

SPECTRA OF COMPUTED FABRIC STRESS AND DEFORMATION VALUES DUE TO A RANGE OF FICTITIOUS ELASTIC CONSTANTS OBTAINED FROM DIFFERENT ESTABLISHED DETERMINATION PROCEDURES

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Summary. The aim of the present paper is to present ways how lower and upper limits for the range of possible fictitious elastic constants (tensile stiffness and Poisson's ratio) can be determined for one fabric material on the basis of different established test and determination procedures. In the structural analysis this range of fictitious elastic constants results in a spectrum of computed stress and deformation values for one and the same structure and load case. The possible range of structural analysis results due to that variety of possible stiffness parameters is demonstrated for one exemplary membrane structure with different magnitudes of curvature.

1 INTRODUCTION

Although woven fabrics show a highly non-linear load-strain-relationship under uniaxial or biaxial tension, it is common in the daily practice, that simple elastic constants are used in the structural analysis of membrane structures. As such a set of maximal three independent elastic constants is not able to cover the complex biaxial load-strain-behaviour of fabrics, elastic constants have to be seen as fictitious stiffness parameters. However, usually biaxial tensile tests are conducted in order to determine these fictitious elastic constants from the resulting load-strain-paths. But: different existing test procedures^{1,12} and different determination procedures^{2,3,4} may lead to a wide range of values for the resulting fictitious elastic constants. Furthermore, different interpretations of the established procedures are common.

In the recent past^{5,10} it could be demonstrated that the material behaviour is of great importance for the structural analysis of woven fabrics, especially with regard to the nowadays more and more minimally curved or even flat structures. The present contribution will give a state of the art report of the currently published test and determination procedures used for the characterization of the material behaviour of woven fabrics. The aim is to present a way how lower and upper limits for the range of possible fictitious elastic constants can be determined for one fabric material based on test data obtained from the different established test and determination procedures. In the structural analysis this range of fictitious elastic constants results in a spectrum of computed stress and deformation values for one and the

same structure. For one exemplary basic form of a tensile structure – a simple hyper – the spectrum of computed stresses and deformations, which can possibly occur for a Glass/PTFE-material in the design practice, will be determined by means of the obtained ranges of fictitious elastic constants.

2 THE ORTHOTROPIC LINEAR-ELASTIC CONSTITUTIVE LAW

For the use in a structural analysis where the membrane is modeled as a continuum – and not as a cable net –, the actual anisotropic highly nonlinear stress-strain-behaviour of woven fabrics is considered as a linear-elastic orthogonal anisotropic plane-stress structure. For practical reasons, nowadays the design engineers are forced to use this assumption in commercial as well as inhouse design software, although it is known that this procedure is a rather rough approximation. One possible mathematical formulation for the load-strain-relationship – known from classical mechanics – is given with the following elementary equations

$$\varepsilon_x = \frac{n_x}{E_x t} - \nu_{xy} \frac{n_y}{E_y t}, \quad (1)$$

$$\varepsilon_y = \frac{n_y}{E_y t} - \nu_{yx} \frac{n_x}{E_x t}. \quad (2)$$

Herein, ε are the strains [-] and n are the loads [kN/m], which are often called stresses in membrane structure analysis. The four elastic constants are: $E_x t$ as the tensile stiffness in warp direction [kN/m] and $E_y t$ in fill direction, respectively. Generally, the axes x and y refer to the warp and the weft (fill) yarn direction of the fabric. The transverse strains are taken into account by the Poisson's ratio ν . ν_{xy} is the Poisson's ratio in x -direction caused by a load in y -direction, ν_{yx} applies analogue in perpendicular direction. Transposed to the loads n and written with matrices this law becomes

$$\begin{bmatrix} n_x \\ n_y \end{bmatrix} = \frac{1}{1 - \nu_{xy} \cdot \nu_{yx}} \begin{bmatrix} E_x t & \nu_{xy} \cdot E_x t \\ \nu_{yx} \cdot E_y t & E_y t \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \end{bmatrix}. \quad (3)$$

The linking matrix between the loads on the left side and the strains on the right side of the equation is the stiffness matrix. The stiffness matrix has to be symmetric, what directly leads to

$$\nu_{xy} \cdot E_x t = \nu_{yx} \cdot E_y t \quad \Rightarrow \quad \nu_{yx} = \nu_{xy} \cdot \frac{E_x t}{E_y t}. \quad (4)$$

