STRUCTURAL MEMBRANES 2011 V International Conference on Textile Composites and Inflatable Structures

E. Oñate, B. Kröplin and K.-U.Bletzinger (Eds.)



Textile Composites and Inflatable Structures V Structural Membranes 2011

Textile Composites and Inflatable Structures V

Volume containing the full length papers accepted for presentation at the V International Conference on Textile Composites and Inflatable Structures, Barcelona, Spain 5-7 October 2011

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PREFACE

This volume contains the full length papers accepted for presentation at the V International Conference on Textile Composites and Inflatable Structures – Structural Membranes 2011, held in Barcelona, October 5-7, 2011. Previous editions of the conference were held in Barcelona (2003), Stuttgart (2005), Barcelona (2007) and Stuttgart (2009). Structural Membranes is one of the Thematic Conference of the European Community in Computational Methods in Applied Science (ECCOMAS www.cimne.com/eccomas/) and is also a Special Interest Conference of the International Association for Computational Mechanics (IACM http://www.cimne.com/iacm/).

Textile composites and inflatable structures have become increasingly popular for a variety of applications in – among many other fields - civil engineering, architecture and aerospace engineering. Typical examples include membrane roofs and covers, sails, inflatable buildings and pavilions, airships, inflatable furniture, airspace structures etc.

The objectives of **Structural Membranes 2011** are to collect and disseminate state-of-the-art research and technology for design, analysis, construction and maintenance of textile and inflatable structures.

The ability to provide numerical simulations for increasingly complex membrane and inflatable structures is advancing rapidly due to both remarkable strides in computer hardware development and the improved maturity of computational procedures for nonlinear structural systems. Significant progress has been made in the formulation of finite elements methods for static and dynamic problems, complex constitutive material behaviour, coupled aero-elastic analysis etc. **Structural Membranes 2011** addresses both the theoretical bases for structural analysis and the numerical algorithms necessary for efficient and robust computer implementation.

A significant part of the conference presents advances in new textile composites for applications in membrane and inflatable structures, as well as in innovative design, construction and maintenance procedures.

The collection of papers includes contributions sent directly from the authors and the editors cannot accept responsibility for any inaccuracies, comments and opinions contained in the text.

The organizers would like to take this opportunity to thank all authors for submitting their contributions.

Barcelona, October 2011

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European Community on Computational Methods in Applied Sciences (ECCOMAS)



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Build Air Ingenieria y Arquitectura S.A.



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PLENARY LECTURES

ZOOMORPHISM AND BIO-ARCHITECTURE: BETWEEN THE FORMAL ANALOGY AND THE APPLICATION OF NATURE'S PRINCIPLES

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Key words: Zoomorphism, Bio-architecture, Biomimetics, Conceptual Design, Nature's Models, Formal Analogy, Principles of Nature

Summary. The environment, ecology and sustainability are important concerns in much modern technological innovation. Since these concerns are of particular relevance to the field of construction, proposals are being made concerning materials, elements and systems, as well as design and conception.

Zoomorphism and bio-architecture, disciplines that look to the design of living things for their inspiration, are two of the directions that have been taken. Nature's models are viable precisely because they are the result of over 500 million years of evolution governed by the principles of economy, efficacy, adaptation and sustainability. However, the use of such models can also produce unwanted outcomes when, rather than applying nature's principles, the designer uses a formal analogy. For this reason, we have investigated some examples of both options and present some key indicators to help distinguish between the two.

1 INTRODUCTION

The evolution of our culture during the twentieth century, and in particular of technology, has made it essential to take into account sustainability and the environmental implications of our actions (A. Cuchí, 2005). In order to reduce the impact of our society on the environment, to address the scarcity of available resources and to prevent the exhaustion of the capacity of natural systems to absorb pollution, it has become clear that we must apply the strategy of the four Rs: reduce, reuse, recycle and rehabilitate (B. Edwards, 2005).

One of the ways this strategy is being implemented is through biomimetics. Biomimetics is the study of living organisms that have been evolving for over 500 million years in harmony with their natural environment and without compromising the general continuity of the system as a whole.

2 THE APPLICATION OF NATURE'S PRINCIPLES

Nature's models are not usually directly applicable to industry because they are the result of a very slow evolutionary process driven primarily by the need to optimize survival and reproduction. To achieve this twofold objective, living beings are based on such principles as energy saving, recycling, optimization of form, economy in the use of locally accessible materials, adaptation to the environment, and sustainability. Although our objectives are not quite the same, all of these principles can also be applied to construction, helping us to save materials and energy, to achieve more efficient and sustainable solutions, and to reduce costs and improve function and durability. Some examples are discussed and the results obtained are presented.

3 BIOMIMETICS IN INDUSTRY



On observing how thistles adhered to the coat of his dog, George de Mestral invented a fastening system based on the use of many small flexible hooks. His invention was patented in 1955 under the name "velcro", made by combining parts of the French words "velours" (velvet) and "crochet" (hook) (Swiss info.ch, 2007).



Based on observation of the manoeuvrability of fish swimming through coral reefs, Mercedes Benz optimised the wind resistance of its vehicles (Daimler Chrysler, 2005).

4 BIOMIMETICS AS APPLIED TO PRODUCTS AND ELEMENTS

Researchers have investigated the self-cleaning mechanism of the water-repellent lotus leaf. When it rains, the droplets do not wet the leaf but rather quickly run off it, carrying away any surface dirt. The results of this research have been applied to the design of textiles and

J.Llorens



self-cleaning paint (Institute of Textile Technology and Process Engineering, Denken-dorf).



Wasps nests and bee hives can support 45 times their own weight thanks to the hexagonal shape of their cell structure. This design has been used to lighten panels and structural elements (Museo de Ciencias Naturales, 2004).



Reducing the amount of material used where less is needed is another principle used to lighten structures. Examples include the vaults used in flooring slabs and the more recent Bubble Deck technology, which uses plastic balls to lighten biaxial reinforced concrete slabs (Bubble Deck, 2006).



Pultruded fibre struts have a structure similar to that of the stems of certain plants (institute of textile technology and process engineering, Denkendorf).



5 BIOMIMETICS AS APPLIED TO SYSTEMS

The use of branching tree structures radically reduces the span required, making the whole structure lighter. (Almuñécar Aquarium, 2007 and Stuttgart Airport, 1990).

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Tensairity pneumatic beams were inspired by the combination of compressive and tensile forces that surround the fluid in plant stems (Tensairity, 2003).



The structure of the biome domes in the Eden Project in Cornwall is optimized by using hexagonal frames to enclose the maximum surface area within the minimum contiguous boundaries. (M. Jackson, 2000).



The passive cooling system used in termite mounds has been used in troglodyte houses in Uchisar and more recently in an office complex in Harare (Zimbabwe), where it replaces mechanical air conditioning (D.G. McNeil, 1997).

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Folding structures can adapt to climatic conditions. The parasols on the Mosque in Medina (1992) and the Venezuela Pavilion at the Hanover World's Fair in 2000.

6 THE LIMITATIONS OF BIOMIMETICS

When transposing solutions from nature, we must take into account the considerable differences between organisms and objects. These are shown in the box below.

LIVING ORGANISMS	OBJECTS			
Geor	netry			
Have no 90° angles.	Have many acute and 90° angles.			
Have curved and rounded surfaces.	Have many flat planes with straight edges.			
Structure and	l Composition			
Are damp and flexible structures.	Are dry, rigid structures.			
Contain no metals.	Contain metals.			
Contain many composite materials.	Include relatively few composite			
Contain abundant microscopic elements.	materials.			
Produce heterogeneous materials, such as	Use homogeneous materials, such as steel.			
wood.				
Mechanisms				
Have folding articulations (the orientation	The orientation of a hinge is changed by			
of a cat's ear is altered through changes in	rotation on an axis.			
its curvatures).	Engines rely on expansion.			
The motor elements (muscles) rely on	Wheels and axles are used.			
contraction.	Energy is stored in various forms:			
No wheels and shafts are used.	gravitational (pendulum and			
Gravitational energy (when walking) and	counterweight), elastic (spring or bow)			
elastic energy (when jumping) are stored.	electrical (battery), and inertial (flywheel).			
A muscle is the sum of a set of identical	The motor is a machine that cannot			
small parts, and the individual functioning	operate if any part is missing or			
of each part is independent of the others.	malfunctioning.			

LIVING ORGANISMS	OBJECTS		
Structural	Behaviour		
The tension-deformation curves are concave. They interact with the demands made on them and adapt. They resist unfavourable forces and take advantage of favourable ones.	The tension-deformation curves are convex or straight. They passively resist and give in to forces that make demands on them. (Sand dunes are formed by action of the wind.)		
Evolution			
They started much earlier and have evolved more slowly. They evolve to enhance their reproductive mechanism and survival. They are created in the moment of reproduction. They constantly renew themselves. After a year, all of the cells have been renewed.	They evolve much more quickly. They evolve by invention, discovery, development and planning. Designed objects work correctly from the very beginning. They do not renew themselves. Unless maintained, they erode or degrade. (The molecules in the pyramids today are the same ones that have been there since the structures were first built)		

7 THE RISKS OF FORMAL ANALOGY IN ARCHITECTURE. CHANGE OF SCALE : THE COLLATERAL EFFECTS OF THE ZOOM

There is a risk in architecture of limiting biomimetics to a formal analogy. Every natural form is the result of a series of interacting factors, including, for example, environment, structural behaviour, function and economy. Imitations that fail to take into account the factors that gave rise to the model, that is, the context in which it evolved and the requirements it satisfies, can easily lead to very different outcomes than those obtained by the original. The results of such designs may be structures that are costly to build and maintain—buildings that devour energy and resources and are inefficient, poorly adapted, unsustainable and unrecoverable.

The change of scale is one of the must common areas of error. Changes in the scale of a model modify the behaviour and characteristics that depend on geometry because they alter the proportion between length, surface area and volume. As the size is reduced, the relationship with the exterior increases. In small animals, gravity is less important than aerodynamic resistance. The relation between the weight of an object and its cross section increases as size increases, so that large animals are proportionally weaker.



Note that nature never changes the scale of a structure. When the size changes, so does the form, the proportions or the material (H.Hossdorf, 1972).



As organisms grow, the proportion between their parts change to preserve functionality.

8 FORMAL ANALOGIES



Carp and slug by M. Sorkin, 1991 and the fish by F. Gehry in the Olympic Port of Barcelona, 1992.

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Lyon-Satolas railway station, 1994 and the Milwaukee Art Museum addition in 2001 by S. Calatrava.



The armadillos in the "Parco della Musica" by R. Piano, Rome 2002 and the zoomorphic structures in the Arca del Mundo Park in Costa Rica, 2002.

9 APPLICATIONS OF PRINCIPLES OF NATURE



Structures: the bison's skeleton and the Forth Bridge, T. Bouch, 1878. The Warren truss from the vulture's wing and the Menil Museum in Houston by R. Piano, 1987.



Protection: the skin as a regulating mechanism. Sendai Mediatheque, Toyo Ito, 2001.Ventilated facade of the United Kingdom Pavilion in the Seville Expo, N. Grimshaw, 1992. Leicester Space Centre, N. Grimshaw, 2002.



Form adapted to the characteristics of the materials: Salginatobel Bridge, Grisons, R. Maillart, 1930. German Pavilion in the World Exhibition in Montreal, F. Otto, 1967.

10 CONCLUSIONS

- The principles of the construction of living beings can provide inspiration and suggestions because for millions of years nature has distilled economy, efficacy, adaptation and sustainability.
- Taking into account the changes in context and requirements, nature's principles can be applied to construction in order to:
 - reduce the weight, amount and cost of materials needed and lightening the constructive elements

- save energy in manufacture and use
- improve thermal behaviour
- recycle materials and reduce the amount of waste generated by the construction and use of the building
- simplify construction, use and maintenance by using simpler solutions (for example, passive solutions).
- reduce maintenance (for example, by increasing the durability of protective elements).
- reduce accidents, responsibilities and litigation arising, for example, from the risks of falls, slips, impact, trapping or the manipulation of heavy elements.
- mere observation and imitation of the form can give rise to outcomes different from those obtained by the original model because the aim is not to copy, but to learn.

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85 WAYS TO MAKE A MEMBRANE MOULD

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Key words: Membranes, morphology, membrane moulds, fabric formwork, form finding, manipulation and adaption.

Summary. The focus of this paper is on the form and materialization of membrane moulding. There has been an impressive and long tradition in research on geometry and form finding of membranes. The use of membranes for moulding has been a growing in the last 10 years. However, if designers ask for the do's and don'ts in membrane moulding it is hard to give a clear and simple answer. This paper provides an overview of all the possibilities for the manipulation of membranes. The overview is presented in a matrix with 85 icons that represent 85 ways to manipulate a membrane. Further a general overview of the techniques and methods for the use of membrane moulds will be given. Case studies by the author and others for the use of glass, ice, concrete and composite will be shown within this overview. The overviews in the paper aim to be a helpful instrument for designers who like to work with membrane moulds. For researchers in membrane moulding the overviews can be helpful to clarify which kind of combinations have been researched and which kind of combinations are still open for further research. In the future new techniques, materials and combinations can be added to the matrixes.

1 INTRODUCTION

Although the title uses the term "membrane", we prefer to use the term "form-active". Form-active is used in the way defined by H. Engel: "*Form-active structure systems are systems of flexible, non-rigid matter, in which the redirection of forces is effected by a self-found Form design and characteristic Form stabilisation*". In practice form-active and flexible structures are pre-stressed membranes, inflatable membranes, chains and cable structures. It is possible to use this type of structures for moulding. They have a double-curved surface and have the ability to make non-repetitive surfaces. This way of moulding is pre-eminently suitable for "free"-formed architecture and all kinds of doubly curved building elements.

In this paper we will consequently connect colors to a certain form. The most important colors are:



- blue for synclastic; and
- red for anticlastic.
- Figure 1 zeroclastic, monoclastic, synclastic and anticlastic surfaces



Pre-stressed membranes are always anticlastic except for

structures having all the corner points in the same plane and all the forces on and in the structure are within the surface of that plane; in that case the form is zeroclastic. Using elastic fabrics the surface of the model is formed within a number of high-points, low-points and borders. The surfaces are not absolutely minimal. The geometry of the plane is determined by the extent to which the fabric is tensioned in one or more directions. The degree of tension in the different directions can be changed and will influence the surface of the membrane. If the ratio of tension in the different directions is more than 1:2, depending on the material properties the membrane might wrinkle.

1.2 Inflatable membranes



Inflatable membrane structures contain at least one direction, a circular cross section. In most cases, they are synclastic (Figure 2). Under certain conditions, it is also possible to contain monoclastic (Figure 3) and even anticlastic (Figure 4) surfaces. The different surfaces that can be made with inflatables depend on the circular cross section in relation to other circular cross sections. The two parameters that influence the form are: *Figure 2 Synclastic balloon*

1 the angle of the cross section to the other cross section; and 2 the radius of the circle.

Figure 2 Synclastic balloon Figure 3: Monoclastic balloon Figure 4: Balloon with an anticlastic part

Both parameters 1 and 2 can be divided into three subgroups:

1a one centre point;1b parallel circular sections perpendicular to a straight center line; and1c non-parallel circular sections perpendicular to a central curvature.

2a fixed radius; 2b linear changing radius; and 2c non-linear changing radius.

Parameters 1 en 2 are combined in Table 1 below and will lead to 7 ways to curve the surface of an inflatable. The table gives an examples of every category.

The form of inflatables	Synclastic	Monoclastic	Synclastic	Monoclastic	Synclastic	Synclastic	Synclastic
			Anticlastic		Anticlastic	Anticlastic	Anticlastic
One centerpoint							
Parallel circular section on a centerline							
Non-Parallel circular section on a central curve							
Fixed radius							
Linear changing radius							
Non-Linear changing radius							
Name	Sphere	Cylinder	Torus	Cone			
Figure	A	В	С	D	E	F	G

Below and in the tables there are drawings of inflatable objects with a particular curved surface.



Figure D, E, F1, F2



Figure G1, G2

Object A with one center is always a sphere. Spheres have a synclastic surface. Objects with one center point in combination with a changing radius do not exist. Object B and D with a parallel circular section on a straight line have always a monoclastic surface. Objects C and E have a synclastic and anticlastic part following the curved centerline. At the "inner site" of the center curve the surface is anticlastic; at the "outer side" the surface is synclastic. Object F has a non-linear changing radius. If the radius changes in a progressive way, the surface is anticlastic. If the radius changes in a regressive way, the surface is synclastic. Object G is a combination of a curved center line and a non-linear changing radius. Both features will give a double curvature. They can stimulate or de-stimulate each other. They stimulate each other in the combination of outer side of the center curve and regressive curvature to a synclastic surface. In the case of the inner site of the center curve and progressive changing radius they stimulate each other to an anticlastic surface. In the other two combinations (inner site of the curve – regressive changing radius and outer side of the curve and progressive changing radius) the feature with the strongest surface curvature will determine the result. Below a scheme with the features and curvatures. The color of the scheme is corresponding to the color of the inflatable object below. The green arrow in object G shows the place where a synclastic surface swaps to and anticlastic surface as a result of the strong curvature of the center line although the change of the radius is repressive. The purple arrow in object G shows the place where an anticlastic surface swaps to synclastic surface as a result of the strong curvature of the center line, although the change of the radius is progressive.

Curvature of inflatable with a non-parallel circular section on a central curve with a non-linear changing radius					
"Innersite" of the center curve	Anticlastic				
"Outersite" of the center curve	Synclastic				
Progressive changing radius	Anticlastic				
Repressive changing radius	Synclastic				
Result of the combination		Anticlastic	Anti/synclastic	Synclastic	Anti/synclastic



Figure G3, G4

2 FORM FINDING

2.1 Physical form finding

Like every design process the form finding of tensile structures is an iterative process, i.e. the conditions can be changed over and over until the most optimal solution is achieved. Finding a final solution is an interplay between the decisions taken by the designer and the form findings process. The form findings process that is used, might affect the results. Table 3 compares two experimental form finding methods.

