Detecting rigid substructures in spatial nucleationfree spherical-spherical bar mechanisms based on kinematical loops

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Abstract

Localization of rigid substructures in multibody systems is still an open research field with many applications in mechanism analysis and synthesis, especially for so-called "frameworks" consisting of spherical-spherical rods. While for planar systems effective methods based on Laman's theorem [1] exist, the 3D case still poses open challenges. In this paper, a new approach for rigidity detection [4] (used for solving the "double-banana" case) based on modeling of multibody systems as systems of interconnected loops [2] is applied to the recently presented challenge of so-called nucleationfree mechanisms [3], which contain intrinsically movable substructures whose rigidity can be detected only from the global couplings.

The regarded system [3] consists of seven topologically identical "roofs" (Fig. 1a), each representing a spherical mechanism that can be deformed by moving the endpoints (P1/P2 or Q1/Q2) along their connecting line (termed "implied edge" in [3]) and which can rotate about the implied edges without deformation. By connecting seven roofs as in Fig. 1b) in a cycle together, a mechanism consisting of altogether 56 rods and 91 spherical-spherical constraints is obtained, yielding $336-273=63$ degrees of freedom (DOF), composed by 56 isolated DOF of the rods plus 6 rigid-body DOF plus one proper DOF for the internal mobility of the whole mechanism. Obviously, this mobility corresponds to the 1-DOF of the 7R loop L_0 in which the roofs are now rigid and can only rotate about their implied edges (Fig. 1c).

While it is very difficult to detect rigidity of the roofs in the cycle by conventional DOF counting algorithms, tracking of isolated DOFs within the loops and their couplings makes this identification easy. To this end, regard the mechanism as a system of smallest independent loops [2], which are here e.g. the four triangle loops $L_1,\ldots,L_4$ for each roof, the loops L_5 between two roofs, and the internal overall loop L_0 (36 in total). Regarding the loops in a first step as unconnected, the loops $L_1,\ldots,L_4$ have three, the loop L_5 has six, and the loop L_0 has 15 internal degrees of freedom, respectively (marked by "f=i" in the upper left sub-box of the loop boxes in Fig. 3). At multiple joints in which the number of incident loops is equal or larger than the number of incident bodies, couplings between the internal variables of the loops occur, corresponding to the condition that the result of the concatenation of relative motions over all incident loops is constant [2]. In the present case, joints "B" induce a coupling between loops L_0 and L_5 and the loops L_1 of two neighboring roofs, while joints A induce two couplings, one between loops L_2, L_4 and L_5, and one between loops L_3, L_4 and L_5, respectively, which interchange cyclically between the roofs. Each coupling implies that the sequence of 3D rotations is equal to the unit matrix, yielding three scalar

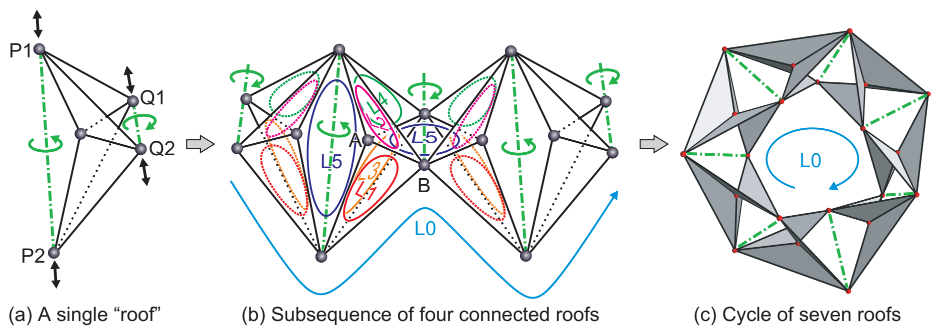

Figure 1: Connecting seven roofs with one DOF each to a cycle with 1 DOF in total

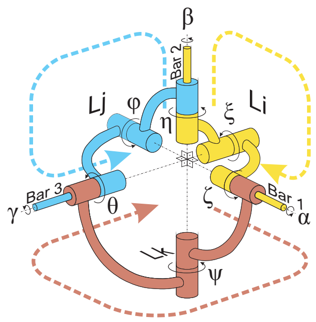

Figure 2: Loop coupling at a spherical joint

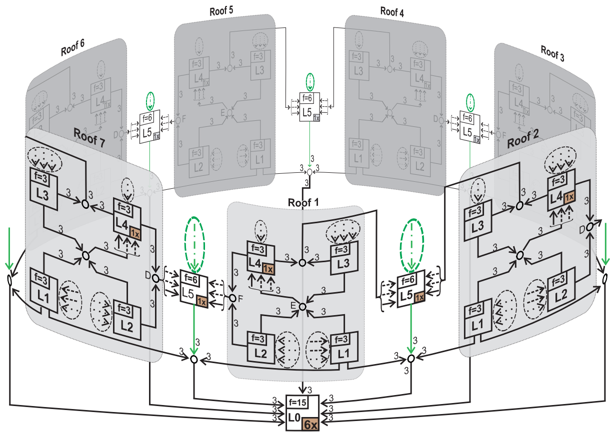

Figure 3: Kinematical network of 7-roof mechanism

constraints [4]. Putting together all loops and their couplings (shown as summing junctions), the cyclic block diagram of Fig. 3 is obtained. Note that by summing up the internal degrees of freedom of all loops and subtracting the coupling conditions (3 per summing junction) yields for the global DOF=57, which corresponds to the 56 isolated rod DOF and 1 DOF of the cycle. Thus the loop connection graph removes automatically the overall 6 rigid DOF and is isomorphic to the multibody representation.

The rest of the algorithm consists in finding proper inputs for the loops such that the local DOF of each loop is respected and that there are no closed cycles in the directed edges [2]. If this is not possible, additional (pseudo) inputs are selected at some loops from which implicit constraints result in some loops downstream (shown as "*n*x" in the lower right sub-box of the loop boxes in Fig. 1). For systems of spherical-spherical rods, there are three types of inputs: (a) "fully" isolated DOFs (dotted lines), which are immaterial and which can be removed without effect from the mechanism; (b) "transmitted" isolated DOFs (dashed lines), which have been already counted once as fully isolated in one loop and become transmitted in a neighboring loop; and "structural" isolated DOFs (dot-dashed lines), which leave adjacent subchains within a loop invariant but operate to the 'outside' of the loop as proper DOFs (here rotations about the induced edges). The difference between fully isolated and transmitted isolated DOFs can be seen from Fig. 2. Assuming that rotations of bars 1 and 2 are counted as fully isolated in loop L_i (α, β), the isolated rotation about bar 2 in loop L_j (η) is now material, and hence termed "transmitted". If loop L_j registers the rotation about bar 3 (γ) as fully isolated, than the isolated rotations about bars 1 and 3 in loop L_k (ζ, θ) are again material, and thus transmitted. Note that the third connected rotation in loop L_k (ψ) is a proper transmitted angle that in the case e.g. of a triangle loop will be subject to an implicit constraint equation. Choosing isolated DOF as inputs for loops L_1,L_2 and L_3 for each roof, yields the transmission structure displayed in Fig. 3. By removing the fully and structurally isolated DOFs and tracking only the transmitted isolated DOF (two at loop L_1 per roof) and the implicit constraint equations (14 in total within the cycle of roofs), it is detected that all loops $L_1, \ldots, L_4$ are locked, and thus that only the structural isolated DOF of rotations about the implied edges are movable, showing that the mechanism is a 7R loop. The developed methods are currently being implemented in Mathematica.

References

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