It can be seen from eq. (4) that only three of the four elastic constants are independent of each other. Furthermore, the stiffness matrix has to be positive definite, which means that the tensile stiffnesses and the determinate of the stiffness matrix have to be positive. The latter constraint leads to

$$\nu_{xy} \cdot \nu_{yx} < 1. \quad (5)$$

Another possible formulation of the constitutive law can be given by

$$\begin{bmatrix} n_{11} \\ n_{22} \end{bmatrix} = \begin{bmatrix} E_{1111} & E_{1122} \\ E_{1122} & E_{2222} \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \end{bmatrix}. \quad (6)$$

Herein E_{1111} and E_{2222} are the tensile stiffnesses in warp and weft direction, respectively, and E_{1122} is the stiffness interaction between warp and weft direction. In the notation of eq. (6) the stiffness matrix is directly symmetric. The two mathematical formulations in eq. (3) and (6) of the same constitutive law result in identical analysis results. Special attention has to be paid, as the numerical values of the elastic constants of both definitions are not equal⁶. But the elastic constants of both definitions can easily be converted by the following equations:

$$v_{xy} = \frac{E_{1122}}{E_{1111}}, \quad (7)$$

$$v_{yx} = \frac{E_{1122}}{E_{2222}}, \quad (8)$$

$$E_x t = E_{1111} \cdot (1 - v_{xy} \cdot v_{yx}), \quad (9)$$

$$E_y t = E_{2222} \cdot (1 - v_{xy} \cdot v_{yx}). \quad (10)$$

3 BIAXIAL TESTS AND THE DETERMINATION OF ELASTIC CONSTANTS

Many different published and unpublished biaxial test procedures and related evaluation procedures exist today. Two common test procedures are described in the Japanese standard MSAJ/M-02-1995² and in the TensiNet European Design Guide for Tensile Surface Structures⁴. Additional test procedures are described in [1,12]^{1,12}. Furthermore, unpublished, office specific as well as project specific test procedures are oftentimes used by the engineering design offices. To every test procedure one or more related evaluation procedures exist to determine elastic constants from the biaxial test results. This situation leads to a confusing variety of elastic constants.

Selected test and evaluation procedures, based on the common recommendations of MSAJ/M-02-1995 and the TensiNet Design Guide, are introduced in the next paragraphs. The objective of both recommendations is to determine one single set of “fictitious” elastic constants from the biaxial test results which are intended to be used for the practical structural analyses of all kinds of structural forms and all load cases.

3.1 The Japanese standard MSAJ/M-02-1995

The main characteristic of the Japanese standard MSAJ/M-02-1995 is that five different predefined load ratios warp:fill – 1:1, 2:1 1:2, 1:0 and 0:1 – are consecutively applied on a cross shaped test specimen with the yarns parallel to the arms of the cross. During the loading and unloading procedure the load ratio warp:fill is held constant. The maximum tensile test load is fixed to ¼ of the maximum strip tensile strength of the material. The result of this test

procedure is a load-strain-diagram as exemplarily shown in figure 1(a). From this complete set of test data ten load-strain-paths can be extracted – one for each yarn direction for the five load ratios –, see figure 1(b).

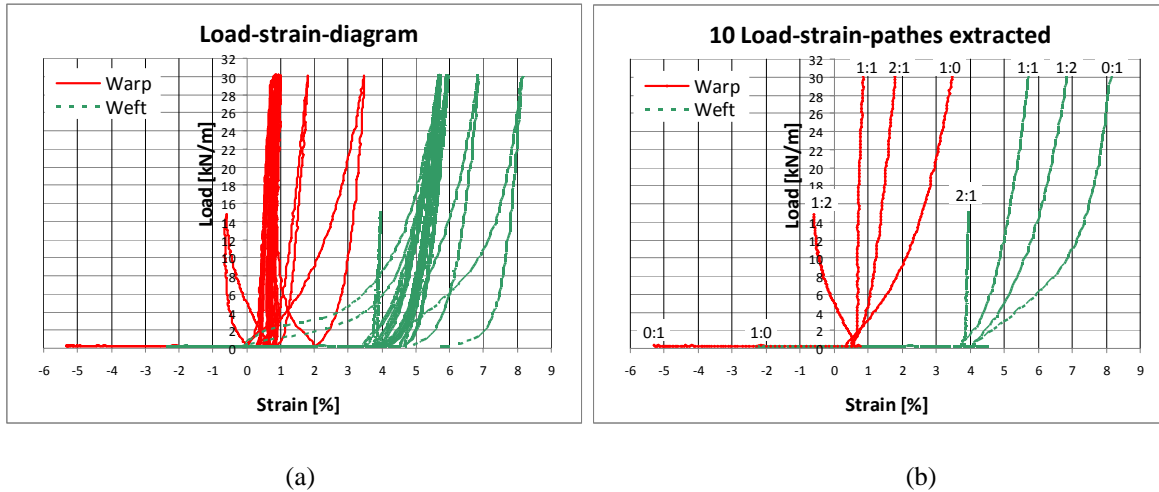


Figure 1: (a) Load-strain-diagram as a result of a MSAJ biaxial test on Glass/PTFE material, (b) ten load-strain-paths (warp/weft at five load ratios), as extracted from the diagram