	Soap film	Elastic membrane/film
Boundary	flexible or rigid closed	flexible or rigid points and
	boundaries	flexible or rigid boundaries
Minimal surface	absolute minimal surface	differ from the minimal
		surface
Form	anticlastic/ synclastic	Anticlastic/synclastic
Surface manipulation	a) depends on the boundary	a) depends on the boundary
_	conditions	conditions
	b) pressure on the surface	b) pressure on the surface
	c) material behaviour	c) material behaviour
		d) depends on the direction and
		force of the pretension in the
		membrane.

Table 3: Comparison between physical methods: soap film versus elastic membrane/film

2.2 Analytical form finding of membrane structures

Until the 70ties of the last century physical form finding was used for engineering tensile structures. The implementation of the model has to be done as precise as possible. It is a time consuming process with the risk of many inaccuracies. After the knowledge gained during the construction of the Olympic Stadium in Munich (1972) computer programs were introduced for the engineering and form finding of membrane structures.

In 1974, H.J. Schek published the paper "The force density method for form-finding and computations of general networks", the fundamental theory for the analytical approach of form finding, the force density method. The computer program Easy and, more recently, other programs use this method for form finding membrane structures. In the analytical form findings process, according to Klaus Linkwitz, two phases can be identified.

2.2.1 Phase1

A number of design studies, non-materialized equilibrium models, have to be done. The process is almost analogous to the soap film method. Within the boundaries a minimal surface will be generated. The membrane can be seen as the discretization of a cable net. There are a number of parameters that influence the geometry of the equilibrium surface. By varying these parameters the geometry (curvature) of the surface can be influenced. The parameters are:

- a) the position of boundaries, the high points and low points;
- b) the curvature of the surface;
- c) proportional force density in the different parts of the surface;
- d) orientation of the network; and
- e) external forces/load cases.

In the first phase the equilibrium geometry of the surface is found. In this phase it is possible to do preliminary studies to load cases, deformation, and stress contribution.

2.2.2 Phase 2

The second phase consists of the materialization of the equilibrium surface. By introduction of the material properties it is possible to have a complete analysis of the load cases, deformation and stress contribution. The geometry of the surface will be influenced by the material properties.

	Phase 1	Phase 2		
conditions	closed boundaries and/or points	equilibrium (Phase 1)		
shape	equilibrium	materialised equilibrium		
shape influence	a) boundaries, high points, low	material properties		
	points			
	b) the curvature of the surface			
	c) proportional force density			
	d) orientation of the network			
	e) external forces/load cases			

Table 4: Process for the analytical form finding of membrane structures

2.2.3 Dynamic relaxation

In the force density method there is a ratio between the force and the length of a line. This is specified by the designer. Beside the force density method it is possible to calculate the deformation of membranes with the dynamic relaxation method. The dynamic relaxation method is based on discretizing the continuum under consideration by lumping the mass at nodes and defining the relationship between nodes in terms

of stiffness. The system oscillates about the equilibrium position under the influence of loads. An iterative process is followed by simulating a pseudo-dynamic process in time, with each iteration based on an update of the geometry.

3 THE MANIPULATION OF MEMBRANES

The surface of a membrane can be manipulated in different ways. This paragraph will give an overview of the different ways a membrane can be influenced.

3.1 85 combinations to manipulate membranes

If single-layered form active surfaces in force equilibrium are brought in relation to their Gaussian curvature, the following five combinations can be made:

- 1. prestressed membrane or cable structure with an anticlastic surface;
- 2. prestressed membrane or cable structure with a zeroclastic surface;
- 3. inflatable with a synclastic surface;
- 4. inflatable with a monoclastic surface; and
- 5. inflatable with an anticlastic surface.

The surfaces in force equilibrium can be manipulated in four different main categories:

- 1. by changing the pre-stress in a certain area of the structure in force equilibrium;
- 2. by an external load on the structure in force equilibrium;
- 3. by pushing other surfaces in force equilibrium against a surface in force equilibrium; and
- 4. by pushing a rigid element in or out the structure in force equilibrium.

It is possible to combine those different ways. In the scheme are 85 ways to manipulate membranes, it leads to an endless number of forms that can be made, for instance the combination of an inflatable pushed against a pre-stressed membrane. Those membranes will still be in force equilibrium (from active). If a membrane is combined with section active, vector active or surface active elements the amount of possibilities does increase further. In this way it is not possible to make every shape you can think of, but it is possible to come close to any shape desired. It is impossible to describe all the surfaces that can be made, but it is possible to give an overview of the different parameters for the manipulation of membranes.

Below is the matrix with all the 85 possibilities. In column 1 to 5, the 5 types of membranes are given. In the rows the ways to manipulate are written divided into the 4 main categories as mentioned above. Please note the legend for the meaning of the colors and marks. The manipulation will increase or decrease the curvature. Not every combination will give a manipulation with a 3D effect on the surface and some of the combinations will give the same result. For instance pushing or pulling against a zeroclastic membrane will give the same deformation if the result is mirrored in the plane of the membrane. There are 5 x 19 combinations, taking out the combinations without a result or with the same result 85 combinations are left.

						2	3	4	5
			Flexible and formactive structures		Prestressed membrane/cablenet	Prestressed membrane/cablenet	Inflatable	Inflatable	Inflatable
×	No manipulation out of given surface		Way to manipulate	Form	Anti d. surface	Zero cl. surface	Syn d. surface	Mono cl. surface	Anti cl. surface
-	Anti cl. surface		Pushed flexible elements						
	Syn cl. surface	1	Prestressed membrane/cablenet	Anti cl. surface	D/E	~	>Z<	~	×.
	Mono cl. surface	2	Prestressed membrane/cablenet	Zero cl. surface	2	x	2	2	2
>75	Decreasing curvature to zero cl	3	Inflatable	Syn cl. surface	224	~	M N	~	220
	then even syn cl. curvature.	4		Mono cl. surface	<u> </u>		324	IVI.	-
>Z<	Decreasing curvature to zero cl.	9	Inflatable	Anti cl. surface		1	725	10	M
dia -	then even anti cl. curvature.	0	Load cases	Point			1	-	-
A	Anti cl. surface to the curved	0	Point load	Point	-	-			
	line in surface		Pulsed linear load	Curve	2	2	0	9	•
5	line in surface	0	Pulled linear load	Curve	272	~		~	
A	Anti cl. surface to the boundaries	10	Pulled surface lead	Surface	574		- MI	-	>7<
	of the element	10	Puiled surface load	Sunace	- 4-	-	~	-	
S	Syn cl. surface to the boundaries		Rigid elements	All kind of houndarion	A1	A1		4	
A/S	Anti or syn cl. surface depence on	12	Pushed out element	All kind of boundaries	AL	AL	A	-	Â
110	specific conditions	13	Pushed in element	Positive boundary curve			A	~	
AI	Anti cl. surface to the boundaries	14	Pushed in element	Negative boundary curve	AL	AL	S	S	S
	the membrane will make an independent		Local prestressed are in surface	Negative boundary corve			-		
	surface in force equilibrium	15	Prestressed cable	Curve	>	×	S	AIS	AIS
<	Increasing curvature	16	Isotroop higher prestressed surface area	Surface	2	x	>7<	4	~
>	Decreasing curvature	17	Anisotroop higher prestressed surface area	Surface	>	×	>Z<	<	<
M	minimal surface whitin the boundaries	18	Isotroop lower prestressed surface area	Surface	<	x	<	<	>Z<
	of the intersection	19	Anisotroop lower prestressed surface area	Surface	<	×	<	<	>7<
>Z<	Decreasing to zero cl - then increasing	19	remember of preservated autobard						

Figure 5: scheme with 85 ways to manipulate membranes divided into 4 main categories.

Below a closer look to the manipulation of membranes divided into the four main categories.
3.2 Changing the pre-stress in a certain area of the structure in force equilibrium

F	Flexible and formactive structures							
			Prestressed Membrane & Cablenet	Prestressed Membrane & cablenet	Inflatable	Inflatable	Inflatable	
			Anti cl. surface	Zero cl. surface	Syn. cl. surface	Mono cl. surface	Anti cl. surface	
Manipulation methods		Form		\diamond				
	prestressed cable	curve						
membrane	Isotroop higher prestressed surface area	surface			,		A/S	
estressed area in r	Anisotroop higher prestressed surface area	surface						
Local pr	Isotroop lower prestressed surface area	surface	V				W N	
	Anisotroop lower prestressed surface area	surface					>76	
Anti. cl. surface Mono cl. surface								

Syn. cl. surface Zero. cl. surface No deformation/manipulation >Z< Decreasing curvature to zero cl. then even sin cl. Curvature >Z< Decreasing curvature to zero cl. then even anti cl. Curvature anti cl. surface to the curved line in surface syn cl. surface to the curved line in surface A anti cl. surface to the boundaries of the element s syn cl. surface to the boundaries of the element A/S anti or sin cl. Surface to the boundaries of the element Μ in case of same pressure/tensile-force minimal surface within the boundaries of the intersection. < Increasing curvature Decreasing curvature >

>Z< pecreasing to Zero cl. then increasing

*Figure 6: The effect of local pre-stressed areas.*3.2.1 The experimtents by Linkwitz

In the specification of the force density in the field and in the boundaries it is possible to influence the geometry of the equilibrium of the membrane. Linkwitz and a group of students did some experimental research to the ratio of the force density and the geometry of the equilibrium. In this research the experiments by Linkwitz have been redone in the program Tess3D.



Figure 7,8. The influence of the size of the force density on the equilibrium shape.

Fig. 4 shows a perpendicular net structure with a force density in the boundary of 1. In Fig. 5 the force density in the boundary and the network is 5. The equilibrium shape of the surface is the same. The conclusion is that if the ratio in force density of the boundary and the network is the same, the shape of the structure will also be the same.



Figure 9: The force density in the borders is 3 times bigger than in the network. Figure 10: The force density in the border is 10 times bigger than in the network.

The forces in the boundary are proportionately bigger than in the network. Therefore the curvature of the boundary is smaller and the surface of the network increases. Conclusion: if the force density in the boundary compared to the network is bigger, the network will become bigger and the curvature in the network and boundary will become smaller.



Figure 11: The force density in the boundary is 3 times smaller than in the network. Figure 12: The density in the boundary is 6 times smaller than in the network.

The force density in the network is relatively big compared to the boundary cables. The curvature of the

boundary cables is lower and the surface of the network is smaller. The above figures show that the shape of the equilibrium depends on the proportional relationship between the force densities in boundaries and network.

3.2.2 The behavior of membranes.

The work of Frei Otto and his team is of importance in a more experimental way to understand the rules for the manipulation of membranes. For a better understanding please see the reference to the work published by F. Otto S. Pellegrino, B. Maurin, K.U. Bletzinger and R. Wagner.

3.2.3 Local pre-stressed areas in anticlastic surfaces



Figure 13 and 14: Two equilibrium surfaces with equal force density proportions in cable (1) and net (1). The force density in the cable in figure 14 is 0. In Figure 15 the force density is 5.



Figure 15, 16, 17, 18 and 19 Anticlastic membranes with a change in local pre-stressed areas

In case of a change in a local pre-stressed force within the surface, there will be a new state of force equilibrium. Figure 15 shows a cable with a higher pre-stress. This cable can be seen as a new border. Figure 10 shows that if the ratio between border and membrane becomes bigger the curvature becomes smaller and the area becomes bigger. In Figure 14 and 15 is happening the same. The higher the stress in the cable the less curvature in membrane and cable. A pre-stressed cable on or in the surface of a mechanical pre-stressed membrane will decrease the anticlastic curvature in the surface and will give a curved folding line at the position of the cable.

Figure 16 shows an area with a higher pre-stress. In that case the curvature perpendicular to the stress will increase within that area. Outside the area and in the direction of the stress the curvature will decrease.

Figure 17 shows a local area with pre-stress in both directions. In the part with the higer pre-stress the curvature will increase in the other parts the curvature will decrease.

Figure 18 and 19 shows a membrane with a local area with a relief of the pre-stress. In those parts will happen the opposite of what happened in the membranes with a local higher pre-stress. In case the stress in a pre-stressed surface is locally lower in one direction, the curvature in that direction will be higher but perpendicular the curvature in that area will be lower to the incensement of the curvature in the surface beside this area;

3.2.4 Local pre-stressed areas in zero clastic membranes



Figure 20, Zero clastic membrane, Figure 21, 22, and 23 Inflatable membranes with a pre-stressed cable

Figure 20 shows a zero clastic membrane. A higher pre-stress within the surface of the membrane will not lead to a curvature besides the wrinkling of the parts with a lower pre-stress. This will happen if the ratio between the tension in the both parts is more than 2.

3.2.5 Inflatable membranes with a pre-stressed cable



Figure 24, 25, and 26: Three equilibrium surfaces with the same equally spread upload. The force density proportions in border cable, cross cable and net have been varied:

- Figure 24 The force density in the border cable is 12, in the cross cable is 1 and net cable is 1
- Figure 25 The force density in the border cable is 12, in the cross cable is 12 and net cable is 1
- Figure 26 The force density in the border cable is 1, in the cross cable is 12 and net cable is 1

Figure 21, 22 and 23 shows a pre-stressed cable on or in the surface of an inflatable. This will always give a synclastic curvature in the surface of the membrane. When the cable is pulled out of the surface of an inflatable, the surface next to the cable will be anticlastic.

3.2.6 Inflatable membranes with a change in anisotropic pre-stressed areas



Figure 27, Synclastic inflatable membrane with an anisotropic higher pre-stressed area Figure 28, Monoclastic inflatable membrane with an anisotropic higher pre-stressed area Figure 29 Anticlastic inflatable membrane with an anisotropic higher pre-stressed area Figure 30, Synclastic inflatable membrane with an anisotropic lower pre-stressed area Figure 31, Monoclastic inflatable membrane with an anisotropic lower pre-stressed area Figure 32, Anticlastic inflatable membrane with an anisotropic lower pre-stressed area

In case the stress in a pre-stressed or inflatable surface is locally higher in one direction, the curvature in that direction will be lower but perpendicular, the curvature in that area will increase to the detriment of the curvature in the surface beside this area.

In case the stress in an inflatable with a synclastic surface is locally higher in one direction (Figure 27, 28 and 29), the curvature in that direction will decrease. The surface in Figure 27 will become zeroclastic and then even anticlastic. The surface in Figure 28 and 28 will be increasingly anticlastic.

In Figure 30 and 31 the stress in an inflatable surface is locally lower in one direction, the synclastic curvature in that direction will increase.

In Figure 32 the stress in an inflatable with an anticlastic surface is locally lower in one direction, the curvature in that direction will decrease to zeroclastic and then become synclastic.

3.2.7 Inflatable membranes with a change in isotropic pre-stressed areas



Figure 33, Synclastic inflatable membrane with an isotropic higher pre-stressed area Figure 34, Monoclastic inflatable membrane with an isotropic higher pre-stressed area Figure 35, Anticlastic inflatable membrane with an isotropic higher pre-stressed area Figure 36, Synclastic inflatable membrane with an isotropic lower pre-stressed area Figure 37, Monoclastic inflatable membrane with an isotropic lower pre-stressed area Figure 38, Anticlastic inflatable membrane with an isotropic lower pre-stressed area

In Figure 33, 34 and 35 the stress in a inflatable is locally higher in both directions, the curvature in that area will increase to, or as, an anticlastic curvature. The inner pressure next to the higher stressed area will form a synclastic area.

In Figure 36, 37 and 38 happens the opposite: the stress in a the inflatable is lower, the curvature in that area will increase to, or as, a synclastic surface. The surface area next to the lower stressed area will be anticlastic.

3.3 External load on the structure in force equilibrium

П	EL	wible and formas	tivo etructuros			•			
ľ	FB	exible and format	sive structures	-					
				Prestressed Membrane & Cablenet	Prestressed Membrane & cablenet	Inflatable	Inflatable	Inflatable	
				Anti cl. surface	Zero cl. surface	Syn. cl. surface	Mono cl. surface	Anti cl. surface	
	1	Manipulation methods	Form		\diamond	\bigcirc			
	,	pointload	point						
	1	Pushed linear oad	Curve			s s			
	1	Pulled linear oad	Curve						
	Load cases	Pushed surface oad	Surface	>Z<		M		>Z<	
	ł	Pulled surface oad	Surface	»Z«				»Z«	
ī		Anti cl sud	face						
í		Mono cl. su	rface						
Sun di surface									
i		Zero. cl. sur	face						
[X	No deformation/manipulation							
 >Z< Decreasing curvature to zero cl. then even sin cl. Curvature >Z< Decreasing curvature to zero cl. then even anti cl. Curvature A anti cl. surface to the curved line in surface 									
S syn cl. surface to the curved line in surface A anti cl. surface to the boundaries of the element									
[1	S syn cl. s	urface to the bou	ndaries of the eler	ment				
,	AIS	anti or sin c	I. Surface to the b	oundaries of the e	element				
1	M	in case of sa	ame pressure/ten	sile-force minimal	surface within the	boundaries of the	intersection.		
<		Increasing curvature							

- Decreasing curvature
- >Z< pecreasing to Zero cl. then increasing

Figure 39: The effect of load cases

3.3.1 Force density, curvature and deformation

Below some more experiments by Linkwitz and a group of students. In this research the experiments by Linkwitz have been redone in the program Tess3D. The influence of the force density on the deformation of the equilibrium surface as a result of a vertical point load is showed in Figure 40 and 41.



Figure 40: The force in the boundary = 5, in the network = 1; on the network is a vertical point load with a size of 6. Figure 41: The force in the boundary = 5, in the network = 5; the vertical point load has a force equivalent of 6. The figures show that the higher the force density in the network, the smaller the deformation will be.

