Quadratic manifolds for reduced order modelling of highly flexible multibody systems

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Abstract

The floating frame of reference (FFR) provides a natural framework for the model order reduction of flexible multibody systems. The global response of a body can be split into a global rigid motion $\mathbf{q}_r$ and a local elastic displacement $\mathbf{q}_f$, which can in turn be linearly mapped into a small subspace spanned by the columns of a matrix $\mathbf{V}$, as

$$\mathbf{q}_f = \mathbf{V}\boldsymbol{\eta} \tag{1}$$

where $\boldsymbol{\eta}$ is the vector of modal amplitudes. This approach is very effective when the elastic deflections are small. In this case, the basis $\mathbf{V}$ can be formed with few vibration modes (VMs). However, when *elastic geometric nonlinearities* have to be considered, VMs fail to correctly reproduce nonlinear couplings and large deflections and their use is therefore of limited applicability.

In an earlier contribution [1] we showed that such reduction basis can be enriched with Modal Derivatives (MDs) stemming from the sensitivity of the eigenvalue problem for free vibration with respect to modal amplitudes:

$$\mathbf{V} = \left[\boldsymbol{\Phi}_1, \ldots, \boldsymbol{\Phi}_m, \ldots \frac{\partial \boldsymbol{\Phi}_i}{\partial \eta_j}, \ldots\right], \tag{2}$$

where $\boldsymbol{\Phi}_i$ and $\frac{\partial \boldsymbol{\Phi}_i}{\partial \eta_j}$ are the VMs and the MDs, respectively. This reduction correctly captures the nonlinear bending-stretching behaviour associated to elastic geometric nonlinearities. Representatives results are shown in Figure 1. While simple and effective, this approach bears the drawback of a relatively large reduction basis, as the number of obtainable MDs scales *quadratically* with the size of the corresponding set of VMs.

We present in this contribution an alternative approach based on a quadratic manifold for the reduction.The local elastic displacement vector $\mathbf{q}_f$ is expanded in the direction of the chosen VMs Φ_i, $i = 1, \ldots, m$,

$$\mathbf{q}_f(t) = \boldsymbol{\Phi}_i \eta_i(t) + \boldsymbol{\Theta}_{ij} \eta_i(t) \eta_j(t), \tag{3}$$

where the quadratic terms $\boldsymbol{\Theta}_{ij}$ are related to the MDs.

The proposed reduction is therefore a manifold of size m. As a consequence of the nonlinearity of the reduction, the projection basis for the elastic displacement $\boldsymbol{q}_f$ is state dependent and generates convective-like terms and configuration dependent mass matrix. The resulting reduced equations for an arbitrary flexible multibody system can be written as

$$\tilde{\mathbf{M}}(\mathbf{q_r}, \boldsymbol{\eta})\ddot{\boldsymbol{\eta}} + \tilde{\mathbf{Q}}_v(\dot{\mathbf{q}}_r, \dot{\boldsymbol{\eta}}) + \tilde{\mathbf{Q}}_{nl}(\boldsymbol{\eta}) + \tilde{\mathbf{C}}_\partial \boldsymbol{\lambda} = \tilde{\mathbf{Q}}_{ext}, \tag{4}$$

complemented with the constraint equation $\mathbf{C}(\mathbf{q}_f, \boldsymbol{\eta}) = \mathbf{0}$. The $\tilde{\star}$ refers to the projected quantities, $\tilde{\mathbf{Q}}_v$, $\tilde{\mathbf{Q}}_{nl}$ and $\tilde{\mathbf{Q}}_{ext}$ are the reduced convective, elastic, and external forces, respectively, and $\boldsymbol{\lambda}$ are the Lagrange multipliers associated to the constraints.

The construction of the reduction basis is illustrated in Figure 2 for a rotating, planar beam in the FFR, with a nodal-fixed frame as described in [2]. In this case, the lower-frequency VMs feature bending displacement only, while the corresponding MDs, which describe the second-order nonlinearities, exhibit longitudinal displacements.

Preliminary attempts for the reduction of the transient analysis of flexible nonlinear structures in a Lagrangian framework show excellent performances of the proposed method. In this contribution, we will present a comparison of reduced order models for highly flexible multibody obtained with a linear and quadratic manifold on representative highly flexible multibody systems featuring elastic geometric nonlinearities and arbitrary rigid rotations.

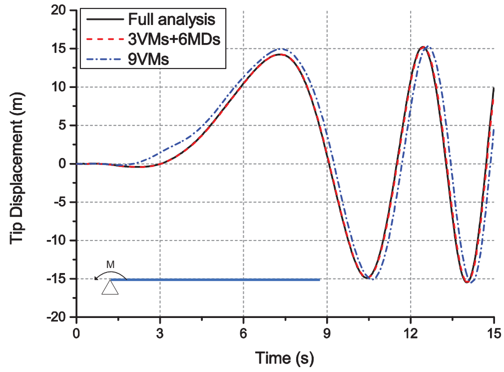

Figure 1: Vertical tip displacement of a rotating beam subjected to constant moment. The full solution is compared to a the reduced solution obtained with (a) 3 VMs and 6 MDs, and (b) 9 VMs, respectively. Although some axial VMs are contained in the basis for case (b), the MDs corresponding to the first 3 VMs (case (a)) yield more accurate results.

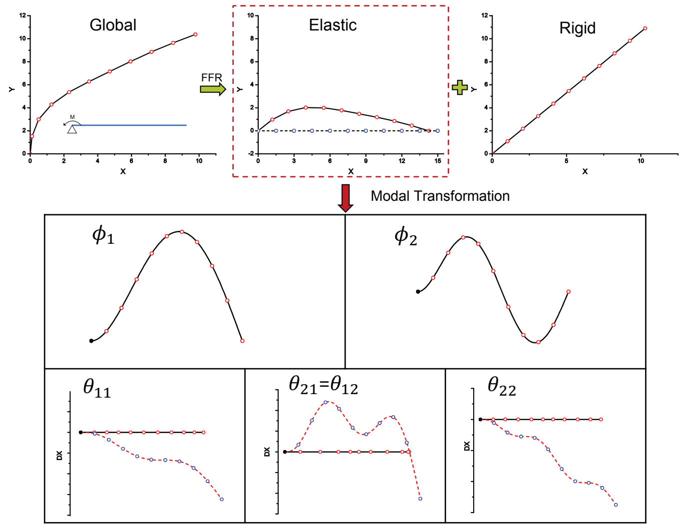

Figure 2: Modal basis extraction procedure for a rotating beam in FFR. The axial displacement DX (red dot line with blue node) is plotted as a function of node position.

References

[1] L. Wu, P. Tiso. Modal derivatives based reduction method for large deflections in floating frame. In E. O˜nate, J. Oliver and A. Huerta (Ed.) Proceedings of 11th World Congress on Computational Mechanics, pp. 3125–3135, Barcelona, Spain, 2013.

[2] A. A. Shabana.Dynamics of Multibody Systems. Cambridge University Press, Cambridge, 2005.