The commentary of MSAJ/M-02-1995, which is an inherent component of the standard, recommends to determine one single design set of elastic constants from the extracted load-strain-paths stepwise in a double step correlation analysis. In the first step each curved loading path has to be substituted by a straight line. In the second step the slopes of the straight lines obtained in the first step have to be modified in such a way that they satisfy the equations of the assumed linear-elastic constitutive law. The MSAJ-commentary uses the formulation of eq. (1) and (2) or eq. (3), respectively. To determine the “optimum” set of elastic constants several methods are proposed, e.g. “least squares method minimizing the sum of squares of the strain term”, “least squares method minimizing the sum of squares of the stress term” and other simplified methods. The resulting sets of elastic constants differ more or less. As this procedure can not be solved “by hand”, a correlation analysis routine has been programmed at the Institute for Metal and Lightweight Structures⁷. However, the MSAJ-commentary recommends to disregard the zero-load-paths, i.e. the weft-path at 1:0 and the warp-path at 0:1, so that only eight out of the ten extracted load-strain-paths are used for the determination of the elastic constants. According to the commentary, this is because the testing method had low repeatability of test results in the low stress range.

3.2 The TensiNet European Design Guide

The TensiNet Design Guide⁴ proposes a completely different loading procedure. An appreciable prestress is initially applied to the test specimen and is hold constant for a – undefined – period of time. After that, one direction of the cross shaped specimen (e.g. warp) is loaded while the perpendicular direction (weft) holds the constant prestress at the same time. This means, that the load ratio warp:weft changes continuously while loading and is not

constant over time like in the MSAJ-procedure. The loading procedure is repeated five times. Afterwards the procedure is inverted, i.e. the weft direction is loaded five times while the warp direction holds the constant prestress. This loading procedure shall approximate a typical loading by wind followed by a snow loading – or the other way around – in an anticlastic structure. The decision on the amount of prestress and the maximum test load is left to the design engineer in each case – although some recommendations are given.

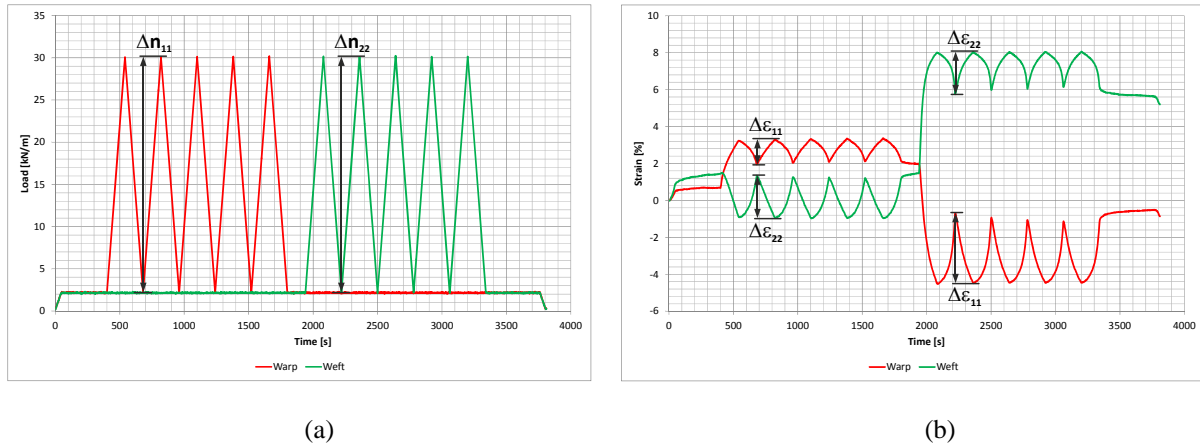


Figure 2: (a) Exemplary loading procedure according to TensiNet Design Guide, (b) strain-time-diagram in warp and weft direction as the result

For the determination of elastic constants the constitutive law as stated in eq. (6) is used. In each loading direction, one of the five load steps is chosen to read out the corresponding strain differences $\Delta \epsilon$ in each fabric direction, see figure 2, firstly for the loading interval Δn_{11} (where $\Delta n_{22}=0$) and secondly for the loading interval Δn_{22} (where $\Delta n_{11}=0$). On the basis of eq. (6), herewith the following eqs. (11) and (12) can be filled for the first part and eqs. (13) as well as (14) for the second part of the loading procedure:

$$\Delta n_{11} = E_{1111} \Delta \epsilon_{11} + E_{1122} \Delta \epsilon_{22} \quad (11)$$

$$0 = E_{1122} \Delta \epsilon_{11} + E_{2222} \Delta \epsilon_{22} \quad (12)$$

$$0 = E_{1111} \Delta \epsilon_{11} + E_{1122} \Delta \epsilon_{22} \quad (13)$$

$$\Delta n_{22} = E_{1122} \Delta \epsilon_{11} + E_{2222} \Delta \epsilon_{22} \quad (14)$$

As the result, four equations with the three unknown elastic constants are established. A usual practical approach to solve this – mathematically unsolvable – problem is to determine two sets of elastic constants and average the results.