The main curvature in the network will be influenced by the relative position of the points and the boundaries. There is a relation to the curvature of the network and the behaviour of the network under load: the lower the curvature, the smaller the deformation and vice versa.



Figure 42 and 43: Two equilibrium surfaces with equal force density proportions and vertical point load but a different curvature.

The experiments by Linkwitz show the possibilities to manipulate the form of pre-tensioned membranes and the behaviour under load. Below some more experiments with the following combinations:

- 1. an equally spread upload in combination with one point load; and
- 2. an equally spread upload in combination with some cables.



Figure 44, 45 and 46: Three equilibrium surfaces with an equally spread upload. The force density proportions in border cable (12) and net (1) are equal. In figure 17 is the surface pulled down in one point with a force of 25. In figure 18 is the surface pulled upwards in one point with a force of 19.



Figure 47, Anticlastic membrane with a point load Figure 48, Zeroclastic membrane with a point load Figure 49, Synclastic inflatable membrane with a point load Figure 50, monoclastic inflatable membrane with a point load Figure 51, Anticlastic inflatable membrane with a point load

In case of an external load, there will be a new state of force equilibrium. A pre-stressed surface in force equilibrium will always make an anticlastic surface to the place of contact with another point.

3.3.3 Linear load on a mechanical pre-stressed membrane



Figure 52, Anticlastic membrane with a linear pushed load Figure 53, Zeroclastic membrane with a linear pushed load Figure 54, Anticlastic membrane with a linear pulled load Figure 55, Zeroclastic membrane with a linear pulled load

For anticlastic pre-stressed structures and zeroclastic surfaces the result for pulling and pushing is the same (Figure 52 and 54 as well as Figure 53 and 55). It will bring a curved line in the surface. The changed surface will be anticlastic.



Figure 56, Synclastic inflatable membrane with a linear pushed load Figure 57, Monoclastic inflatable membrane with a linear pushed load Figure 58, Anticlastic inflatable membrane with a linear pushed load Figure 59, Synclastic inflatable membrane with a linear pulled load Figure 60, Monoclastic inflatable membrane with a linear pulled load Figure 61, Anticlastic inflatable membrane with a linear pushed load

For a linear load on an inflatable the result will bring a curved line in the surface of the membrane. Pushed from the outside (Figure 56, 57 and 58) the changed surface will be synclastic. Pulled from the outside (Figure 59, 60 and 61) the surface will be anticlastic.

3.3.6 Local surface load mechanical pre-stressed membrane



Figure 62, Anticlastic membrane with a local pushed surface load Figure 63, Zeroclastic membrane with a local pushed surface load Figure 64, Anticlastic membrane with a local pulled surface load Figure 65, Zeroclastic membrane with a local pulled surface load

A surface load on an anticlastic surface will always decrease an anticlastic surface, if the force on the surface is strong enough, it will lead to a synclastic surface. A zeroclastic membrane will always form a synclastic surface. For anticlastic pre-stressed structures and zeroclastic surfaces the result for pulling and pushing is the same (Figure 62 and 64 as well as Figure 63 and 65).

3.3.7 Load surface load inflatable membrane



Figure 66, Synclastic membrane with a local pushed surface load Figure 67, Zeroclastic membrane with a local pushed surface load Figure 68, Anticlastic membrane with a local pushed surface load Figure 69, Synclastic membrane with a local pulled surface load Figure 70, Zeroclastic membrane with a local pulled surface load

Figure 71, Anticlastic membrane with a local pulled surface load

Figure 72 Sections of a synclastic membrane with a local pushed surface load

The surface load on an inflatable will always end with a local synclastic surface if the force is strong enough. In Figure 68 and 71 the surface will decrease to zero clastic before it becomes anticlastic. In Figure 66 the surface decreases to zero clastic and becomes synclastic in the opposite direction. At the borders of the "negative" synclastic surface to the "positive" curvature there will be a folding line or anticlastic area. See

Figure 72.

This is an overview with the results of the manipulation of membranes. For a better understanding of the behavior of membranes with a variation in stress and load cases please see the referred papers by F. Otto S. Pellegrino, B. Maurin, K.U. Bletzinger and R. Wagner.



3.4 Pushing surfaces in force equilibrium against another surface in force equilibrium

Figure 73: The effect of pushed flexible elements

Pushing surfaces in force equilibrium against another surface in force equilibrium will transform the two surfaces in a new state of force equilibrium based on the forces and form of the two individual surfaces. Below the general rules:

- Figure 74
- 1. In the place of connection between both surfaces the angle between both surfaces is 0 as shown in Figure 74.
- 2. If membrane structures are combined, this will lead to a fluid surface to the place where both surfaces are connected.
- 3. In the place of connection of the surfaces, the membrane with the highest tension demands the form of the surface.
- 4. If the tension in both surfaces is the same, the surface will form a minimal surface within the boundaries of the connected area. That will be an anticlastic or zeroclastic surface;
- 5. In all the cases where a membrane is pushed against a zeroclastic membrane (Figure 76, 86, 91 and 96) the curvature of the membrane will be decreased.
- 3.4.1 Pushing an anticlastic mechanical pre-stressed membrane against another membrane



Figure 75, Anticlastic mechanical pre-stressed membrane pushed against another anticlastic mechanical pre-stressed membrane

Figure 76, Anticlastic mechanical pre-stressed membrane pushed against a zeroclastic mechanical pre-stressed membrane

Figure 77, Anticlastic mechanical pre-stressed membrane pushed against a synclastic inflatable membrane Figure 78, Anticlastic mechanical pre-stressed membrane pushed against a monoclastic inflatable membrane Figure 79, Anticlastic mechanical pre-stressed membrane pushed against an anticlastic inflatable membrane

In Figure 75 the connected area will be anticlastic. If the pre-stress in both membranes is the same the surface will form a minimal surface within the boundaries of the connected area.

In Figure 78 and 79 the connected area will be anticlastic. If the pre-stress in both membranes is the same the surface will differ from a minimal surface due to the overpressure at one side.

In Figure 77 the connected area is anticlastic and decreses to zeroclastic to become synclastic if the tension in the synclastic inflatable membrane and pushing force is strong enough.

3.4.2 Pushing a zeroclastic pre-stressed membrane against another membrane



Figure 80, Zeroclastic mechanical pre-stressed membrane pushed against an anticlastic mechanical pre-stressed membrane

Figure 81, Zeroclastic mechanical pre-stressed membrane pushed against another zeroclastic mechanical prestressed membrane

Figure 82, Zeroclastic mechanical pre-stressed membrane pushed against a synclastic inflatable membrane

Figure 83, Zeroclastic mechanical pre-stressed membrane pushed against a monoclastic inflatable membrane

Figure 84, Zeroclastic mechanical pre-stressed membrane pushed against an anticlastic inflatable membrane

In all the figures the zeroclastic membrane will adapt the surface of the membrane pushed to. It will also decrease the curvature. In Figure 81 there is no manipulation.





Figure 85, Synclastic inflatable membrane pushed against an anticlastic mechanical pre stressed membrane Figure 86, Synclastic inflatable membrane pushed against a zeroclastic mechanical pre stressed membrane Figure 87, Synclastic inflatable membrane pushed against another synclastic inflatable membrane Figure 88, Synclastic inflatable membrane pushed against a monoclastic inflatable membrane Figure 89, Synclastic inflatable membrane pushed against an anticlastic inflatable membrane

In Figure 87 the surface decreases to zeroclastic and becomes synclastic in the opposite direction. At the borders from the "negative" synclastic surface to the "positive" curvature there will be a folding line or anticlastic area (Figure 72).

In Figures 85, 88 and 89 the surface decreases to zeroclastic and becomes anticlastic.





Figure 90, Monoclastic inflatable membrane pushed against an anticlastic mechanical pre-stressed membrane Figure 91, Monoclastic inflatable membrane pushed against a zeroclastic mechanical pre-stressed membrane Figure 92, Monoclastic inflatable membrane pushed against a synclastic inflatable membrane Figure 93, Monoclastic inflatable membrane pushed against another monoclastic inflatable membrane Figure 94, Monoclastic inflatable membrane pushed against an anticlastic inflatable membrane

In Figure 90 and 94 the surface decreases the anticlastic surface of the surface pushed in. In Figure 92 the curvature decreases to zeroclastic and becomes synclastic in the opposite direction. At the boundary of the "negative" synclastic surface to the "positive" curvature there will be a folding line or anticlastic area (Figure 72).

In Figure 93 there are two cases: 1 the mono the surfaces have the same parallel direction. In that case the monoclastic curvatures decreases to zero clastic and becomes monoclastic in the opposite direction, at the boundary of the "negative" monoclastic surface to the "positive" curvature there will be a folding line or monoclastic area. (Figure 72). In the other case the monoclastic surfaces are not parallel. In that case the monoclastic curvatures decreases to anticlastic and becomes monoclastic in the opposite direction, at the boundary of the "negative" monoclastic surface to the "positive" curvature there will be a folding line or anticlastic area.

3.4.5 Pushing an anticlastic inflatable against another membrane





Figure 95, Anticlastic inflatable membrane pushed against an anticlastic mechanical pre-stressed membrane Figure 96, Anticlastic inflatable membrane pushed against a zeroclastic mechanical pre-stressed membrane Figure 97, Anticlastic inflatable membrane pushed against a synclastic inflatable membrane Figure 98, Anticlastic inflatable membrane pushed against a monoclastic inflatable membrane Figure 99, Anticlastic inflatable membrane pushed against another anticlastic inflatable membrane

In Figure 99 the connected area will be anticlastic. If the pre-stress in both membranes is the same the surface will be a minimal surface.

In Figure 95 and 98 the connected area will be anticlastic. If the pre-stress in both membranes is the same the surface will differ from a minimal surface due to the overpressure at one side.

In Figure 97 the connected area is anticlastic and decreases to zeroclastic to become synclastic if the tension in the synclastic inflatable membrane and pushing force is strong enough.

This is an overview with the results of the manipulation of membranes. For a better understanding of the mechanical behavior of membranes in force equilibrium against another surface in force equilibrium please read more about this in the referred papers.



3.5 Pushing a rigid element into or out of a membrane structure in force equilibrium

Figure 98: Pushing a rigid element into or out of a membrane structure in force equilibrium

3.5.1 General rules for pushing rigid elements

General rules for the surfaces in force equilibrium that will be manipulated by pushing a rigid element into or out of the structure.

- 1. if a rigid element is pushed against the surface in force equilibrium, the structure will adapt the surface of the rigid element unless the boundaries/surface of the rigid element allow the flexible surface to form a new surface in force equilibrium released from the surface of the rigid element. This is shown below in Figure 99E;
- 2. if there is contact between two surfaces, the angle between them is zero;
- 3. if the angle between the surfaces fluently increases from zero, the release between the surfaces is within a place on the two surfaces. In that case there is a smooth transition between the two surfaces into a joint surface. This is shown below in Figure 99A, 99C and F;
- 4. if the angle between the surfaces changes suddenly, the release between the surfaces is at one of the boundaries of the surface. This will give a (curved) line within the other flexible surface. This is shown below in Figure 99B and 99D and 99E.



Figure 99: The section of a surface in force equilibrium pushed against a rigid element.

3.5.2 Pushed-out elements from the inside of an inflatable



Figure 100, Synclastic inflatable membrane with a pushed-out element Figure 101, Monoclastic inflatable membrane with a pushed-out element Figure 102, Anticlastic inflatable membrane with a pushed-out element

An inflatable structure will make an anticlastic surface to the boarders of any rigid element pushed out from the inside.

3.5.3 Pushed-in elements against mechanical prestressed membranes



Figure 103, Anticlastic mechanical pre-stressed membrane with a pushed-in element Figure 104, Zeroclastic mechanical pre-stressed membrane with a pushed in-element Figure 105, Anticlastic mechanical pre-stressed membrane with a pushed-in element with a positive boundary Figure 106, Zeroclastic mechanical pre-stressed membrane with a pushed-in element with a positive boundary Figure 107, Anticlastic mechanical pre-stressed membrane with a pushed-in element with a negative boundary Figure 108, Zeroclastic mechanical pre-stressed membrane with a pushed-in element with a negative boundary Figure 108, Zeroclastic mechanical pre-stressed membrane with a pushed-in element with a negative boundary

A zeroclastic mechanical prestressed membrane and an anticlastic mechanical prestressed membrane will make an anticlastic surface to the boundaries of any rigid element pushed-in or out as long as the boundaries are not parallel.



3.5.4 Pushed-in elements with a parallel boundary curve



Figure 109, Anticlastic mechanical pre-stressed membrane with a pushed-in element with parallel boundaries Figure 110, Zeroclastic mechanical pre-stressed membrane with a pushed-in element with parallel boundaries Figure 111, Monoclastic inflatable with a pushed-in element with parallel boundaries Figure 112, Anticlastic inflatable with a pushed-in element with parallel boundaries

If the boarders of a zeroclastic mechanical pre-stressed membrane and an anticlastic mechanical pre-stressed membrane are parallel with the element pushed-in, it will form a zeroclastic surface.

If the direction of a monoclastic or synclastic inflatable surface is parallel to a monoclastic or zeroclastic element or straight boundary line the surface will be monoclastic.

3.5.5 Pushed in elements with a positive boundary curve against an inflatable



Figure 113, Synclastic inflatable membrane with a pushed-in element with a positive boundary Figure 114, monoclastic inflatable membrane with a pushed-in element with a positive boundary Figure 115, Anticlastic inflatable membrane with a pushed-in element with a positive boundary

An inflatable structure will make an anticlastic surface to the boundary of any rigid element pushed-in from the outside with a positive boarder curvature, also if the positive boundary-line is parallel to the boundary-line of the membrane.

3.5.6 Pushed-in elements with a negative boundary curve against an inflatable



Figure 116, Synclastic inflatable membrane with a pushed-in element with a negative boundary Figure 117, monoclastic inflatable membrane with a pushed-in element with a negative boundary Figure 118, Anticlastic inflatable membrane with a pushed-in element with a negative boundary

An inflatable structure will make a synclastic surface when a rigid element with a negatively curved boundary is pushed in from the outside.

4 **Rigidizing membrane moulds.**

There are many types of form-active structures and many ways to make a rigid surface with the help of a form-active structure. The diagram below will give an overview of all the possibilities known for the materials: concrete, water, polymer composites and glass. The possibilities are characterized in 6 different aspects. This overview gives the opportunity to look for new combinations. The product and function of the product is in this overview of minor importance. In the last column case studies are mentioned. Of course this is not a final list. If new techniques will be developed the list can be extended and new combinations can be made.

1 The four materials to make the transition from fluid to solid are:

- concrete
- ice/water
- polymer composites
- glass.

2 The form-active structure: the structural typology of the form active structure

- cable net
- woven fabric
- knitted fabric
- textile composite
- foil
- the material of the shell in "fluid" condition
- hinged plate structure

3 Form-active typology: Amount of layers and connection between the layers

- single layer
- double layer
- connected double layer

4 Way to stabilize the mould: How the from-active mould is stabilized

- (pre-)stressed surface
- inflate
- bending stiffness of a material
- hydraulic pressure
- under pressure

•

- 5 Technique to handle the rigidizing material:
- hand layup
- spraying
- vacuum injection
- submersion
- casting
- pumping
- prefab elements

6 Reinforcement of the material:

- non
- single fibers
- ropes/cables
- bars
- fabric (woven)
- woven fabric

7 Surface treatment:

- non
- surface tension of the material
- plastering
- polishing
- coating
- drape
- melting
- spraying



Figure 119, Diagram of materials and techniques for ridgidizing form-active structure, all the possibilities available for glass, composite, ice and concrete

Figure 119 gives an overview of all the technical possibilities. In the diagram below (Figure 120) are all the techniques known in relation to the material glass. In the last-column case studies will be mentioned. Figure 121, 122 and 123 will deal with the materials Ice, Composite and Concrete.



Figure 12, Glass and the techniques to make doubly curved surfaces with a form-active mould

4.2 Ice

Figure 121 Ice and the techniques to make doubly curved surfaces with a form-active mould



Figure 122 Composite and the techniques to make doubly curved surfaces with a form active mould **4.4 Concrete**



5 Conclusions

Even well-trained researchers in the field of membranes have problems to predict the outcome of the manipulation of membranes. The aim of this paper is to give an overview of all the possibilities for the manipulation of membranes. The result is a matrix with 85 ways to manipulate a membrane. With this matrix we like to give insight in the effect and possibilities in an early stage during the form-finding process. The technical possibilities for rigidizing are not further clarified in this paper. In the near future more papers about the materials concrete, ice, polymer composites and glass in relation to membrane moulding will be published. The overviews in the paper aim to be a helpful instrument for designers who like to work with membrane moulds. For researchers in membrane moulding the overviews can be helpful to clarify what kind of combinations have been made and which kind of combinations are still open for further research. In the future new techniques, materials and combinations can be added to the matrixes. The author will publish the matrixes on internet and invites researchers and producers to add their work to the matrixes.

7 Acknowledgments

The experiments in the diagrams above by the author took a period of 10 years. The authors gratefully acknowledge the support of many students, college's, funds, companies and the generous and visionary policy of the Eindhoven University of Technology who gave the confidence to do so many experiments is such a wide field.

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SPACE–TIME FSI MODELING OF RINGSAIL PARACHUTE CLUSTERS

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Key words: Fluid–structure interaction, Parachute clusters, Ringsail parachute, Space– time technique, Geometric porosity, Contact

Abstract. The computational challenges posed by fluid–structure interaction (FSI) modeling of ringsail parachute clusters include the lightness of the membrane and cable structure of the canopy compared to the air masses involved in the parachute dynamics, geometric complexities created by the construction of the canopy from "rings" and "sails" with hundreds of ring gaps and sail slits, and the contact between the parachutes. The Team for Advanced Flow Simulation and Modeling (T \star AFSM) has successfully addressed these computational challenges with the Stabilized Space–Time FSI technique (SSTFSI), which was developed and improved over the years by the T \star AFSM and serves as the core numerical technology, and a number of special techniques developed in conjunction with the SSTFSI. We present the results obtained with the FSI computation of parachute clusters and the related dynamical analysis.

