dependent coordinates, $\ddot{\mathbf{q}}$ the vector of accelerations, $\boldsymbol{\Phi}_{\mathbf{q}}$ the Jacobian matrix of the constraints, $\boldsymbol{\Phi}$ the vector of constraints, $\boldsymbol{\lambda}^*$ the Lagrange multipliers, $\mathbf{Q}$ the vector of generalized external forces and $\boldsymbol{\alpha}$ is the diagonal matrix of penalty factors. Due to the explicit form of the difference equations shown in Equation (2), explicit integrators have to be used to discretize the equations of motion.

Nonlinear state and input observer

A state observer can only be applied if all the inputs of the system model are known. However, this assumption is seldom verified for mechanical systems. For multibody systems, the inputs of the observer are usually external forces while the sensors required for the estimation provide positions, velocities and/or accelerations measurements [6]. Frequently only a subset of all external forces is known thus preventing to perform a forward dynamic simulation in the estimation process. A state and input observer solves this problem by estimating the unknown inputs based on the sensor information. The authors investigated previously the use of an augmented discrete Extended Kalman filter with a multibody formulation in independent coordinates [7]. This research proposes the use of a constrained discrete UKF with a multibody formulation in dependent coordinates. The discrete nonlinear system model of the observer is given in Equations (2) and (3).

$$\mathbf{x}_{n+1} = \mathbf{f}(\mathbf{x}_n, \mathbf{u}_n) + \mathbf{G}_n\mathbf{d}_n + \boldsymbol{\omega}_n \tag{2a}$$

$$\mathbf{y}_n = \mathbf{C}_n\mathbf{x}_n + \mathbf{H}_n\mathbf{d}_n + \boldsymbol{\nu}_n \tag{2b}$$

subject to

$$\mathbf{g}(\mathbf{x}) = \mathbf{0} \tag{3}$$

where $\mathbf{x}$ is the state vector, $\mathbf{f}$ contains the nonlinear system equations, $\mathbf{u}$ is the control input vector, $\mathbf{d}$ is the unknown input vector, $\boldsymbol{\omega}$ is the process noise, $\mathbf{y}$ is the measurement vector, $\mathbf{C}$ is the measurement matrix, $\boldsymbol{\nu}$ is the measurement noise, $\mathbf{G}$ and $\mathbf{H}$ are known mapping matrices and $\mathbf{g}$ are equality constraints. The multibody model of the MacPherson suspension mentioned earlier has been used as a numerical example to demonstrate the validity of the approach. In order to match the form of the equations of motion shown in Equation (1) with the form of the discrete nonlinear system model shown in Equation (2), it is necessary first to transform the second-order differential equations into first-order differential equations through variable duplication and then to combine the equations of the integrator with the transformed equations of motion. Multiple explicit integrators have been implemented in order to evaluate their influence on the accuracy of the estimates and on the computational efficiency of the algorithm. Instead of using a real suspension, a virtual one has been simulated (including the sensors) in order to provide a true reference to evaluate the accuracy of the state and input estimates. The sensors considered for the estimation are an elongation sensor for the damper and accelerometers for the sprung and unsprung masses. The efficiency and applicability of the algorithm have also been evaluated by implementing it in C++ and executing it in an ARM-based embedded system.

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