4 SPECTRUM OF FICTITIOUS DESIGN STIFFNESS PARAMETERS

Recently a discussion on the determination of elastic constants had been started and modifications of existing evaluation procedures have been proposed, resulting in a great

variety of values for the “fictitious” elastic constants. In this paragraph, some selected sets of elastic constants are presented, which can be obtained for one and the same exemplary material. For this purpose, a Glass/PTFE-material with a tensile strength of 140/120 kN/m in warp and weft direction has been tested in the Essen Laboratory for Lightweight Structures (ELLF). On the basis of these tests, different sets of elastic constants have been determined at the Institute for Metal and Lightweight Structures (IML) at the University of Duisburg-Essen. All test specimens were taken from one batch. The results are given in tables 1 and 2, starting with the set of elastic constant obtained from the original MSAJ-test and determination procedure, see determination option (DO) 1.

Table 1: Different sets of elastic constants obtained by different determination options from one and the same set of MSAJ-test data for a Glass/PTFE-material with a tensile strength of 140/120 kN/m

Determination option (DO)		Tensile stiffness [kN/m]		Poisson's ratio [-]		$\nu_{xy}\nu_{yx}$
		E_{xt}	E_{yt}	ν_{xy}	ν_{yx}	
1	Original MSAJ-determination: 8 load-strain-paths evaluated (zero-load-paths omitted)	1300	770	0.55	0.93	$0.51 < 1$
2	MSAJ modified: All ten load-strain-paths evaluated (Bridgens&Gosling) ⁸	930 (752)*	590 (611)*	0.82 (0.88)*	1.29 (1.08)*	$1.06 > 1$ ($0.95 < 1$)
3	Particular for plane structure: MSAJ-load-ratios 1:1, 2:1, 4 load-strain-paths (IML, Univ. of Duisburg-Essen) ⁷	1920	1020	0.42	0.79	$0.33 < 1$
4	Particular for anticlastic structure and load case with warp stressing: MSAJ-load-ratios 1:0, 2:1, 4 load-strain-paths (IML, Univ. of Duisburg-Essen) ⁷	890	240	0.49	1.82	$0.89 < 1$
min/max		890/ 1920	240/ 1020	0.42/ 0.82	0.79/ 1.82	0.33/ 1.06

* Values in brackets are directly taken from literature⁸. These values were determined by Bridgens & Gosling on the basis of biaxial tests conducted by themselves on the same material but from another batch.

Bridgens&Gosling⁸ have emphasized, that the zero-load-paths of the load ratios 1:0 and 0:1 – which are omitted in the MSAJ determination procedure, see above – are highly relevant for the critical design case of anticlastic membrane structures. Based on the biaxial test procedure of the MSAJ/M-02-1995 they have discussed results, which were obtained when taking the zero-load-paths into account. Due to mathematical reasons, the tensile stiffnesses decrease and the Poisson's ratios increase compared to the original MSAJ procedure (omitting the zero-load-paths), see DO 2. In the present determination, the product of the Poisson's ratios exceeds 1.0 and therefore, this set of elastic constants cannot be used in a structural analysis. Due to that the exemplary analysis in paragraph 5 will be conducted using the values in the brackets for DO2.

As for the load ratios 1:0 and 0:1 a good correlation between measured load-strain-paths

and calculated straight lines can (for Glass/PTFE-materials) only be obtained with big values for the Poisson's ratio ($\nu > 1$) while for other load ratios (1:1, 2:1) considerable smaller values are required (e.g. $\nu < 0.5$)⁹, it is impossible to model all load-strain-paths with only one single set of elastic constants. This problem can be solved if the elastic constants are determined particularly for a specific structure and a specific load case⁷. This means, e.g. for an anticlastic membrane structure with predominant warp stressing under one load case, that the load ratios 2:1 and 1:0 might be reasonable. In this case, the load ratios 1:2 and 0:1 have to be picked out for opposite loading. For plane and synclastic structures as well as anticlastic structures with very small curvature, the load ratios 1:1 and 2:1 fit best. For the exemplary analysed structures in paragraph 5 this proposal leads to the set of elastic constants shown under DO 3 for the plane structure and DO 4 for the two anticlastic structures.