1 INTRODUCTION

Fluid-structure interaction (FSI) modeling of ringsail parachute clusters poses a number of computational challenges. The membrane and cable structure of the canopy is much lighter compared to the air masses involved in the parachute dynamics, and this requires a robust FSI coupling technique. This challenge is of course not limited to ringsail parachutes but is common to all parachute FSI computations. The geometric challenge created by the construction of the canopy from "rings" and "sails" with hundreds of ring gaps and sail slits requires a computational model that makes the problem tractable. Contact between the parachutes requires an algorithm that protects the fluid mechanics mesh from excessive deformation, and this computational challenge might also be encountered in other classes of FSI problems where two solid surfaces come into contact. The Team for Advanced Flow Simulation and Modeling (T \star AFSM) has successfully addressed these computational challenges with the Stabilized Space–Time FSI technique (SSTFSI), which was developed and improved over the years by the T \star AFSM and serves as the core numerical technology, and special techniques developed in conjunction with the SSTFSI.

The SSTFSI technique was introduced in [1]. It is based on the new-generation Deforming-Spatial-Domain/Stabilized Space-Time (DSD/SST) formulations, which were also introduced in [1], increasing the scope and performance of the DSD/SST formulations developed earlier [2, 3, 4, 5] for computation of flows with moving boundaries and interfaces, including FSI. This core technology was used in a large number of parachute FSI computations (see, for example, [1, 6, 7, 8, 9, 10, 11, 12]). The direct and quasi-direct FSI coupling techniques, which are generalizations of the monolithic solution techniques to cases with incompatible fluid and structure meshes at the interface, were introduced in [13]. They provide robustness even in computations where the structure is light compared to the fluid masses involved in the dynamics of the FSI problem and were also used in a large number of parachute FSI computations (see, for example, [1, 6, 7, 8, 9, 10, 11, 12]). Computer modeling of large ringsail parachutes by the T*AFSM was first reported in [6, 7]. The geometric challenge created by the ringsail construction was addressed with the Homogenized Modeling of Geometric Porosity (HMGP) [6], adaptive HMGP [8] and a new version of the HMGP that is called "HMGP-FG" [9]. Additional special techniques the $T \star AFSM$ introduced in the context of ringsail parachutes include the FSI Geometric Smoothing Technique (FSI-GST) [1], Separated Stress Projection (SSP) [6], "symmetric FSI" technique [8], a method that accounts for the fluid forces acting on structural components (such as parachute suspension lines) that are not expected to influence the flow [8], and other interface projection techniques [14].

The T \star AFSM recently started addressing (see [10, 11]) the challenge created by the contact between the parachutes. In a contact algorithm to be used in this context, the objective is to prevent the structural surfaces from coming closer than a predetermined minimum distance we would like to maintain to protect the quality of the fluid mechanics mesh between the structural surfaces. The Surface-Edge-Node Contact Tracking (SENCT) technique was introduced in [1] for this purpose. It had two versions: SENCT-Force (SENCT-F) and SENCT-Displacement (SENCT-D). In the SENCT-F technique, which is the relevant version here, the contacted node is subjected to penalty forces that are inversely proportional to the projection distances to the contacting surfaces, edges and nodes. For FSI problems with incompatible fluid and structure meshes at the interface, it was proposed in Remark 1 of [6] to formulate the contact model based on the fluid mechanics mesh at the interface. This version of the SENCT was denoted with the option key "-M1". The contact algorithm used in the parachute cluster computations reported in [10] has some features in common with the SENCT-F technique but is more robust. Also, compared to the SENCT-F technique, the forces are applied in a conserva-

tive fashion. We call the new technique "SENCT-FC", where the letter "C" stands for "conservative". The new technique was used with option M1 in [10]. The SENCT-FC technique was described in detail in [11] and was used with option M1 also in the cluster computations reported in that article. This short article uses material from [11]. We present the computational results together with the related dynamical analysis.

2 CLUSTER COMPUTATIONS

A series of two-parachute cluster computations were carried out in [11] to determine how the parameters representing the payload models and starting conditions affect longterm cluster dynamics. The parachute clusters reported in [11] were used with a 19,200 lb payload. Each parachute has 80 gores and 4 rings and 9 sails, with 4 ring gaps and 8 sail slits. Figure 1 shows, for an inflated ringsail parachute from [9], the ring and sail construction and the ring gaps and sail slits. More information on the parachutes can be



Figure 1: A four-gore slice of a ringsail parachute in its inflated form. The pictures on the right illustrate the ring and sail construction of the canopy and show the shapes of the ring gaps and sail slits.

found in [7, 8, 9]. The parameters selected for testing were the payload-model configurations and initial coning angles (θ_{INIT}) and parachute diameters (D_{INIT}) (for readers not familiar with the term "coning angle", see [11]). We also investigated two scenarios to approximate the conditions immediately after parachute disreefing. This is explained in more detail in a later paragraph. In all cases, the θ_{INIT} is the same for both parachutes

The first set of computations were carried out to investigate the effect of the payload model. In drop tests, the parachutes are connected to a rectangular pallet that is weighted to represent the mass and inertial properties of a proposed crew capsule. The preliminary parachute cluster computations reported in [10] modeled the payload as a point mass located at the confluence of the risers. We will refer to this as the payload at the confluence (PAC) configuration. Two new computational payload models were created to see how they would influence parachute behavior. The payload lower than the confluence (PLC) configuration adds another cable element below the confluence and models the payload as a point mass at the location of the pallet center of gravity. The payload as a truss element (PTE) configuration further enhances the model by distributing the payload mass at 9 different points to match the mass, center of gravity, and six components of the inertia tensor of the pallet. This is accomplished by adding 5 cable elements and 26 truss elements below the confluence. In all of the payload comparison computations, $\theta_{\text{INIT}} = 35^{\circ}$.

The second set of computations were carried out to investigate the effect of θ_{INIT} . Three values of θ_{INIT} were tested: 15°, 25°, and 35°. It should be noted that 35° is greater than the θ values seen in drop tests. The average θ during normal descent is around 15°, and the maximum θ does not usually exceed 25°. We used $\theta_{\text{INIT}} = 35°$ only to cause a large perturbation in order to analyze the dynamic response of the parachute cluster. All of the θ_{INIT} comparison computations used the PTE configuration.

The parachute described in [11] uses a reefing technique to permit incremental opening of the canopy. The parachute skirt is initially constricted by reefing lines and the reefing lines are cut at predetermined time intervals to allow the canopy to "disreef" to larger diameters. In the third set of computations, two scenarios were computed to analyze how conditions immediately after disreefing could have an effect on long-term dynamics. In the first scenario, which we call "simulated disreef", $\theta_{\text{INIT}} = 10^{\circ}$, and for both parachutes $D_{\text{INIT}} = 70$ ft. These values represent the approximate θ during final disreefing and the average minimum D during nominal descent. The second scenario represents an "asynchronous disreef" by using for one parachute $D_{\text{INIT}} = 70$ ft, and for the other $D_{\text{INIT}} = 90$ ft. These values represent the average minimum parachute diameters during nominal descent, respectively. Both scenarios used the PTE configuration.

2.1 Starting conditions

Before an FSI computation is started, a series of pre-FSI computations are carried out to build a good starting condition. For the process of building the starting condition, we refer the reader to [11].

2.2 Computational conditions

Figure 2 shows, for a single parachute, the canopy structure mesh and the fluid mechanics interface mesh. The fluid mechanics mesh is cylindrical with a diameter of 1,740 ft and a height of 1,566 ft. It consists of four-node tetrahedral elements, while the fluid interface mesh consists of three-node triangular elements. The number of nodes and elements are given in Table 1. The porosity model is HMGP-FG. The interface-stress projection is based on the SSP. For more information on the computational conditions, we refer the reader to [11]. We computed each parachute cluster for a total of about 75 s, with remesh as needed to preserve mesh quality. The frequency of remeshing varies for each compu-



Figure 2: Canopy structure mesh (left) and fluid mechanics interface mesh (right) for a single parachute. The structure has 30,722 nodes, 26,000 four-node quadrilateral membrane elements, and 12,521 two-node cable elements. There are 29,200 nodes on the canopy. The fluid mechanics interface mesh has 2,140 nodes and 4,180 three-node triangular elements.

tation and usually depends on how often the parachutes collide, how much the cluster rotates about the vertical axis, and how much each parachute rotates about its own axis. Depending on the computation, remeshing was needed every 170 to 370 time steps.

2.3 Results

Figures 3–6 show the descent speed U and the drag coefficient, which is calculated as $C_{\rm D} = W/(\frac{1}{2}\rho U^2 S_{\rm o})$, where W is the payload weight, ρ is the density of the air, and $S_{\rm o}$ is the nominal area of the parachute.



Figure 3: Cluster computations for different payload models and $\theta_{INIT} = 35^{\circ}$.

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Volume (35°, 70/90 ft) $\frac{nn}{ne}$ 289,221 $\frac{1}{1,795,542}$			ne	$2,\!199,\!472$
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		volume (55, 70/30 ft)	ne	1,795,542

Table 1: Number of nodes and elements for the two-parachute clusters before any payload modifications. Here nn and ne are number of nodes and elements, respectively. The fluid mechanics volume mesh is tabulated for different combinations of θ_{INIT} and D_{INIT} values. The PLC configuration has 1 more structure node and 1 more cable element. The PTE configuration has 10 more structure nodes, 5 more cable elements, 26 more truss elements, and 8 more payload elements.

Figures 7–8 show the contact between two parachutes from the asynchronous-disreef computation. Figures 9–12 show the vent-separation distance (L_{VS}) for all cluster computations. The horizontal black line on each plot shows the approximate vent-separation distance when the parachutes are in contact. Tables 2–4 summarize the payload descent speeds and drag coefficients for all of the cluster computations.

Payload Model	U (ft/s)	$C_{\rm D}$
PAC	28.1	0.97
PLC	30.1	0.85
PTE	29.5	0.88

Table 2: Average U and $C_{\rm D}$ for different payload models with $\theta_{\rm INIT} = 35^{\circ}$. Statistical analysis begins 20 s after the start of the computation.

3 CONCLUDING REMARKS

We have presented our FSI computations of clusters of large ringsail parachutes, which are constructed from membranes and cables with hundreds of ring gaps and sail slits. The core technology is the SSTFSI technique, supplemented with special FSI techniques. Many



Figure 4: Cluster computations for PTE and different θ_{INIT} values.



Figure 5: Cluster computations for simulated disreef.

of the special techniques were developed to address the challenges involved in computer modeling of ringsail parachutes. They include the homogenization techniques that make the problem tractable despite hundreds of gaps and slits. Another special technique addresses the computational challenge created by the contact between the parachutes of a cluster. We have also presented a dynamical analysis of the computed results.

ACKNOWLEDGMENT

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Figure 6: Cluster computations for asynchronous disreef.

$ heta_{ ext{INIT}}$	$U ({\rm ft/s})$	C_{D}
15°	29.9	0.86
25°	31.4	0.78
35°	29.5	0.88

Table 3: Average U and C_D for PTE and different values of θ_{INIT} . Statistical analysis begins 20 s after the start of the computation.

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	U (ft/s)	C_{D}
Simulated Disreef	30.6	0.82
Asynchronous Disreef	30.8	0.81

Table 4: Average U and $C_{\rm D}$ for the disreef cases. Statistical analysis begins 5 s after the start of the computation for the simulated-disreef case, and 20 s after the start of the computation for the asynchronous-disreef case.



Figure 7: Parachutes at t = 52.20 s, t = 53.36 s and t = 54.52 s during the asynchronous-disreef computation modeling the contact between parachutes.

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Figure 8: Parachutes at t = 55.68 s, t = 56.84 s and t = 58.00 s during the asynchronous-disreef computation modeling the contact between parachutes.

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Figure 9: Vent-separation distance. Left: PAC and $\theta_{INIT} = 35^{\circ}$, Right: PLC and $\theta_{INIT} = 35^{\circ}$.



Figure 10: Vent-separation distance. PTE and $\theta_{\text{INIT}} = 35^{\circ}$.

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Figure 11: Vent-separation distance. Left: PTE and $\theta_{INIT} = 15^{\circ}$, Right: PTE and $\theta_{INIT} = 25^{\circ}$.



Figure 12: Vent-separation distance. Left: Simulated-disreef, Right: Asynchronous-disreef.

PRESSURIZED MEMBRANES FOR STRUCTURAL USE: INTERACTION BETWEEN LOCAL EFFECTS AND GLOBAL RESPONSE

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Key words: Inflatable beams, Wrinkling, Non-linear elastic behaviour.

Summary. This paper is aimed to assess the non-linear elastic response of an inflatable cylindrical beam through a simple mechanical model recently proposed by the authors¹ to study the equilibrium shapes of highly pressurized elastic membranes. The local geometric nonlinearities due to the wrinkling of the membrane are taken into account by means of an equivalent physical non-linearity, assuming a two-states constitutive law for the material: when a fiber is stretched (the active state), its response is elastic, while when the fiber is contracted, no compressive force can be engendered in it (the passive state). The evolution of the wrinkled regions and the distribution of longitudinal and transverse stresses in the membrane are accurately determined for increasing levels of loads, up to the collapse. The numerical results, obtained through an expressly developed incremental-iterative algorithm, are then compared with the experimental ones available in the literature.

1 INTRODUCTION

In recent years the number of applications where use has been made of textile structures under form of pressurized membranes has seen a steady growth in many areas of engineering. Compared to ordinary structures, inflatable ones are lighter and economic, are easily portable, and enable their rapid set up and final removal.

Beams and arches, made of highly pressurized very flexible membranes, properly strengthened by textile or glass fibers, are a viable alternative to more traditional choices in all those cases in which the speed of execution or the lightness constitute primary requirements (consider, for example, all the emergency situations where shelters need to be erected as soon as possible, or the structures designed to operate in the space). However, some basic aspects characterizing the mechanical response of the inflatable elements have not yet entirely cleared and may therefore be placed among the current topics of mechanics of structures².

The equilibrium states of these elements are characterized by strong geometrical nonlinearities (local buckling and wrinkling phenomena) due to the smallness of membrane thickness values, which may interest large portions of their surface. Moreover, with increasing loads, the size of the corrugated zones, whose position constitutes one of the main unknowns of the problem, tends to increase, thus strongly influencing the overall behaviour of the whole element and, what it is most important, its actual carrying capacity.

In this paper we assess the state of stress in the wall of an inflatable cylindrical beam through a simple mechanical model recently proposed by the authors to study the equilibrium shapes of highly pressurized elastic membranes¹. The aforesaid local geometric nonlinearities are taken into account by means of an equivalent physical non-linearity, assuming a two-states constitutive law for the material: when a fiber is stretched (the active state), its response is elastic, while when the fiber is contracted, no compressive force can be engendered in it (the passive state). The evolution of the corrugated regions and the distribution of longitudinal and transverse stresses in the membrane are determined accurately for increasing levels of loads, up to the collapse. The numerical results, obtained by an expressly developed incremental-iterative algorithm, are then compared with the numerical and experimental ones available in the literature^{3, 4}.

2 THE BENDING OF AN INFLATABLE BEAM

In the following, we will investigate the bending response of a cylindrical inflatable beam, variously supported at its ends, subjected to a transversal concentrated load 2F acting in the middle section (Figure 1). The beam is made up of two rectangular elastic sheets of initial length 2L and height h, joined together along their common boundaries.

In all the cases examined the dimensions of the beam are kept fixed, as well as the mechanical parameters of the material. The equilibrium problem will be solved for different values of the pressure p and of the transversal load 2F for each of the two cases corresponding to simply-supported and built-in ends, respectively.



Figure 1: the inflatable beam.

The solutions will be obtained numerically by making use of an expressly developed incremental-iterative algorithm implemented in a FEM code, already proposed by the authors, which accounts for the local geometric nonlinearities resulting from the wrinkling of the membrane by means of an equivalent physical non-linearity.

To improve the rate of convergence and the stability of the numerical algorithm, the load process is subdivided into three separate phases. The membrane starts the load process in its

reference configuration where it is flat and unstressed. During the first phase, the pressure is kept equal to zero while a uniform antagonist plane traction is assumed to act along the boundary until a final value for it is reached. During the successive phase two, an increasing pressure p acts internally, up to its established final value. Finally, in the following phase three the boundary traction progressively reduces to zero and wrinkling may freely develops. In all the three phases the analysis is performed by the same incremental-iterative procedure although the first pre-tensioning phase may be performed within a single load step.

At the end of each incremental step, equilibrium is imposed via the virtual work principle. Large displacements and strains are considered, while a nonlinear elastic constitutive law which makes uses of a relaxed energy is included into the analytical model to account for wrinkling.

In all the numerical analyses the membrane material is assumed to behave as linear elastic under tension with a Young's modulus of E = 2.5 GPa and a Poisson's ratio of v = 0.3; the membrane thickness is set equal to 0.125 mm, the height and the length of the beam are set equal to 126 mm and 660 mm, respectively.

2.1 A simply-supported beam

The first case taken under investigation is that of the simply-supported inflatable beam showed in Figure 2. It is assumed that the beam, initially lying in the *x*-*y* plane, is first inflated up to the final pressure p (accordingly, the central part of the beam becomes a cylinder of radius approximately equal to R = 40 mm); then, it is subjected to the action of the load 2F. By virtue of symmetry, only one half of the beam has been modeled (Figure 3a).



Figure 2: the simply-supported inflatable beam.

The inflated configuration together with three equilibrium configurations is showed in Figure 3. In the picture, wrinkled elements are represented in red colour, taut ones in bleu.

