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On the numerical efficiency of model reduction and high-order discontinuous Galerkin for waves and fluid problems

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ABSTRACT

Nowadays, many problems in engineering and computational science are particularly hard to deal with, and in some cases remain intractable, with standard techniques. Due to the physical and numerical complexity, multidimensional character and other restrictions (requirements coming from optimization purposes, real-time constraints, high fidelity of simulations...), new methods are investigated. In particular, improving the numerical efficiency of the computations is mandatory if accurate solutions are needed.

The presented work studies the efficiency of the following numerical techniques:

1. *Discontinuous Galerkin (DG) methods*, and in particular, hybridizable discontinuous Galerkin (HDG). Specifically, a p-adaptive strategy is proposed, in combination with the (a) HDG superconvergence property for the primal unknown, (b) high-order approximations and (c) hybridization of the globally coupled degrees of freedom. The crucial point remains in using cheaper error estimations based on the HDG postprocessed solution.
2. *Reduced order models.* For those problems defined in highly multidimensional spaces, standard finite elements basis results in massive systems of equations. Here an a priori reduced order modeling via proper generalized decompositions (PGD) of the unknown field is considered. The efficiency of several PGD solvers is compared when low-fidelity approximations of the multidimensional solution are of interest.

Firstly, unbounded wave propagation problems in frequency domain are studied, thus Helmholtz-type equations are considered. In the context of scattering problems with multiple incident wave parameters (i.e. frequency and direction of incidence), the parametrized 4D solutions are approximated with PGD. Interesting engineering problems, such as agitation in harbors, can be modeled in this framework. Application of PGD is particularly challenging here because it implies difficulties for the a priori decomposition due to the loss of elliptic character in the Helmholtz operator, specially when dealing with high frequencies.

Secondly, when solutions for a single incident wave are of concern, p-adaptive HDG provides accurate solutions with important improvements on the efficiency compared with p-homogeneous techniques. Moreover, high-order approximations are crucial to reduce the dispersion error and to guarantee efficient computations for real applications.

Thirdly, the p-adaptive HDG is also used to solve challenging fluid problems. The adaptive algorithm greatly simplify the design of the initial computational mesh, reducing for instance the need of highly distorted elements for capturing the boundary layer, and it is particularly suited for moving geometries and reference frames. Applications to examples of engineering interest, such as the flow around a rotating wind turbine, are shown.