To determine elastic constants according to the TensiNet Design Guide, two tests have been conducted in the Essen Laboratory for Lightweight Structures. To enable a direct comparability to the MSAJ-procedure, firstly, a biaxial test with the same maximum tensile load of 30 kN/m as for the MSAJ-test has been chosen. The biaxial test has been conducted with material from the same batch as for the MSAJ-test. The prestress has been chosen to be 2 kN/m in each fabric direction so that it equals the minimum load of the load-strain-path on which the MSAJ-determination is based on. This value is fixed by the MSAJ-commentary to be 2 kN/m for Glass/PTFE-materials. Elastic constants have been determined with the second loading cycle. The results are shown in table 2 for both: as defined in eq. (6) and for a better comparability as defined in eq. (3), too. The determination results reveal much bigger elastic constants than those obtained by the MSAJ procedure, for the tensile stiffness as well as for the Poisson's ratios. The latter ones give a product $\nu_{xy}\nu_{yx} > 1$, which means, that this set of constants is unfeasible for a structural analysis, see explanations above.

In the second test, the maximum test load range was much smaller. Oriented towards the expected maximum membrane stress, a value of 13 kN/m was chosen. The prestress has been chosen to be 3 kN/m in each fabric direction as supposed to be in the exemplary structure analysed in paragraph 5. From the results in table 2 it can be seen that all values of the elastic constants decrease compared to the first test procedure. However, the product of the Poisson's ratios still clearly exceeds 1.0.

Table 2: Two sets of elastic constants obtained by the TensiNet Design Guide test and determination procedure with prestress 2 kN/m (Test N° 1) and 3 kN/m (Test N° 2) and maximum test loads of 30 kN/m (Test N° 1) and 13 kN/m (Test N° 2) for a Glass/PTFE-material with a tensile strength of 140/120 kN/m

Test N°	Elastic constants according to eq. (6) [kN/m]			Elastic constants according to eq. (3)				
	E ₁₁₁₁	E ₂₂₂₂	E ₁₁₂₂	Tensile stiffness [kN/m]		Poisson's ratio [-]		ν _{xy} ν _{yx}
				E _{x,t}	E _{y,t}	ν _{xy}	ν _{yx}	
1	-1180	-640	-1560	2460	1340	1.32	2.42	3.19 >> 1
2	-3015	-1190	-2360	1660	660	0.78	1.98	1.54 > 1

This problem occurs especially for Glass/PTFE-materials, which show considerable transverse strains, and especially for very big and very small values of load ratios warp:weft,

where the transverse strains show big absolute values. In the second conducted test the load ratio at maximum test load was $13:3 = 4.3$ (and $3:13 = 0.23$, respectively). For the analysed Glass/PTFE-material, the test and determination procedure of the TensiNet Design Guide leads even for that relatively low load differences between warp and weft to very high values of Poisson's ratios. Similarly high Poisson's ratios could be obtained from MSAJ test results, if tried to determine one set of elastic constants from the four load-strain-paths of the load ratios 1:0 and 0:1. It can be realized from this comparison, that it is difficult and probably quite often inappropriate to try to cover all loading situations (wind *and* snow) of a membrane structure with only one single set of elastic constants.

This enormous spectrum of elastic constants could be used by design engineers for one and the same material – excluding the unfeasible ones (table 1 DO2 and table 2) of course. Consequently, the question arises whether this spectrum of elastic constants has a significant influence on the stress and deformation results in the structural analysis or whether the influence is negligible. This question shall be answered in the following paragraph.

5 INFLUENCE OF THE FICTITIOUS STIFFNESS PARAMETER SPECTRUM ON THE STRUCTURAL ANALYSIS RESULTS

The quantitative influence of the spectrum of fictitious elastic constants obtained in paragraph 4 is exemplarily examined by means of a 10×10 m square hypar with two high points and two low points (a saddle shaped example is given by the authors, too¹⁰). The edges are fixed. Prestress is chosen to be isotropic with $p = 3.0$ kN/m in the main anisotropic fabric directions. The shear modulus is supposed to be $G = 50$ kN/m. The structural analysis is conducted with the finite element software package SOFiSTiK 2012¹¹ applying a third order analysis. The structure is vertical loaded downwards with $q = 0.60$ kN/m².

Three different curvatures are analysed, from $h = 0$ m (plane structure) up to $h = 4$ m, see figures 3 and 4. The warp direction is running between the high points, so that for the curved variations of the structure the warp direction is stressed for a downward load while in the weft direction the prestress decreases. Load ratios of approximately 4:1 and greater occur in the center of the structure. Thus, the four measured load-strain-paths of the MSAJ load ratios 1:0 and 2:1 are picked out to determine the elastic constants for DO 4 in table 1. The plane variation of the structure is characterized by load ratios between 1:1 and 2:1. Correspondent to that, elastic constants for DO 3 are determined based on the four load-strain-paths of these two load ratios.