After the beam is inflated some first wrinkled zones appear in correspondence to the simply-supported end; in the following loading phase a second wrinkled region spreads itself in the middle of the beam as the concentrated load increases. In the case where the internal pressure is kept fixed, the wrinkled region at the end of the inflated beam turns out to be almost unaffected by the magnitude of the load F. The effects of end wrinkled regions, measured in terms of strains and stresses, are detectable only in the lateral parts of the beam¹

while the standard uniform tensile state of stress persists in the cylindrical central part of the inflated configuration (see Figure 3b).



Figure 3: a) the FEM mesh; b) the inflated configuration; c) three equilibrium configurations corresponding to F = 20.2 N, 26.0 N and 22.4 N respectively (left half of the beam; red elements are in a wrinkled state; $p = 0.1 \text{ N/mm}^2, E = 2500 \text{ N/mm}^2, v = 0.3, t = 0.125 \text{ mm}$)

On the contrary, the extension of the wrinkled region under the point load increases with F and the wrinkling produces effects which are not confined to a local scale but influence the load-displacement response of the inflated beam as a whole.
Figure 4 shows the central part of the beam. A noteworthy mechanical phenomenon emerges from a closer observation of the evolution of the shape of the wrinkled region under the point load. As one may easily verify, the spreading of wrinkles due to the compressive stresses produced by the bending moment (which is maximum in the middle section) is hindered by an increase in the tensile stresses at the extrados of the beam due to the local punching action of the concentrated load 2F.



Figure 4: wrinkled region in the middle of the beam for F =20.2 N, 26.0 N and 22.4 N. a) side view; b) axonometric view ($p = 0.1 \text{ N/mm}^2$, $E = 2500 \text{ N/mm}^2$, v = 0.3, t = 0.125 mm).

As a result, the wrinkled region is smaller than that which would be forecasted by means of a one-dimensional beam model; consequently, the stiffness of the inflated beam in the three-dimensional model will be higher. Notwithstanding this, it is reasonable to think that the load-displacement response of the present 3d model would be in good agreement with that

obtained by a one-dimensional model (a confirmation of this may be found, for example, in reference²). In fact, the increase in the stiffness due to the reduction of the wrinkled region is balanced by an increase in the vertical displacements due to the local punching action of the force 2F.

To highlight this feature two load-displacement curves, one regarding the application point of the load and the other the corresponding point on the axis of the inflatable beam, are showed in Figure 5. Both are obtained by using the present three-dimensional model in the case where the pressure *p* is kept fixed to the value of 0.1 N/mm², the Young's modulus and the Poisson's ratio of the membrane are set equal to E = 2,500 MPa and v = 0.3, respectively, and the membrane thickness is assumed equal to t = 0.125 mm.

In the same figure, a straight line starting from the origin and whose slope is the stiffness:

$$k_{el} = \frac{24E\pi R^3 t}{L^3},$$

represents the response of a standard linear elastic beam. As it can be seen, this straight line results tangent to the load-displacement curve of the inflated beam (obviously, the one regarding the displacement of the point of the line of axis in correspondence to the loaded point) within a good approximation. The tangent stiffness here obtained, equal to 0.66 N/mm, results quite close to that obtained numerically³ (0.68 N/mm) for a cantilever inflated beam having half the span of the beam considered here and the same geometrical and mechanical properties.



Figure 5: load-displacement curve for p = 0.1 N/mm² (E = 2500 N/mm², v = 0.3, t = 0.125 mm).

It is worth observing that the differences between the two load-displacement curves of Figure 5 increase linearly for low values of the load, i.e. when the central part of the inflated beam is slightly wrinkled only in proximity of the loaded point. The same difference results instead to be quite constant when the load approaches its limit value and a spreading of wrinkles in the central part of the inflated beam is observed. The increase in the vertical displacement due to the punching effect may also explain the commonly observed difference



between experimental and theoretical displacements⁴.

Figure 6: principal stresses in the membrane for vertical displacement of the point of application of the load equal to 164 mm (left half of the beam; $p = 0.1 \text{ N/mm}^2$, $E = 2500 \text{ N/mm}^2$, v = 0.3, t = 0.125 mm).

The stress distribution in the membrane obtained by the same non-linear FEM analysis confirms the presence of the wrinkled region in the middle of the beam. Moreover, it provides with useful hints about the magnitude of the transversal stresses and about the transmission of the shear force. As an example, the case illustrated in Figure 6 clearly shows that in the central part of the beam, where a wide wrinkled region is present, the circumferential tensile stresses are far from being uniform so the shear force is transmitted almost exclusively by the unwrinkled part of the cross-section.

The stiffening effect produced by an increasing internal pressure is showed in Figure 7. Here, the load vs. pressure diagram is obtained by imposing to the loaded point a fixed displacement of 84 mm and by checking the magnitude of the load for increasing pressure. Two different responses are recognizable depending on whether the pressure is lower or higher than a threshold value, which in the case examined turns out to be equal to about 0.4 N/mm^2 . In the low pressure range (p < 0.4 N/mm^2), the central part of the beam is in a wrinkled state and a small increase in the pressure results in a strong stiffening of the beam (to this regard, in Figure 7 the dashed straight line representing the separation curve between wrinkled and unwrinkled responses of the beam is plotted). For higher pressures instead (p > 0.4 N/mm^2), the beam is unwrinkled and the stiffening effect is due to the decrease in the



displacements in the small punched region just under the load.

Figure 7: load vs. pressure diagram for vertical displacement of the loaded point equal to 84 mm ($p = 0.1 \text{ N/mm}^2$, $E = 2500 \text{ N/mm}^2$, v = 0.3, t = 0.125 mm).

2.2 The built-in beam

The second case taken under consideration is that of the built-in inflatable beam showed in Figure 8. It is assumed that the built-in constraint is first imposed to both the ends of the beam, which initially lies in the *x*-*y* plane. Then the beam is inflated up to the final pressure *p* (accordingly, the central part of the beam becomes a cylinder of radius approximately equal to R = 40 mm) and finally the load 2F is applied. Once again, by virtue of symmetry, only one half of the beam has been modeled.



Figure 8: the built-in inflatable beam.

The inflated configuration together with the loaded one corresponding to a vertical displacement of the loaded point equal to 100 mm is showed in Figure 9.



Figure 9: a) the inflated configuration; b) equilibrium configuration corresponding to F =192 N (left half of the beam; red elements are in a wrinkled state; $p = 0.1 \text{ N/mm}^2$, $E = 2500 \text{ N/mm}^2$, v = 0.3, t = 0.125 mm)

Contrary to the simply-supported case examined in the preceding section, here a very small increase in the extension of the wrinkling regions (located under the point load and at the built-in ends) is observed as the force F grows.



Figure 10: load-displacement curve (a) and axial force-displacement curve (b) for $p = 0.1 \text{ N/mm}^2$ ($E = 2500 \text{ N/mm}^2$, v = 0.3, t = 0.125 mm).

The strongly different behaviour of the built-in beam is highlighted in the loaddisplacement diagram showed in Figure 10a. The beam shows progressive stiffening as the vertical displacement of the loaded point increases and no softening phase is observed. This may be easily explained by considering the relevant increase in the tensile longitudinal force which takes place in this loading process (Figure 10b) and by remembering that the limit value for the bending moment which corresponds to the onset of wrinkling in the crosssection of the beam is proportional to the axial force.

The influence of the pressure level on the load-displacement response of the beam is showed in Figure 11. It is worth observing that, contrary to the previous case of a simplysupported beam, here an increase in the pressure does not always have a beneficial effect. In effect, an optimum value for the pressure, whose value will depend upon the magnitude of the vertical displacement of the loaded point, seems to exist as shown in the figure.



Figure 11: load vs. pressure diagram for vertical displacement of the loaded point equal to 50 mm ($p = 0.1 \text{ N/mm}^2$, $E = 2500 \text{ N/mm}^2$, v = 0.3, t = 0.125 mm).

3 CONCLUSIONS

- Some features of the static behaviour of an inflated cylindrical beam subjected to a transversal concentrated load have been analyzed by means of a non-linear model which accounts for both large displacements and wrinkling of the membrane.
- The effects in terms of stress and of strain/displacements for increasing pressure and load levels have been examined for two beams: the first one simply-supported at its ends, the other built-in. The two beams showed completely different behaviours; moreover, in the case of the built-in beam, an optimum value for the pressure seems to exist for imposed load or displacement.

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AIR VOLUME ELEMENTS FOR DISTRIBUTION OF PRESSURE IN AIR CUSHION MEMBRANES

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Key words: Membrane Analysis, Inflatable membranes, Air Pressure, Air Cushion.

Summary. This paper describes techniques and presents examples for nonlinear finite element analysis on membranes interacting with an air volume.

1 INTRODUCTION

Extended use of air cushion membranes requires detailed analysis of pressure distribution and pressure transmission from one membrane to another. Standard finite element analysis, coupling membrane shell with real volume brick elements with specific air material properties, produces good results for semi-scaled deformations. The following figure shows a cushion loaded only on top-left side. The inner brick volume elements move to the right and distribute the pressure to the whole upper as well as to the lower membrane. A constant air pressure in the cushion volume elements (yellow) appears.



Figure 1: Air cushion with brick air volume elements

However, this technique is not applicable for larger deformations and air movements. Also the need of brick elements is not comfortable.

A technique without brick elements with a single stiffness bubble connecting all nodes of the membrane is implemented into a common software package and is presented in this paper. A similar approach is described in [1] with further references.

2 TECHNIQUE FOR AIR VOLUME BUBBLE

The finite element, connecting all nodes of the membrane with a single stiffness bubble, will be called VOLU element in this paper.

2.1 Principle of load stiffness method

Looking to a closed air volume with contact nodes on the boundary, we can easily determine the effect of the displacement of one node. If we move one node in z direction in the sketch below (v_i) , we get a reduction of the total volume. This volume reduction results in an air pressure increase inside. This pressure now acts on all nodes of the boundary with forces P_{ik} .



Figure 2: Load stiffness method: increased air pressure induced by deformation V_i of one node

This means that a displacement v_i in node i produces forces P_{ik} at all other nodes or one row in the air volume stiffness bubble S:





Figure 3: A single displacement V_i creates one row in the volume stiffness bubble S

As the volume reduction is v_i multiplied with the corresponding area a_i of the node, the pressure increase is $k*v_i*a_i/V0$, with V0 = total volume and k = compression modulus (100 kN/m2 for air). So S_{ik} is then pressure*area: $S_{ik} = k*v_i*a_i/V0$.

Of course in 3D the surface normal vector direction has to be taken into account. For the coarse finite element mesh in 3.1 (tennis court), we have 361 nodes with 3*361=1083 unknowns and get a triangular stiffness matrix containing n*(n+1)/2 = 586986 entries.

2.2 Global finite element analysis

Using the system of figure 2 we now get:

- 10*20 = 400 quad elements for the membrane with each 3*4=12 unknowns
- 1 VOLU air volume element with 3*10*20=600 unknowns

With these 401 elements the FE program can now solve the equation system for external loads. The quad elements create membrane forces while the VOLU yields only one single result – the air pressure inside.

2.3 Nonlinear effects

Geometric nonlinear behavior leads to the effect that the volume increase is not linear with the deformation because the quad element size may change as extremely shown in the balloon example 3.5. These effects must be balanced in a nonlinear iteration. The nonlinear iteration is necessary anyway for the geometric nonlinear membrane and the wrinkling effects. Also an air pressure on the membrane changes the direction with increasing deformations and must be updated (nonlinear effect).

2.4 Similar problem on slip cables

A similar approach is also used for slip cables in the SOFiSTiK software. Using a load stiffness method, all cables form one single slip cable stiffness. In the following example a displacement in one node of the slip cable ensemble (yellow) results in a length change and thus in a force change in all cable parts. Using the stiffness bubble technique this also works in just one single linear equation step. All cable parts work as one element!



Figure 4: One single slip cable element (yellow) connecting the upper and lower beams (blue)

3 EXAMPLES

3.1 Water bed

Starting with a plain mesh we first apply a prestress on the membrane. Then we apply a partial area load as shown in figure 5. We compare the analysis of the pure membrane with the analysis of the membrane plus a VOLU air element with a starting volume of 5000 m3. Input: VOLU NO 1 GRP 1 V0 5000. GRP 1 just defines the quad elements used for the air volume boundary.



Figure 5: Straight membrane: left without VOLU; right with VOLU air volume: uplift water bed effect

With the air volume element VOLU, the load induces vertical deformation and thus an air pressure increase in the volume below, uplifting the right part of the water bed. This already happens in a first linear equation step!

3.2 Tennis court

With the VOLU element we can also blow up the membrane over a tennis court. Defining a fixed pressure we now do not need the VOLU stiffness, because we do not want a pressure change due to the deformations. But the pressure load has to be updated during the nonlinear iteration because the load direction as well as the load area changes – the membrane area and so the volume surface increases. The membrane itself can be defined without stress change leading to a perfect formfinding with given membrane prestress.



Figure 6: Membrane formfinding with given membrane prestress and given VOLU air pressure

Using this formfinding shape and stress state we can now activate the real stiffness of the membrane as well as of the VOLU element and apply additional external loads as wind or snow. Also an inner air volume increase or an air temperature increase can be applied on the VOLU air volume element. The following result shows the effect of such a temperature loadcase in the VOLU element:

QUAD VOLUME RESULTS (VOLU) Loadcase 24 +Temperatur Luft +20 Gra NO V0 V-PLC V-now P-PLC P-now area-PLC area-now [m3] [m3] [m3] [kN/m2] [kN/m2] [m2] [m2] 1 5000.000 11164.30 11388.47 0.40 0.67 1536.901 1555.427 P: positive values = pressure P-start = P-PLC + k*DP/V-PLF + k*DT*alphaT with k=compression modulus

The starting volume defined for the initial mesh was 5000 m3 (4 m height under the straight membrane). The formfinding increases this initial volume to 11164 m3, the additional air temperature increases it further to 11388 m3. The initial formfinding air pressure of 0.40 kN/m2 increases to 0.67 N/m2, of course corresponding to a stress increase in the membrane. The actual geometric nonlinear surface of the VOLU element reaches 1555 m2 (plain mesh at the beginning was 37.5*33.5 = 1256 m2).

3.3 Air cushion

The same technique is now applied on two membranes. The VOLU element uses both membranes as surface. The starting volume is just the membrane area 4m*8m multiplied with the initial 10 mm distance = 0.32 m3.



Figure 7: Air cushion formfinding with given membrane prestress and given VOLU air pressure

In the formfinding step with 0.2 kN/m2 air pressure and 0.6 kN/m2 membrane prestress we get a volume of 21.472 m3:

QUAE) VOLUME	RESULTS (VOLU)			
Loadcase		1 Formfindung				
NO	V0	V-now	PO	P-now	area-0	area-now
	[m3]	[m3]	[kN/m2]	[kN/m2]	[m2]	[m2]
1	0.320	21.472	0.00	0.20	64.000	67.616
P:	positive	e values =	pressure			

This formfinding system is now frozen and can be loaded with real external actions. A wind gust loading applied only to the upper left part of the membrane, as shown in the brick example in figure 1, now behaves similar (higher load used). In comparison to figure 1, the analysis is now done without brick elements but with a not visible VOLU element.



Figure 8: Air pressure effect in an air cushion - air volume element VOLU acts but is not visible

The air pressure increases up to 0.44 kN/m2 and as in a real air cushion. Also the lower membrane is a part of the flexible system and acts with a stress increase. The air volume only decreases slightly from 21.472 to 21.420 m3 due to the given air compressibility:

```
OUAD VOLUME RESULTS (VOLU)
Loadcase
        11 Wind gust on upper folio
NO
        V0
              V-PLC
                      V-now P-PLC P-start P-now area-PLC area-now
                      [m3] [kN/m2] [kN/m2] [kN/m2]
      [m3]
              [m3]
                                                    [m2]
                                                             [m2]
     0.320 21.472 21.420
                                    0.20
                                                     67.616
                                                            67.921
1
                              0.20
                                            0.44
P: positive values = pressure
P-start = P-PLC + k*DP/V-PLF + k*DT*alphaT
                                         with k=compression modulus
```

3.4 Independent air surfaces

In the example above the upper and lower membranes are only connected by the VOLU element. This behavior can be shown more crass, if the two membranes are really separated widely. In the following system only the left membrane is loaded but also the right membrane reacts due to the given air volume inside the building.



Figure 9: Air pressure effect in a closed building - right membrane goes up on loading the left membrane

3.5 Blown up balloon

In a last example the VOLU element is used to blow up a membrane. In a normal membrane analysis without volume control this causes problems, because with increasing radius the necessary inner air pressure does not increase anymore. Here we get:



Figure 10: Balloon

4 CONCLUSIONS

The presented air volume technique allows numerous applications on membranes and inflatable structures. Simple examples explain the use of this exotic finite element.

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Advanced cutting pattern generation – Consideration of structural requirements in the optimization process

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Summary. This paper presents extensions to optimized cutting pattern generation through inverse engineering regarding structural requirements. The optimized cutting pattern generation through inverse engineering is a general approach for the cutting pattern generation which is based on the description of the underlying mechanical problem. The three dimensional surface, which is defined through the form finding process, represents the final structure after manufacturing. For this surface the coordinates in three dimensional space Ω_{3D} and the finally desired prestress state $\sigma_{\text{prestress}}$ are known. The aim is to find a surface in a two dimensional space Ω_{2D} which minimizes the difference between the elastic stresses $\sigma_{\text{el},2D\to 3D}$ arising through the manufacturing process and the final prestress. Thus the cutting pattern generation leads to an optimization problem, were the positions of the nodes in the two dimensional space Ω_{2D} are the design variables. In this paper various improvements to the method will be shown. The influence of the seam lines to the stress distribution in the membrane is investigated. Additionally, the control of equal edge length for associated patterns is an example for important enhancement.

