Quadratic manifolds for reduced order modelling of highly flexible multibody systems

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Abstract
The floating frame of reference (FFR) provides a natural framework for the model order reduction of flexible multibody systems. The global response of a body can be split into a global rigid motion \( q_r \) and a local elastic displacement \( q_f \), which can in turn be linearly mapped into a small subspace spanned by the columns of a matrix \( V \), as

\[
q_f = V \eta
\]

where \( \eta \) is the vector of modal amplitudes. This approach is very effective when the elastic deflections are small. In this case, the basis \( V \) can be formed with few vibration modes (VMs). However, when elastic geometric nonlinearities have to be considered, VMs fail to correctly reproduce nonlinear couplings and large deflections and their use is therefore of limited applicability.

In an earlier contribution [1] we showed that such reduction basis can be enriched with Modal Derivatives (MDs) stemming from the sensitivity of the eigenvalue problem for free vibration with respect to modal amplitudes:

\[
V = [\Phi_1, \ldots, \Phi_m, \ldots, \partial \Phi_i / \partial \eta_j, \ldots]
\]

where \( \Phi_i \) and \( \partial \Phi_i / \partial \eta_j \) are the VMs and the MDs, respectively. This reduction correctly captures the nonlinear bending-stretching behaviour associated to elastic geometric nonlinearities. Representatives results are shown in Figure 1. While simple and effective, this approach bears the drawback of a relatively large reduction basis, as the number of obtainable MDs scales quadratically with the size of the corresponding set of VMs.

We present in this contribution an alternative approach based on a quadratic manifold for the reduction. The local elastic displacement vector \( q_f \) is expanded in the direction of the chosen VMs \( \Phi_i \), \( i = 1, \ldots, m \),

\[
q_f(t) = \Phi_i \eta_i(t) + \Theta_{ij} \eta_i(t) \eta_j(t),
\]

where the quadratic terms \( \Theta_{ij} \) are related to the MDs. The proposed reduction is therefore a manifold of size \( m \). As a consequence of the nonlinearity of the reduction, the projection basis for the elastic displacement \( q_f \) is state dependent and generates convective-like terms and configuration dependent mass matrix. The resulting reduced equations for an arbitrary flexible multibody system can be written as

\[
\ddot{M}(q_r, \eta) \dot{\eta} + \dot{Q}_v(q_r, \dot{\eta}) + \dot{Q}_{el}(\eta) + \dot{C}_\lambda \lambda = \dot{Q}_{ext},
\]

complemented with the constraint equation \( C(q_f, \eta) = 0 \). The \( \dot{\cdot} \) refers to the projected quantities, \( \dot{Q}_v \), \( \dot{Q}_{el} \) and \( \dot{Q}_{ext} \) are the reduced convective, elastic, and external forces, respectively, and \( \lambda \) are the Lagrange multipliers associated to the constraints.

The construction of the reduction basis is illustrated in Figure 2 for a rotating, planar beam in the FFR, with a nodal-fixed frame as described in [2]. In this case, the lower-frequency VMs feature bending displacement only, while the corresponding MDs, which describe the second-order nonlinearities, exhibit longitudinal displacements.

Preliminary attempts for the reduction of the transient analysis of flexible nonlinear structures in a Lagrangian framework show excellent performances of the proposed method. In this contribution, we will present a comparison of reduced order models for highly flexible multibody obtained with a linear and quadratic manifold on representative highly flexible multibody systems featuring elastic geometric nonlinearities and arbitrary rigid rotations.
Figure 1: Vertical tip displacement of a rotating beam subjected to constant moment. The full solution is compared to a the reduced solution obtained with (a) 3 VMs and 6 MDs, and (b) 9 VMs, respectively. Although some axial VMs are contained in the basis for case (b), the MDs corresponding to the first 3 VMs (case (a)) yield more accurate results.

Figure 2: Modal basis extraction procedure for a rotating beam in FFR. The axial displacement DX (red dot line with blue node) is plotted as a function of node position.

References