Figure 4 shows the resulting membrane warp stress n_w as the result of the structural analyses for the three sets of elastic constants of DO 1 to DO 3 (DO3 is replaced by DO 4 for the curved structures, respectively) as warp stress (n_w)-Poisson's ratio (v_{xy})-diagrams. The stress value is always given for the middle of the membrane, although the maximum stress occurs sometimes at other locations. The n_w - v_{xy} -diagrams emphasize the importance of the Poisson's ratio. The Poisson's ratio v_{xy} corresponding to each set of elastic constants in table 1 are marked in the diagrams.

In the plane structure, the set of constants of DO1 results in $n_w = 12.3$ kN/m while DO2 results with $n_w = 22.5$ kN/m in an over 80% greater stress value, although the tensile stiffness is considerable smaller. The reason can immediately be identified in the n_w - v_{xy} -diagram as the

influence of the high value of Poisson's ratio ν_{xy} . In the curved structures with $h = 2.0$ m and $h = 4.0$ m the set of elastic constants from DO2 also results in 60%-75% greater stresses compared to the results from DO1. The results of DO3 (plane structure) and DO4 (curved structures) lay in between.

On the one hand, it can be seen from the curves closing ranks that with increasing curvature the influence of the material stiffness parameters decrease. But on the other hand, the concrete sets of elastic constants demonstrate their enormous importance, especially the high magnitudes of Poisson's ratios. This emphasizes the role of Poisson's ratio as part of a whole set of elastic constants. A comparison or assessment only of the tensile stiffnesses – as done sometimes – is not sufficient.

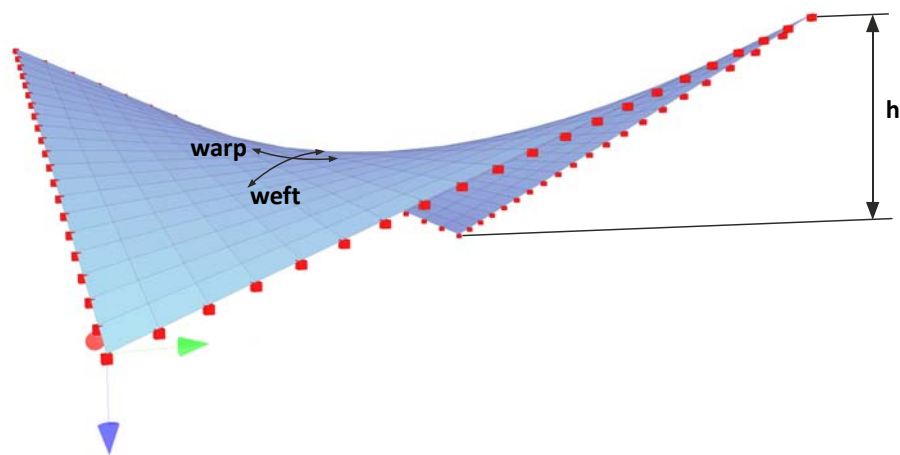


Figure 3: Simple hyper with 2 low points and 2 high points and fixed edges for the exemplary analyses

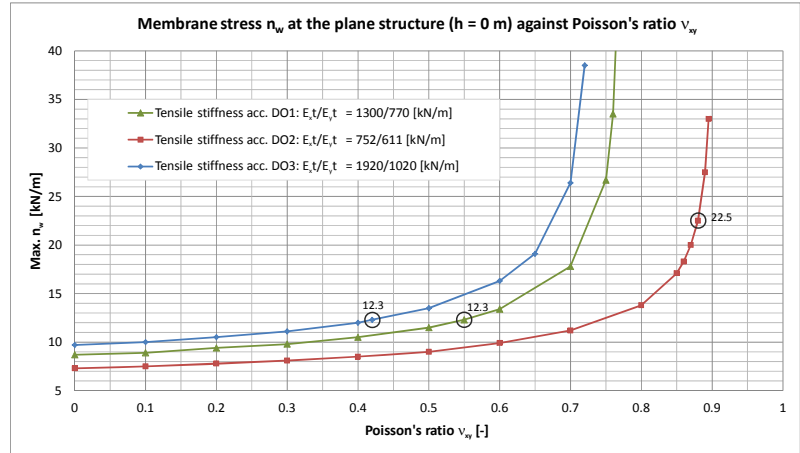
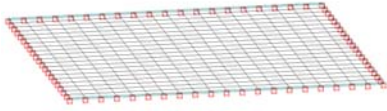
Figure 5 shows the influence of the spectrum of fictitious elastic constants on the deflection results. In the plane structure $\max f_z$ varies between 20 cm (DO2) and 39 cm (DO1 and 3), which is a variation of almost factor 2. For the curved structures, the deflections decrease considerably as expected. But the results also show a variation of 60%-70%. That means, that the deflections may possibly be underestimated by a factor of up to 2, which can lead to damages of the membrane in case of hitting the primary structure.