1 INTRODUCTION

Membrane Structures are lightweight structures, which combine optimal stress state of the material with an impressive language of shapes [1]. The shape of membrane structures is defined by the equilibrium of surface stress and cable edge forces in tension. The process of find the shape in equilibrium is known as form finding. In the past a various number of methods are developed to solve the inverse problem of form finding [2] – [11]. Recent researches in the field of Fluid-Structure-Interaction accomplish the design process with the strong possibility of simulating the coupled problem of wind and membrane structure [12]. Throughout the whole design process of membrane structures the variation of prestress constitutes the main shaping parameter. Details of cutting pattern and compensation are affected by residual stresses from developing curved surfaces into the plane and anisotropic material properties. With the knowledge of this, numerical methods for the design and analysis of membrane structures should be able to deal with all sources of stress state in a proper way. In the next sections, methods for a proper cutting pattern generation of membrane

structures, which is able to treat the prestress and residual states of stress in a correct continuum mechanical way.

2 CUTTING PATTERN GENERATION

It is well known that a general doubly curved surface cannot be developed into a plane without compromises which results in additional residual stresses when the structure will be erected. In addition, the elastic deformation due to pre-stress has to be compensated. Typically, a two stage procedure is applied consisting of (i) forced flattening of the curved surface into a plane by pure geometrical considerations and (ii) compensation of both, the intended pre-stress and the additional elastic stresses of the flattening procedure [13]. Usually, an additional problem occurs if the flattening strains are determined using the curved, final surface as undeformed reference geometry, thus neglecting the correct erecting procedure. In highly curved regions the error might be remarkably large.

2.1 Optimized Cutting Pattern Generation

An alternative approach is suggested which uses ideas from the inverse engineering [14], [15]. The idea is to correctly simulate the erection procedure from a plane cutting pattern as undeformed reference geometry to the final deformed surface as defined by the design stresses. Consequently, the definition of strains is non-conventional, i.e. inverse, as the coordinates of the undeformed cutting pattern are introduced as unknowns. The proper cutting pattern layout is found by an optimization technique by minimizing the deviation of the stresses due to pre-stress and elastic deformation from the defined stress distribution of form finding. The advantages of this procedure are that (i) the true erection process is modelled, (ii) automatically, all sources of stress deviation are correctly resolved, e.g. residual stresses from development, (iii) all mechanical and geometrical reasons of compensation are considered, and (iv) again, the procedure is consistently embedded into non-linear continuum mechanics allowing for a formulation as close as possible to reality. Figure 1 roughly visualizes the optimization procedure. The challenge in this way of doing cutting pattern generation is to handle the numerical solution strategies for the optimization problem. In [15] two different methods of solving the optimization problem are reviewed. Both of them are well known methods for solving unconstrained optimization problems. In the following, both methods are briefly discussed.

Method I: Least-squares optimization

$$\min_{\mathbf{X}} \to \Pi = \frac{1}{2} t \int_{\Omega_{3D}} (\boldsymbol{\sigma}_{el,2D\to3D} - \boldsymbol{\sigma}_{ps}) : (\boldsymbol{\sigma}_{el,2D\to3D} - \boldsymbol{\sigma}_{ps}) da_{3D}$$
(1)



Fig. 1: Optimization problem for cutting pattern generation

A necessary condition for a minimum in the least squares is a stationary point in the functional Π w.r.t. a variation in the reference geometry. This results in the following variational form of the optimization problem:

$$\delta w^{I} = t \int_{\Omega_{3D}} \left(\boldsymbol{\sigma}_{el,2D \to 3D} - \boldsymbol{\sigma}_{ps} \right) : \frac{\partial \boldsymbol{\sigma}_{el,2D \to 3D}}{\partial \mathbf{X}} \partial \mathbf{X} da_{3D} = 0$$
(2)

To solve the nonlinear field equation (2) standard procedures like the finite element method in combination with the Newton-Raphson method are applied. It turned out in [15] that equation (2) shows a very small convergence radius due to the non-convex character of the optimization problem. This disadvantage could be avoided by method II.

Method II: Minimization of the "stress difference" energy

The idea of method II is to solve the optimization of the stress difference in a "mean way". To do so a Galerkin approach is used. In this case the stress difference is integrated over the domain and a weighting function is applied to the stress difference. Again, a variation of the equation should lead to the optimum cutting pattern (see eq. (3)).

$$\delta w^{II} = t \int_{\Omega_3} \left(\boldsymbol{\sigma}_{el,2D \to 3D} - \boldsymbol{\sigma}_{ps} \right) : \delta \boldsymbol{\eta} da_{3D} = 0$$
(3)

To avoid the non-convexity of the optimization the weighting function is chosen as the Euler-

Almansi strains since they are energetically conjugated to Cauchy stresses (see eq. (4)).

$$\delta w^{II} = t \int_{\Omega_3} (\mathbf{\sigma}_{el,2D\to 3D} - \mathbf{\sigma}_{ps}) : \frac{\partial \mathbf{e}_{el,2D\to 3D}}{\partial \mathbf{X}} \delta \mathbf{X} da_{3D} = 0$$
(4)

With both methods the resulting cutting patterns are only slightly different. A detailed comparison and more details about the numerical issues of both methods could be found in [14] - [15].

2.2 Consideration of seam stiffness in the computation

After the computation of the cutting patterns, an important step is quantification of the residual stresses in the membrane. These are occurring due to the fact that a non-developable surface could not be developed into a flat surface without any compromises. This results in residual stresses after the simulation of the erection process. From a continuum mechanical point of view, this means that the cutting patterns are the reference configuration and the configuration from form finding is an intermediate stage which is not totally in equilibrium. Performing a geometrical nonlinear analysis to this configuration will result in a slight displacement which ends up in the residual stresses. Starting with this in mind, the exact description of the erection process is crucial. There are several of influences to the computation. One of them is the correct description of the geometry, respectively the structural members of the structure. From this point of view it is totally clear that the modelling of the seams is an important step to the "right" structural model. The influence of the seams is related to the fact that the membrane is doubled in these regions since overlapping material is needed for joining the adjacent strips together. To take this issue into account in the computation, cable elements are included which consider the stiffness from this doubled material. The question which arises is how large is the influence of this doubled material in the membrane? There is no unique answer this. Typical examples for a noticeable effect of the concentration of material are highpoints of membranes were a concentration of seam lines is located. Figure 2 shows the well known Chinese hat membrane were the seam lines are concentrated towards the upper ring. The diagram in figure 2 shows the change in stresses σ in the membrane and change in forces F in the cable elements by increasing the cross section area A of the seam line cable elements. It can be seen that the membrane stresses are decreasing while the forces in the cable elements increase for larger seam line cross section areas. What is very interesting in this example is the amount of stress decrease in the membrane. For a ratio of two for A to Amax the membrane stresses decrease to 65% of the initial stresses. This effect is not equal for each kind of membrane and each situation but this example shows that the effect should be investigated and included in each computation of a membrane structure.



Fig.2: Influence of seam line stiffness

2.3 Control the same length of adjacent edges in cutting pattern generation

When building a membrane structure the process of erection is structured like (i) take the patterns of the membrane and join them together, (ii) erect the primer structure (e.g. steel frames) and (iii) mount the membrane into it. In the first step of joining the patterns it is obvious that the adjacent edges of the pattern need to fit together or at least must have the same length. When neglecting this constraint the resulting membrane will show wrinkles along the seam line due to the fact that the initial strains are included into the membrane by joining unequal edges together. From the numerical point of view the requirement of same edge length of adjacent edges shows up in an equality constraint in the optimization problem. In figure 3 the requirement of same edge length of adjacent edges is shown for the example of a four point tent consisting of 6 patterns with 5 seam lines.



Fig. 3: Same length of adjacent patterns

From a mathematical point of view the description of the equality constraint is rather simple. The differences of the lengths should be 0 for all seams. That allows the formulation of an equality constraint for each seam which is shown in equation (5).

$$h_i(\mathbf{X}) = \Delta L_i = 0 \tag{5}$$

All differences of the lengths are added to get a single equation of the equality constraint. To avoid cancellations from positive and negative length differences the square of the values is chosen. With this equality constraint the optimization problem by using the Least-Squares approach is stated in equation (6).

$$\min \to f(\mathbf{X}) = \frac{1}{2} t \int_{\Omega_{3D}} (\boldsymbol{\sigma}_{el,2D\to3D} - \boldsymbol{\sigma}_{ps}) : (\boldsymbol{\sigma}_{el,2D\to3D} - \boldsymbol{\sigma}_{ps}) da_{3D}$$
such that : $h_i(X) = \Delta L_i = 0$
(6)

Knowing from section 2.1 that the objective function has a relatively small convergence radius the starting point is a crucial issue to the optimization problem. Additionally we know from 2.1 that the method II (Minimization of the "stress difference" energy) is a suitable related problem and provides a very good starting point for the optimization problem. In that sense a staggered approach is used to solve the optimization problem: First solve the unconstraint problem with method II. Use the resulting cutting pattern as the starting

geometry for the constraint optimization problem which can be tackled by various optimization algorithms. The chosen solution strategy herein is the Lagrange method. The Lagrangian function for the unconstraint problem is given in equation (7).

$$L(\mathbf{X}, \boldsymbol{\mu}_i) = f(\mathbf{X}) + \boldsymbol{\mu}_i \boldsymbol{h}_i(\mathbf{X})$$
⁽⁷⁾

The corresponding Karush-Kuhn-Tucker conditions are given in equation (8).

$$\nabla f(\mathbf{X}) + \mu \nabla h_i(\mathbf{X}) = 0$$

$$h_i(\mathbf{X}) = 0$$
(8)

For solving this nonlinear system of equations the Newton-Raphson method can be used.

3 CONCLUSIONS

Computational methods for the optimized cutting pattern generation have been presented. The governing equations for cutting pattern generation were derived by using the idea of inverse engineering from a full continuum mechanical approach. Improvements regarding structural requirements have been included into the optimization process. Concerning seam line stiffness and controlling length of adjacent patterns are tackled directly in optimization process of the cutting pattern

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A ROTATION FREE SHELL TRIANGLE WITH EMBEDDED STIFFENERS

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Abstract. In this paper a rotation free shell element with embedded stiffeners is presented. The element is based on a previous one where the membrane and bending strains are obtained using a patch of four triangular elements centered on the analyzed one. The stiffener is located between two adjacent elements, thus its position is defined by the two end nodes of the corresponding triangle side. The curvature of the stiffener in the tangent plane to the surface is disregarded as it is assumed that the surface is quite rigid in its plane. The torsion and surface normal curvature of the stiffener are computed from the curvatures of its two adjacent elements. A classical beam theory is used for the stiffener disregarding shear strains while the axial strain is standard. An example is presented for a preliminary assessment of the developed element.

1 INTRODUCTION

Rotation-free thin shell elements are being used with success for the simulation of different problems. Three of the main advantages of rotation-free elements over standard elements are: a)as rotation DOFs are not included, the total number of degrees of freedom is drastically reduced (typically to 50% or 60%) with important savings in both storage and CPU time, b)problems associated with rotation vectors or local triads (non-symmetric matrices for instance), that are in general costly and difficult to parametrize and update do not appear, and c)no special techniques are necessary to deal with problems appearing in the thin shell limit (e.g. shear locking). Some drawbacks also exist, we can mention: a) sensitivity to irregular nodes (a regular node is one shared by 6 elements), b) a direct combination with other finite element types, like beam or solid elements, is not straightforward and c) coding may be more involved. Probably thin sheet metal forming is the most extended application of rotation-free elements but it is not by no means the only one. They have been used for general shell analysis and with special success to assess the behavior of elastic membranes and fabrics where the inclusion of bending is necessary to obtain detailed deformed configurations, see for example references [1, 2, 3, 4] to mention just a few. In many situations the shells include stiffeners and presently it is not possible to join rotation-free shells with standard beam elements.

In this paper a rotation free shell element with embedded stiffeners is presented. The element is based on a previous one[5] where the membrane and bending strains are obtained using a patch of four triangular elements centered on the analyzed one. The stiffeners are located between two adjacent elements and can-not be located across an element, thus its position is defined by the two end nodes of the corresponding triangle side. The curvature of the stiffener in the tangent plane to the surface is disregarded as it is assumed that the surface is quite rigid in its plane. The torsion and and surface normal curvature of the stiffener ares computed from the curvatures of the its two adjacent elements. A classical beam theory is used for the stiffener disregarding shear strains while the axial strain is standard.

The initial target of this element is to simulate the behavior of insect wings with orthotropic properties with large displacements and small strains, but the possible applications are wide. An example is presented for a preliminary assessment of the developed element.

2 ENHANCED ROTATION-FREE SHELL TRIANGLE

In this section a brief summary of the rotation-free shell triangle used in this work. More details can be found in the original references [5, 6, 7]. The starting point of the rotation-free so-called basic shell triangle (BST) is to discretize the shell surface with a standard 3-node triangular mesh. The difference with a standard finite element method is that, for the computation of strains within an element, the configuration of the three adjacent triangular elements is also used. Then, at each triangle a, four-element-patch formed by the central triangle and the three adjacent ones is considered (see Figure 1.a).

In the original rotation-free BST element the displacement field was linearly interpolated from the nodal values within each triangle [7] leading to a constant membrane field. The curvature field over each triangular element was computed using information from the displacements of the three adjacent triangles [5]. In this work we use the enhanced basic shell triangle (EBST) formulation as described in [6]. The displacement field in the EBST element is interpreted quadratic for the nodal displacement values at the six nodes of the four-element patch of Figure 1.



Figure 1: Patch of triangles for computation of strains in the EBST element M

2.1 Membrane strains computation

We use a standard quadratic approximation of the shell geometry over the 6-node patch of triangle (Figure 1) as

$$\boldsymbol{\varphi} = \sum_{i=1}^{6} L^{i} \boldsymbol{\varphi}^{i} = \sum_{i=1}^{6} L^{i} (\boldsymbol{\varphi}_{0}^{i} + \mathbf{u}^{i})$$
(1)

where $\varphi^i = [x_1^i, x_2^i, x_3^i]^T$ is the position vector of node i, φ_0^i is the position vector at the initial configuration, $\mathbf{u}^i = [u_1^i, u_2^i, u_3^i]^T$ is the displacement vector and

$$L^{1} = \eta^{1} + \eta^{2}\eta^{3} \qquad L^{2} = \eta^{2} + \eta^{3}\eta^{1} \qquad L^{3} = \eta^{3} + \eta^{1}\eta^{2}$$

$$L^{4} = \frac{\eta^{1}}{2}(\eta^{1} - 1) \qquad L^{5} = \frac{\eta^{2}}{2}(\eta^{2} - 1) \qquad L^{6} = \frac{\eta^{3}}{2}(\eta^{3} - 1)$$
(2a)

with η^1 and η^2 the natural coordinates (also area coordinates) in the parametric space (see Figure 1.b) and $\eta^3 = 1 - \eta^1 - \eta^2$.

Note in Figure 1 that, as usual, for the numeration of the sides and the adjacent elements the opposite local node is used, and naturally the same numeration is used for the mid-side points G. Note also that the numeration of the rest of the nodes in the patch begins with the node opposite to local node 1, then each extra node and each mid-side point can be easily referenced.

From Eq.(1) the gradient at each mid-side G point of the central triangle M with

respect to a local in-plane Cartesian system $(x_1 - x_2)$ can be written as:

$$\begin{bmatrix} \varphi_{'1} \\ \varphi_{'2} \end{bmatrix}^{(I)} = \begin{bmatrix} L_{'1}^1 & L_{'1}^2 & L_{'1}^3 & L_{'1}^{I+3} \\ L_{'2}^1 & L_{'2}^2 & L_{'2}^3 & L_{'2}^{I+3} \end{bmatrix}^{(I)} \begin{bmatrix} \varphi^1 \\ \varphi^2 \\ \varphi^3 \\ \varphi^3 \\ \varphi^{I+3} \end{bmatrix}$$
(3)

Note that the gradient depends on the 3 nodes of the main element and (only) on the extra node (I+3), associated to the side (I). This fact implies that a unique value will be obtained for the gradient when it is evaluated from any of the two neighbor elements. In Eq. (3) the super index surrounded by brackets indicate evaluated at the center of side I, while the super index on nodal shape functions and nodal coordinates indicate the node.

Defining the metric tensor at each mid-side point

$$g_{\alpha\beta}^{(I)} = \varphi_{\prime\alpha}^{(I)} \cdot \varphi_{\prime\beta}^{(I)} \tag{4}$$

a linear interpolation can be defined over the element as

$$\mathbf{g} = (1 - 2\eta^1) \,\mathbf{g}^{(1)} + (1 - 2\eta^2) \,\mathbf{g}^{(2)} + (1 - 2\eta^3) \,\mathbf{g}^{(3)}$$
(5)

and any convenient Lagrangian strain measure \mathbf{E} can be computed from it

$$\mathbf{E} = f\left(\mathbf{g}\right) \tag{6}$$

We note that the definition of \mathbf{g} in Eq.(5) is equivalent to using a linear "assumed strain" approach [5, 7].

In our case a unique point is used at the element center with the average of the metric tensors computed at mid-side points. This is equivalent to using one point quadrature for the assumed strain field.

$$\mathbf{g}\left(\boldsymbol{\eta}\right) = \frac{1}{3} \left(\mathbf{g}^{(1)} + \mathbf{g}^{(2)} + \mathbf{g}^{(3)}\right) \tag{7}$$

The Green-Lagrange strain tensor on the middle surface is used here. This can be readily obtained from Eqs. (3), (4) and (7)

$$\mathbf{E}_{GL} = \frac{1}{2} \begin{bmatrix} g_{11} - 1 & g_{12} \\ g_{12} & g_{22} - 1 \end{bmatrix}$$
(8)

The membrane strains variation is:

$$\delta \begin{bmatrix} E_{11} \\ E_{22} \\ 2E_{12} \end{bmatrix} = \frac{1}{3} \sum_{I=1}^{3} \sum_{J=1}^{4} \begin{bmatrix} L_{\prime_{1}}^{J(I)} \varphi_{\prime_{1}}^{(I)} \cdot \delta \mathbf{u}^{J} \\ L_{\prime_{2}}^{J(I)} \varphi_{\prime_{2}}^{(I)} \cdot \delta \mathbf{u}^{J} \\ L_{\prime_{2}}^{J(I)} \varphi_{\prime_{1}}^{(I)} \cdot \delta \mathbf{u}^{J} + L_{\prime_{1}}^{J(I)} \varphi_{\prime_{2}}^{(I)} \cdot \delta \mathbf{u}^{J} \end{bmatrix} = \mathbf{B}_{m} \delta \mathbf{a}^{p} \qquad (9)$$

where for each mid-side point (G = I) there are contributions from the 4 nodes (J). In Eq.(15) \mathbf{B}_m is the membrane strain-displacement matrix and \mathbf{a}^p is the patch displacement vector. The form of \mathbf{B}_m can be found in [5, 7]. The element is then non-conforming. However, it satisfies the "patch test", and the approach can be used for large displacement problems [5].

2.2 Computation of curvatures

Curvatures will be assumed to be constant within each element. An averaging of the curvatures $\kappa_{\alpha\beta}$ is made over the element in a mean integral sense as

$$\kappa_{\alpha\beta} = \frac{-1}{A} \int_{A} \mathbf{t}_{3} \cdot \boldsymbol{\varphi}_{\beta\alpha} \, dA \tag{10}$$

Integrating by parts the right hand side of previous equation gives

$$\kappa_{\alpha\beta} = \frac{1}{A} \oint_{\Gamma} n_{\alpha} \mathbf{t}_{3} \cdot \boldsymbol{\varphi}_{\beta} d\Gamma$$
(11)

$$\begin{bmatrix} \kappa_{11} \\ \kappa_{22} \\ 2\kappa_{12} \end{bmatrix} = \frac{-1}{A} \oint_{\Gamma} \begin{bmatrix} n_1 & 0 \\ 0 & n_2 \\ n_2 & n_1 \end{bmatrix} \begin{bmatrix} \mathbf{t}_3 \cdot \boldsymbol{\varphi}_{\prime_1} \\ \mathbf{t}_3 \cdot \boldsymbol{\varphi}_{\prime_2} \end{bmatrix} d\Gamma$$
(12)

Adopting one-point integration on each side and using the standard area coordinates (η^i) derivatives we have

$$\begin{bmatrix} \kappa_{11} \\ \kappa_{22} \\ 2\kappa_{12} \end{bmatrix} = -2\sum_{I=1}^{3} \begin{bmatrix} \eta_{I_1}^I & 0 \\ 0 & \eta_{I_2}^I \\ \eta_{I_2}^I & \eta_{I_1}^I \end{bmatrix} \begin{bmatrix} \boldsymbol{\varphi}_{I_1} \cdot \mathbf{t}_3 \\ \boldsymbol{\varphi}_{I_2} \cdot \mathbf{t}_3 \end{bmatrix}^{(I)}$$
(13)

where A is the element area and \mathbf{t}_3 is the normal to the central triangle M. The gradient $\varphi_{\prime_{\alpha}}$ at each mid-side point G is computed from Eq.(2). Other alternatives for computing $\varphi_{\prime_{\alpha}}$ are possible as discussed in [5]. The stretching of the shell in the normal direction is defined by a parameter λ as

$$\lambda = \frac{h}{h^0} = \frac{A^0}{A} \tag{14}$$

where h and h^0 are the actual and original thickness, respectively.

The second equality assumes that the deformation is isochoric (and elastic). The assumption that the fiber originally normal to the surface in the reference configuration is also normal to the surface in the current configuration (Kirchhoff hypothesis) is adopted herein.

Curvature-displacement variations are more involved. The resulting expression is (see[5, 7] for details)

$$\delta \boldsymbol{\kappa} = \delta \begin{bmatrix} \kappa_{11} \\ \kappa_{22} \\ 2\kappa_{12} \end{bmatrix} = 2 \sum_{I=1}^{3} \begin{bmatrix} \eta_{I_{1}}^{I} & 0 \\ 0 & \eta_{I_{2}}^{I} \\ \eta_{I_{2}}^{I} & \eta_{I_{1}}^{I} \end{bmatrix} \sum_{J=1}^{4} \begin{bmatrix} L_{I_{1}}^{J(I)} \left(\mathbf{t}_{3} \cdot \delta \mathbf{u}^{J} \right) \\ L_{I_{2}}^{J(I)} \left(\mathbf{t}_{3} \cdot \delta \mathbf{u}^{J} \right) \end{bmatrix} - 2 \sum_{I=1}^{3} \begin{bmatrix} \left(\eta_{I_{1}}^{I} \rho_{11}^{1} + \eta_{I_{2}}^{I} \rho_{11}^{2} \\ \eta_{I_{1}}^{I} \rho_{12}^{1} + \eta_{I_{2}}^{I} \rho_{22}^{2} \\ \eta_{I_{1}}^{I} \rho_{12}^{1} + \eta_{I_{2}}^{I} \rho_{12}^{2} \end{bmatrix} \left(\mathbf{t}_{3} \cdot \delta \mathbf{u}^{I} \right) = \mathbf{B}_{b} \delta \mathbf{a}^{p}$$
(15)

where the projections of the vectors $\mathbf{h}_{\alpha\beta}$ over the contravariant base vectors $\tilde{\boldsymbol{\varphi}}_{\prime\alpha}$ have been included

$$\rho^{\delta}_{\alpha\beta} = \mathbf{h}_{\alpha\beta} \cdot \tilde{\boldsymbol{\varphi}}_{\prime\delta} \tag{16}$$

where

$$\mathbf{h}_{\alpha\beta} = \sum_{I=1}^{3} \left(\eta^{I}_{\prime\alpha} \, \boldsymbol{\varphi}^{(I)}_{\prime\beta} + \eta^{I}_{\prime\beta} \, \boldsymbol{\varphi}^{(I)}_{\prime\alpha} \right) \tag{17}$$

The form of the bending strain matrix \mathbf{B}_b can be found in [5, 7].

3 EMBEDDED STIFFENERS

The stiffener as a beam element may include the following forces and moments:

- Normal force N
- Shear forces T_2 and T_3
- Twisting Moment M_1
- Bending Moment with two components:
 - a) on the tangent plane of she surface M_2
 - **b)** normal to the surface M_3

For the formulation of the embedded stiffeners some simplifications are adopted and some limitations appear as a consequence of the lack of rotational DOFs, essential in any beam theory:

- The stiffener axis is on the shell middle surface, i.e. eccentric beams are precluded. This is a very important restriction as it excludes many structural elements present in ship, plane or car structures.
- The surface where stiffeners are embedded is assumed smooth, without kinks or branching.
- The stiffeners are located along the common side of two shell triangular elements. Stiffeners between two arbitrary nodes are not allowed. It is possible to include a stiffener along the shell boundary (with just one adjacent shell element)
- The classical Bernoulli beam theory is used, disregarding transverse shear strains due to shear forces T_2 and T_3 .
- The beam curvature in the surface tangent plane is disregarded, so the influence of the bending moment component M_3 is assumed negligible. This simplification steams from the assumption that the shell membrane stiffness is high compared with the beam bending stiffness.

Then the geometry of the stiffener is defined by its two end nodes that have to coincide with two element sides or one element side along the shell boundary. These two nodes (J and K) are enough to define the axial behavior (similar to a truss element). The Green-Lagrange strain in terms of the axial stretch λ is:

$$\lambda = \frac{L}{L_0} = \frac{\left\| \mathbf{x}^K - \mathbf{x}^J \right\|}{\left\| \mathbf{X}^K - \mathbf{X}^J \right\|}$$
(18)

$$\varepsilon = \frac{1}{2} \left(\lambda^2 - 1 \right) \tag{19}$$

For the bending and twisting curvatures the shell adjacent elements are considered. On each element (I = 1, 2) the surface curvature tensor κ^{I} defined on a arbitrary local Cartesian system is

$$\kappa^{I} = \begin{bmatrix} \kappa_{11} & \kappa_{12} \\ \kappa_{21} & \kappa_{22} \end{bmatrix}^{I}$$
(20)

If the side direction in that local system is defined by components (s_1, s_2) , then the curvature vector associated to the side can be written as:

$$\kappa_s^I = \begin{bmatrix} \kappa_{11} & \kappa_{12} \\ \kappa_{21} & \kappa_{22} \end{bmatrix}^I \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$$
(21)

Projecting this vector along the side (twisting component) and in the normal direction (bending), the bending and twisting curvatures are:

$$\begin{bmatrix} \kappa_b^I \\ \kappa_t^I \end{bmatrix} = \begin{bmatrix} s_1 & s_2 \\ -s_2 & s_1 \end{bmatrix} \begin{bmatrix} \kappa_s^I \end{bmatrix} = \begin{bmatrix} s_1 & s_2 \\ -s_2 & s_1 \end{bmatrix} \begin{bmatrix} \kappa_{11} & \kappa_{12} \\ \kappa_{21} & \kappa_{22} \end{bmatrix}^I \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$$
(22)

$$= \begin{bmatrix} s_1^2 & s_2^2 & s_1 s_2 \\ s_1 s_2 & -s_1 s_2 & \frac{s_2^2 - s_1^2}{2} \end{bmatrix} \begin{bmatrix} \kappa_{11} \\ \kappa_{22} \\ 2\kappa_{12} \end{bmatrix}$$
(23)

With the curvature values obtained from each adjacent element a weighted average can be computed to define the stiffener curvatures. Here the inverse of each element area has been used as weighting factor:

$$\begin{bmatrix} \kappa_b \\ \kappa_t \end{bmatrix} = \left\{ \frac{1}{A^1} \begin{bmatrix} \kappa_b \\ \kappa_t \end{bmatrix}^1 + \frac{1}{A^2} \begin{bmatrix} \kappa_b \\ \kappa_t \end{bmatrix}^2 \right\} \left[\frac{1}{A^1} + \frac{1}{A^2} \right]^{-1}$$
(24)

$$= c_1 \begin{bmatrix} \kappa_b \\ \kappa_t \end{bmatrix}^1 + c_2 \begin{bmatrix} \kappa_b \\ \kappa_t \end{bmatrix}^2$$
(25)

$$= \begin{bmatrix} \kappa_b \\ \kappa_t \end{bmatrix}_1 + \begin{bmatrix} \kappa_b \\ \kappa_t \end{bmatrix}_2$$
(26)

The force and moments are computed assuming a linear elastic material behavior:

$$\begin{bmatrix} N\\ M_b\\ M_t \end{bmatrix} = \begin{bmatrix} EA & & \\ & EI & \\ & & GJ \end{bmatrix} \begin{bmatrix} \varepsilon\\ \kappa_b\\ \kappa_t \end{bmatrix}$$
(27)

For the computation of the equivalent nodal force and the stiffness matrix we have:

• Axial Force: the Green-Lagrange strain variation can be written as:

$$\delta \varepsilon = \lambda \ \delta \lambda = \frac{1}{L_0^2} \begin{bmatrix} -\Delta \mathbf{x}^T, \Delta \mathbf{x}^T \end{bmatrix} \begin{bmatrix} \delta \mathbf{u}^J \\ \delta \mathbf{u}^K \end{bmatrix}$$
(28)

where $\Delta \mathbf{x} = \mathbf{x}^K - \mathbf{x}^J$. The equivalent nodal forces are:

$$\begin{bmatrix} \mathbf{r}^{J} \\ \mathbf{r}^{K} \end{bmatrix} = \frac{N}{L_{0}} \begin{bmatrix} -\Delta \mathbf{x}, \\ \Delta \mathbf{x} \end{bmatrix}$$
(29)

For the stiffness matrix, the material and geometric parts may be distinguished:

$$\mathbf{K}_{M} = \frac{EA}{L_{0}^{3}} \begin{bmatrix} \mathbf{\Delta} \mathbf{x} \, \mathbf{\Delta} \mathbf{x}^{T} & -\mathbf{\Delta} \mathbf{x} \, \mathbf{\Delta} \mathbf{x}^{T} \\ -\mathbf{\Delta} \mathbf{x} \, \mathbf{\Delta} \mathbf{x}^{T} & \mathbf{\Delta} \mathbf{x} \, \mathbf{\Delta} \mathbf{x}^{T} \end{bmatrix}$$
(30)

$$\mathbf{K}_{G} = \frac{N}{L_{0}} \left\{ \begin{bmatrix} \mathbf{1} & -\mathbf{1} \\ -\mathbf{1} & \mathbf{1} \end{bmatrix} \right\}$$
(31)

where ${\bf 1}$ is the unit tensor .

• Moments: the contributions from each adjacent element are summed as:

$$\begin{bmatrix} M_b \\ M_t \end{bmatrix} = \begin{bmatrix} M_b \\ M_t \end{bmatrix}_1 + \begin{bmatrix} M_b \\ M_t \end{bmatrix}_2$$
(32)

For the curvature variations the bending \mathbf{B} matrix corresponding to each element are consistently used:

$$\left[\delta\kappa^{I}\right] = \mathbf{B}^{I}\delta\mathbf{a}^{I} \tag{33}$$

where the vector $\delta \mathbf{a}^{I}$ gathers the virtual displacements of the patch of elements associated to element *I*. Where it is possible to write:

$$\delta \begin{bmatrix} \kappa_b \\ \kappa_t \end{bmatrix}^I = \begin{bmatrix} s_1^2 & s_2^2 & s_1 s_2 \\ s_1 s_2 & -s_1 s_2 & \frac{s_2^2 - s_1^2}{2} \end{bmatrix} \mathbf{B}^I \delta \mathbf{a}^I$$
(34)

Here the equivalent nodal forces will be computed as the sum of the independent contributions of each adjacent element. This is not consistent but quite easier to implement. Using the curvatures variations the contributions from each shell element is:

$$\mathbf{r}_{1}^{I} = \left(\mathbf{B}^{I}\right)^{T} \begin{bmatrix} s_{1}^{2} & s_{2}^{2} & s_{1}s_{2} \\ s_{1}s_{2} & -s_{1}s_{2} & \frac{s_{2}^{2}-s_{1}^{2}}{2} \end{bmatrix}^{T} \begin{bmatrix} M_{b} \\ M_{t} \end{bmatrix}^{I} L_{0} = \left(\mathbf{B}^{I}\right)^{T} \begin{bmatrix} s_{1}^{2} & M_{b}^{I} + s_{1}s_{2}M_{t}^{I} \\ s_{2}^{2} & M_{b}^{I} - s_{1}s_{2}M_{t}^{I} \\ s_{1}s_{2} & M_{b}^{I} + \left(\frac{s_{2}^{2}-s_{1}^{2}}{2}\right)M_{t}^{I} \end{bmatrix} L_{0}$$

$$(35)$$

Finally the contribution to the material part of the stiffness matrix is (the geometric part is assumed negligible):

$$\mathbf{K}_{M}^{I} = \left(\mathbf{B}^{I}\right)^{T} \begin{bmatrix} s_{1}^{2} & s_{2}^{2} & s_{1}s_{2} \\ s_{1}s_{2} & -s_{1}s_{2} & \frac{s_{2}^{2}-s_{1}^{2}}{2} \end{bmatrix}^{T} \begin{bmatrix} EI \\ GJ \end{bmatrix} \begin{bmatrix} s_{1}^{2} & s_{2}^{2} & s_{1}s_{2} \\ s_{1}s_{2} & -s_{1}s_{2} & \frac{s_{2}^{2}-s_{1}^{2}}{2} \end{bmatrix} \mathbf{B}^{I} L_{0}c_{i}$$

$$(36)$$

$$= \left(\mathbf{B}^{I}\right)^{T} \begin{bmatrix} EIs_{1}^{4} + GJs_{1}^{2}s_{2}^{2} & (EI - GJ)s_{1}^{2}s_{2}^{2} & (EIs_{1}^{2} + GJ\frac{s_{2}^{2}-s_{1}^{2}}{2})s_{1}s_{2} \\ EIs_{2}^{4} + GJs_{1}^{2}s_{2}^{2} & (EIs_{2}^{2} - GJ\frac{s_{2}^{2}-s_{1}^{2}}{2})s_{1}s_{2} \\ symmetric & EIs_{1}^{2}s_{2}^{2} + GJ\left(\frac{s_{2}^{2}-s_{1}^{2}}{2}\right)^{2} \end{bmatrix} \mathbf{B}^{I} L_{0}c_{i}$$

$$(37)$$

4 BOUNDARY CONDITIONS

The restrictions on nodal translations do no present any difficulty. On the other side the restrictions on nodal rotations have influence on the computation of the shell element curvatures used by the stiffeners. The details for the treatment of boundary conditions on rotation-free shell elements can be seen in the references [5, 6].

When a stiffener is located along the shell boundary, it will have just one adjacent element. Figure 2 shows a stiffener defined by nodes J - K along an element boundary with one of its nodes clamped. If the shell is restrained to rotate along line J - K3 the influence on the stiffener is weaker than if it were side I - K the constrained one. This fact may imply different behaviors when imposing clamped or symmetry boundary conditions, obtaining non symmetrical fields with identical discretization but with triangles in different orientations.

To alleviate this effect, the computation of curvatures for the stiffeners on the boundary is also computed using a weighted average, but now between the only adjacent element and the nearest non-adjacent element.

5 NUMERICAL EXAMPLE

A thin square plate (side a = 10m and thickness t = 0.05m), reinforced with beams (square cross section with b = 0.20m) every 1.25m in both directions, is subjected to a



Figure 2: Stiffener at the shell boundary

uniform transversal step load q = 1KN/m². Material properties considered are (for both plate and stiffeners) E = 200 GPa, $\nu = 0.3$ and $\rho = 1000$ Kg/m³.

The discretization includes 128 triangular shell elements over one quarter of the plate (doble symmetry is considered) and 64 stiffeners. For comparison the same problem is discretized (same nodes) using a 4-node quadrilateral shear deformable shell element (SHELQ) and a 2-node shear deformable beam element (BEAME).