This exemplary structural analysis demonstrates the immense range of stress and deflection results due to a great variety of fictitious elastic constants that could be used by design engineers for one and the same material product. None of the underlying determination options is validated by static load tests on curved structural components, which means that the real stresses and deflections are left unknown to the engineer.

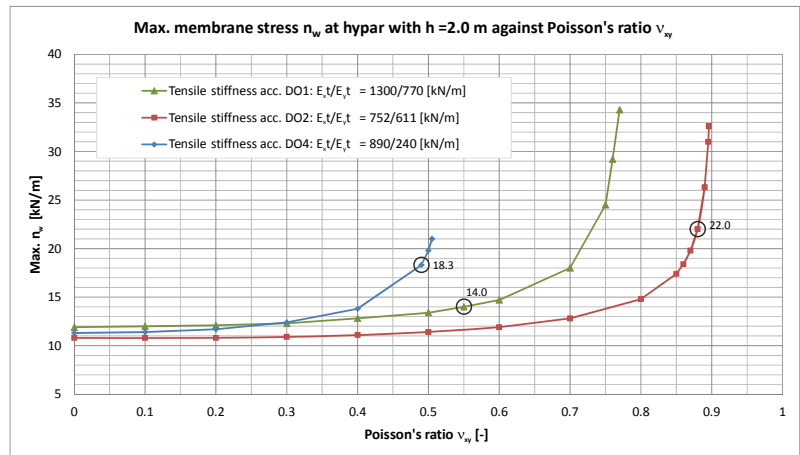
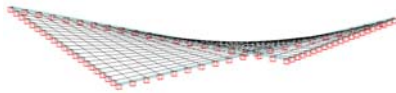
6 CONCLUSIONS

Nowadays, the highly nonlinear anisotropic material behaviour of woven fabrics is simplified to a linear-elastic orthotropic plane stress material in order to conduct the structural analysis. The existing variety of recommendations to determine elastic constants from biaxial test results – which was shown to result sometimes in unfeasible sets of elastic constants –

Hypar h = 0 m



Hypar h = 2.0 m



Hypar h = 4.0 m

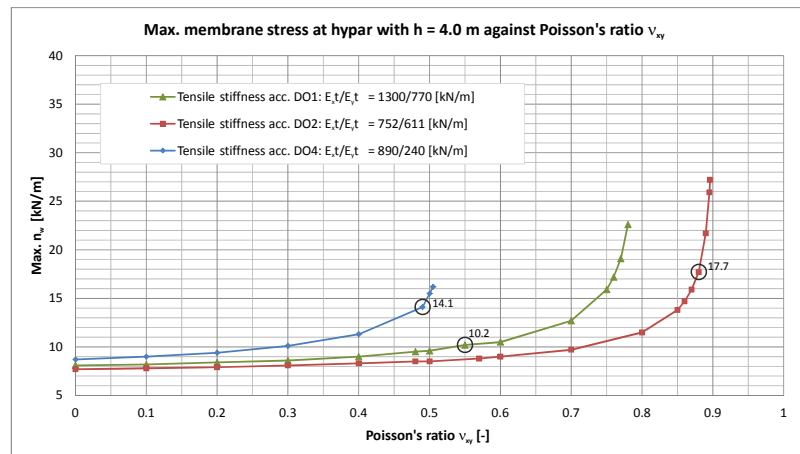
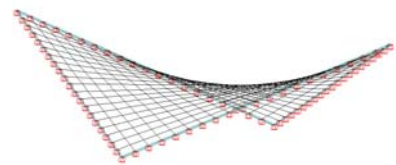
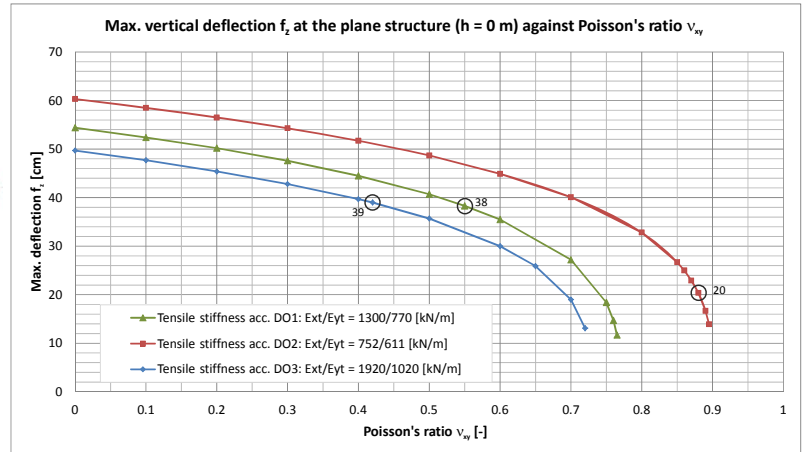
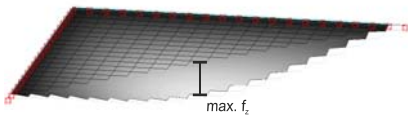
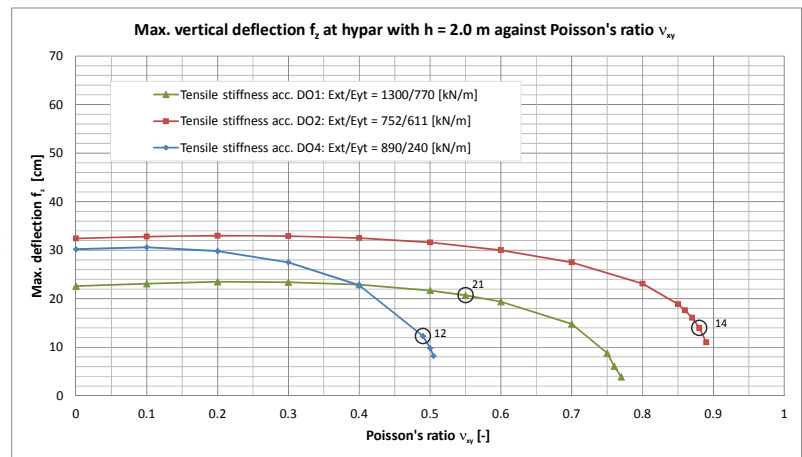
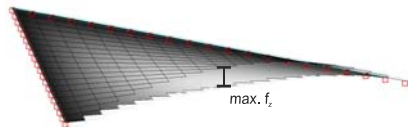


Figure 4: Maximum membrane stress n_w in the middle of hypar structures with three different curvatures obtained with three different sets of elastic constants from table 1

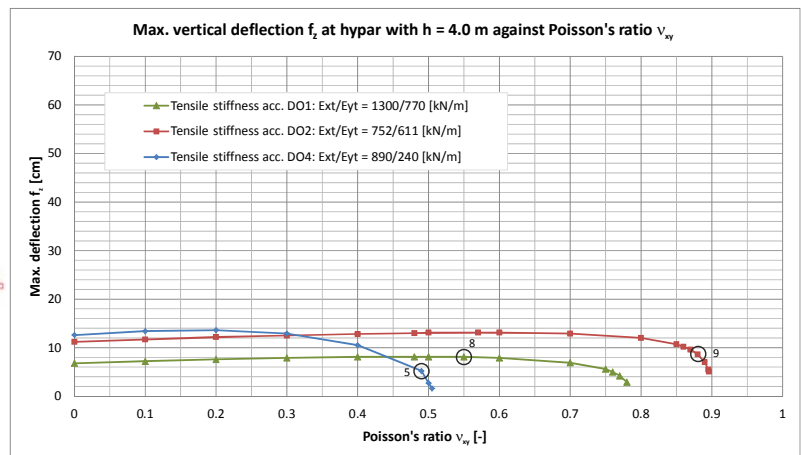
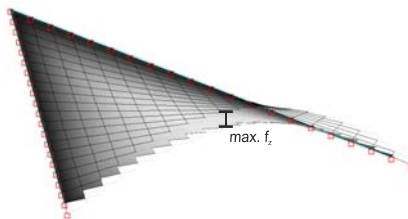
Hypar $h = 0$ m



Hypar $h = 2.0$ m



Hypar $h = 4.0$ m



Displacement illustrations are exaggerated

Figure 5: Maximum deflection f_z in the middle of hypar structures with three different curvatures obtained with three different sets of elastic constants from table 1

leads to a great spectrum of values, which design engineers can possibly use for their calculations for one and the same material. It was the aim of the present contribution to demonstrate the importance of this stiffness parameter spectrum on the stress and deformation results, which were found to be considerably high. Design engineers should have this issue on mind. The development of an European design standard for membrane structures as well as a European standard for biaxial testing – in which the authors are involved – is currently under way. This, together with the related research, hopefully leads to a better understanding for the determination of elastic constants and a more unified approach in the structural analysis.

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