For reference Figure 3.a shows the displacement of the center of the plate as a function of time for the un-reinforced model (plate only). It can be seen the the rotation-free triangle provides a more flexible model than the quadrilateral. The Figure 3.b plots the displacement of the center of the plate as a function of time for both the present formulation (RBST) and the standard model including rotational degrees of freedom (SHELQ-BEAME). Again the rotation-free model shows a more flexible behavior than a standard model including rotational DOFs.

Finally Figure 4 shows contour-fills of the transversal displacement for t = 0.1. On the left the model with rotational DOFs and the present formulation on the right.

CONCLUSIONS

A rotation free shell element with embedded stiffeners has been presented. Presently the formulation has same limitations but can be applied to a large class of problems. A simple example is shown for a preliminary assessment of the element bur more detailed evaluations are still necessary.

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Figure 3: clamped square plate under step uniform load. (a)plate model (b)reinforced model



Figure 4: Reinforced square plate. Normal displacement for t = 0.1

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SHAPE ANALYSIS FOR INFLATABLE STRUCTURES WITH WATER PRESSURE BY THE SIMULTANEOUS CONTROL

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Summary.

Simultaneous control is an incremental technique that can be applied to shape analysis with soap film elements, when the shape has large volume and high rise. Furthermore, the tangent stiffness method is a clear and strict analytical theory to be able to solve the problems with large deformational behavior. The simultaneous control brings out the best performance for form-finding of isotonic surfaces in combination with the tangent stiffness method. This study proposes a significant modification of the simultaneous control. The modification realizes a wide application range regarding load conditions and boundary conditions for soap film shape analyses by calculating the average of surface tension. Results are presented as some computational examples in which the behavior of soap film structures under air pressure and /or water pressure becomes evident.

Key words: Shape analysis, soap film surface, simultaneous control, tension averaging, approximation of surface, polyhedral equilibrium solution, tangent stiffness method.

1 INTRODUCTION

Recently, membrane structure are used not only for roof of stadiums, but also underground structures such as geo-membrane or coastal membrane structures. However, compared to the huge application of this type of structure, studies and examples are significantly few for form finding analysis, such as embedded oil tank in coastal area by Ishii¹, rubber dam by Ogiwara etc.². Furthermore, the field of studies is only limited to the issues of axial symmetrical or two dimensional problems, it is not sufficient for the cases of complicated boundary shape or loading condition such as follower load.

Until recently, authors had executed form finding for soap film surface (Ref.^{3 to 5}), the application of the tension force as a constant value for triangular shape soap film element using tangent stiffness method algorithm which solves geometrically non linear problems with high convergence. In Ref.⁴, to analyze the shape of soap film inflated by internal air

pressure up to extra large volume, it is suggested to apply compulsory displacement on a node by incremental method using simultaneous control. In addition, when observing the behavior of the volume consisted in soap film, an unstable condition occurs which has the similarity to elastic buckling. Furthermore, according to Ref.⁵, it was clear that the soap film surface satisfy the isotonic condition not only in loading condition of the internal pressure which is distributed uniformly, but also in case of the location dependency loading such as water pressure.

In this paper, based on the average internal pressure suggested by the previous authors, average tension by simultaneous control is proposed and in this concept, it is focused on the physical characteristic of soap film which is 'constant value of surface tension'. In this method, the application of location dependency loading in shape analysis for soap film is widely executed, as for example, it was able to perform as a simulation of a volume expansion phase in a soap film surface by water pressure which works from the inside of the structure. In addition, the input of loading condition (air, water or earth load), boundary condition and control point coordinate could produce output results which are surface morphology and tension of soap film, and by this result, the rational design for real membrane structure could be carried out by achieving the surface prototype and initial tension simultaneously.

2 SOAP FILM SHAPE ANALYSIS USING TANGENT STIFFNESS METHOD

TANGENT STIFFNESS AND TANGENT GEOMETRICAL STIFFNESS EQUATION FOR TRIANGULAR MEMBRANE ELEMENT

An element constituted by two edges with element edge force and the force vector for both edges is considered as S, displayed as external force for the nodal force vector, U, in a three dimensional coordinate system, and the equilibrium matrix is J, the equilibrium relation is shown as the following equation;

$$\mathbf{U} = \mathbf{J}\mathbf{S} \tag{1}$$



Figure-1: Triangular membrane element and element edge force along the element direction

With the differentiation of the equation above, the tangent stiffness equation could be represented as;

$$\delta \mathbf{U} = \mathbf{J} \delta \mathbf{S} + \delta \mathbf{J} \mathbf{S} = (\mathbf{K}_0 + \mathbf{K}_G) \tag{2}$$

Here, K_0 represents the element behavior, correspondent to the element stiffness in the coordinate system while K_G , represents the element displacement originated by the tangent geometrical stiffness. In addition, **u** is the nodal displacement vector displayed in the coordinate system. With the definition of the statement above, the tangent stiffness method is able to evaluate the geometrically non-linear factor caused by the rigid body displacement strictly.

Therefore, we can apply the same geometrical stiffness to soap film elements as real elements with material stiffness. Furthermore, in the case that an element edge force vector which constituents have the direction of triangular sides, the K_G in Eq. (2) has the same form as a triangular truss block.

Furthermore, **e** is a 3 x 3 unit matrices, α_i is the cosine vectors for the respective node of the element.

$$\mathbf{K}_{G} = \begin{bmatrix} \mathbf{k}_{G2} + \mathbf{k}_{G3} & -\mathbf{k}_{G3} & -\mathbf{k}_{G2} \\ -\mathbf{k}_{G3} & \mathbf{k}_{G3} + \mathbf{k}_{G1} & -\mathbf{k}_{G1} \\ -\mathbf{k}_{G2} & -\mathbf{k}_{G1} & \mathbf{k}_{G1} + \mathbf{k}_{G2} \end{bmatrix}$$
(3)
$$\mathbf{k}_{Gi} = \frac{N_{i}}{l_{i}} \left(\mathbf{e} - \boldsymbol{\alpha}_{i} \boldsymbol{\alpha}_{i}^{\mathrm{T}} \right) \quad (i = 1, 2, 3)$$
(4)

ELEMENT FORCE VECTOR FOR SOAP FILM ELEMENT

It is convenient to apply element force equation, achieved from the differentiation of the element measurement potential P, in the element measurement. As for performing shape analysis of an element without any material stiffness such as soap film element. Here, assume the area of a triangular element as A and tension of soap film as σ t (constant), the proportion of element potential to the cross section could be defined as;

$$P = \sigma t A \tag{5}$$

The element measurement potential of soap film elements, which is the function of element area, gives the minimal surface of an isotonic surface, while the stationary of the potential energy gives an equilibrium state. It showed that the element measurement potential of soap film elements is equivalent to the strain energy of real members. If the direction of force is parallel to sides of a triangle and defined as element edge force, the treatment of geometrical stiffness becomes more simple as shown in Figure-1 and if the element constituents are grouped into a triangular element, the element edge force;

$$N_1 = \frac{\partial P}{\partial l_1} = \sigma t \frac{l_1 (l_2^2 + l_3^2 + l_1^2)}{8A}$$
(6)

$$N_2 = \frac{\partial P}{\partial l_2} = \sigma t \frac{l_2 (l_3^2 + l_1^2 + l_2^2)}{8A}$$
(7)
$$N_3 = \frac{\partial P}{\partial l_2} = \sigma t \frac{l_3(l_1^2 + l_2^2 + l_3^2)}{8A}$$
(8)

could be achieved by differentiating the element potential in the length and the element edge force vector could be defined as the following equation;

$$\mathbf{S} = \begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix}^T \tag{9}$$

Therefore, the tangent stiffness method to perform form-finding for soap film analysis calculates the nodal force vector from a constant value of air pressure or static water pressure and NR method produces the perfect equilibrium shape by convergence of the unbalanced forces.

In addition, based on Ref.³, the element stiffness $\mathbf{K}_{\mathbf{0}}$, is relatively small compared to the tangent geometrical stiffness $\mathbf{K}_{\mathbf{G}}$ for soap film element and $\mathbf{K}_{\mathbf{0}} = 0$ is substituted into Eq. (2) of the calculation. Moreover, based in Ref.³, the modification to one degree of freedom analysis has been made and the displacement of the node is to be set in the normal direction of the surface.

INCREMENTAL PROCESS BY SIMULTANEOUS CONTROL

REVIEW OF THE SIMULTANEOUS CONTROL

The idea of 'the simultaneous control averaging internal pressure' is explained specifically in Ref.^{4,5}, and this study described 'the simultaneous control averaging tension'.

In order to obtain a soap film surface using a mechanical approach, the following determinations have to be prepared other than the boundary condition as the input information;

- 1) The tension of soap film σt , which is stress multiplied by thickness of membrane, is constant along the surface.
- 2) The internal pressure p, which is the air pressure or static water pressure.

However, assuming an isotonic surface which involves air or water, if the value p is larger than the value of σt relatively, convergence result could not be achieved. This suggests that the maximum value exists on the P-V curve which represents the relation between pressure and volume, and additional geometrical restriction is required in order to get the equilibrium solutions.

'The simultaneous control averaging internal pressure' is to get the equilibrium shape and the internal pressure of a constant value along the surface simultaneously from input of tension force and the displacement of a control point. However, in the case of the water pressure acts on the soap film, the tension distributed on surface, of course, should be a constant value, but the magnitude of the pressure is depending on the position. Therefore, it is more rational to inverse the input and the output to obtain equilibrium shape with large volume.

In this study, the procedure of form-finding is as follows;

1) A control point is displaced with small incremental step.

- 2) 'The converted tension', which is balanced with the sum total of 'the element edge forces' gathering at a node, is calculated at every node.
- 3) 'The converted tension' is averaged out to all nodes, and adopted to tension of soap film for next iteration step. We call this, 'the converted average of tension'.
- 4) 'The converted average of tension' is renewed in every iteration step, and finally the unbalanced forces of all nodes and the reaction force at the control point are converged to zero. Furthermore, the tensions of all elements are equalized on the obtained equilibrium shape.

According to this procedure, the pressure acting on the soap film surface do not have to be distributed uniformly. Therefore, for example, it became possible to search a shape of soap



Figure-2 : Elements connected to node *i* and force in the normal direction

bubble involving liquid with extremely large volume. In this paper, this incremental procedure is called 'the simultaneous control averaging tension'.

CONVERTED AVERAGE OF TENSION

Assume that the total number of element, m are connected to node i as shown in Figure-2. In the case of the total area vector for element j, the pressure loading that subjected on node i is assumed as 1/3, the corresponding total area vector for each element are shown in the following equation.

$$\mathbf{A}_{ij} = \frac{\mathbf{a}_{ij}}{3} \tag{10}$$

Furthermore, the normal cosine vector for node *i* on the surface could be expressed as;

$$\mathbf{H}_{\mathbf{i}} = \frac{\sum_{j=1}^{m} \mathbf{A}_{ij}}{\left|\sum_{j=1}^{m} \mathbf{A}_{ij}\right|} \tag{11}$$

Meanwhile, if the element edge force for edge *i* in element *j* is assumed as S_{ij} , the conversion to the standard coordinate system is expressed in equilibrium matrix J_{ij} which can be expressed as the following equation.

$$\mathbf{U}_{\mathbf{S}_{\mathbf{i}\mathbf{i}}} = \mathbf{J}_{\mathbf{i}\mathbf{j}}\mathbf{S}_{\mathbf{i}\mathbf{j}} \tag{12}$$

Therefore, the resultant force for the element edge force for node *i* is;

$$\mathbf{U}_{\mathbf{S}_{\mathbf{i}}} = \sum_{j=1}^{m} \mathbf{U}_{\mathbf{S}_{\mathbf{i}j}} \tag{13}$$

Since the stiffness is only applied in the normal direction of the soap film surface, the component of resultant force for the element edge force in the normal direction is;

$$\overline{U}_{S_i} = \mathbf{H}_i^T \mathbf{U}_{S_i} \tag{14}$$

Then, the equilibrium equation at node *i* is expressed as follows;

$$p_i(\mathbf{u}_i) \left| \sum_{j=1}^m \mathbf{A}_{ij} \right| - \overline{U}_{S_i} = 0$$
⁽¹⁵⁾

where, $p_i(\mathbf{u}_i)$ is the pressure at node *i*, which is depending on the position of the node.

Referring to Eq. (6) until Eq. (8), element edge force S_{ij} and tension of soap film σ t have a linear relation, with that Eq. (14) can be rewritten as following;

$$\overline{U}_{S_i} = \sigma t \mathbf{H}_i^{\mathrm{T}} \mathbf{V}_i \tag{16}$$

Therefore, Eq. (15) can also be rewritten as;

$$\sigma t_{c_i} = p_i(\mathbf{u}_i) \frac{\left|\sum_{j=1}^{m} \mathbf{A}_{ij}\right|}{\mathbf{H}_i^{\mathsf{T}} \mathbf{V}_i}$$
(17)

Eq. (17) provides 'the converted tension' σt_{Ci} at node *i*, which is balanced with the pressure referring to the current geometry.

By averaging the converted tension σt_{Ci} will establish the tension of soap film to become constant along the whole surface. However, as the following calculation example (Figure-3) clearly shows, the tension of soap film can have an infinite value when the shape of surface is close to plane under the designation of non-zero pressure. Therefore, by applying the average value of inverse number for converted membrane stress, 'the converted average of tension' σt_{AV} could be determined and be renewed in each iteration step.

$$\frac{1}{\sigma t_{AV}} = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{\sigma t_{C_i}} = \frac{1}{n} \sum_{i=1}^{n} \frac{\mathbf{H}_i^{\mathsf{T}} \mathbf{V}_i}{p_i(\mathbf{u}_i) |\sum_{j=1}^{m} \mathbf{A}_{ij}|}$$
(18)

The unbalanced force for all nodes and the reaction force for the control point converge simultaneously, then the morphology of the soap film surface including the control point and the equalized tension of soap film could be determined.

3 NUMERICAL ANALYSIS EXAMPLE

CASE 1 : STATIC WATER PRESSURE SUBJECTED ON SOAP FILM



Figure-3 : Front view and side view for Case 1

Here is an example of a soap film subjected by water pressure. As shown in Figure-3, the orientation of the rectangular shape boundary frame forms a 90 degree angle to the horizontal direction with a water level, equal to the respective height. Compulsory displacement is applied on the control node in the normal direction of the surface, with 0.01 m of incrementation for each iteration step. The whole structure consists of triangular soap film elements and the initial tension of soap film value is 20 kN/m.

Table-10 shows the relation between volume, V and tension of soap film σt for selected incremental step of compulsory displacement. According to the graph, it is clear that the minimal value of tension for soap element (marked as **D**) exists, thus the result which surpassed the minimal value could also be traced (marked as **E** and **F**).

In this analysis, when the surface volume reaches approximately 1.9 m³, divergence occurs and there was no result for the following phase. This is due to the 'extra large' deformation for the soap film element and to solve this, it is proposed to increase the value of mesh to obtain better result.

Label	Compulsory displacement value, <i>dcp</i> (m)	Volume, $V(m^3)$	Tension of soap film, σt (kN/m)
А	-0.4159	0.0883	25.2982
В	-0.4127	0.1948	11.8968
С	-0.3728	0.6776	5.0219
D	-0.3313	1.0531	4.6543
Е	-0.2971	1.4345	4.8185
F	-0.2772	1.9052	5.2943

Table 1 : Analysis of soap film subjected to water pressure



Figure-4 : The relation of tension of soap film, σt (kN/m) and volume, V (m³)



Figure-5 : Deformation diagram for soap film structure (Case 1)

CASE 2 : STATIC WATER PRESSURE SUBJECTED ON SOAP FILM

As shown in Figure-6, a hexagonal shape soap film is fixed along the border, subjected with a water loading. The water pressure is equivalent to 12 m of water level, pumped from beneath the structure. Using averaging surface tension by simultaneous control, a control point is located in the middle of the hexagonal structure, applied in the upward direction with the increment value of 0.1 m for each iteration step. Figure-7 shows the relation between tension of soap film, σt (kN/m) and surface volume, V (m³), and Figure-8 shows the deformation diagram of the structure as water pressure is applied throughout the analysis.



Figure-6 : Analysis example for Case 2



Figure-7 : The relation of tension of soap film, σt and volume, V

Similar to the previous analysis, the minimal value of tension for soap film also exists (marked as \mathbf{D}) and so as the solution that surpasses it. As the structure is continuously deforming and the surface volume increases, it is clear that the deformed hexagonal shape structure seems to turn into a 'droplet' shape. However, in this analysis, the water pressure that works on the nodes which are located on the surface could not be assumed as zero. This is due to the approximation of polyhedron structure where all nodes are scattered in a three dimensional area, it is not possible to assume the external force as zero which assumes that all nodes are equal in a plane area. Thus, a real 'droplet' could not be achieved exactly as seen in nature.







Figure-8 : Deformation diagram for soap film structure (Case 2)

In conclusion, by applying this method, an isotonic tension on an equilibrium surface could be obtained even though the water pressure is applied from beneath the structure.

4 CONCLUSION

In this paper, the idea of 'the simultaneous control averaging tension' is proposed based on the increment method by the simultaneous control.

The proposed method is the improvement of 'the simultaneous control averaging internal pressure' proposed by the authors, and in this method, the mean value of converted tension for all nodes are calculated, the 'converted average of tension' is renewed and converged in each iteration step.

Therefore, in the proposed method, the tension of soap film, obtained from given value of loading condition, is able to be applied as the initial tension for the surface. Compared to 'the simultaneous control averaging internal pressure' which calculates nodal force as an unknown quantity, it is more rational analysis flow if it is applied in the design calculation.

In addition, for the proposed method, the adaptability of location dependency loading was improved gradually. With this method, cases such as isolating fresh water from sea water by using embedded membrane structure tank or a gas container made out by a giant membrane structure in outer space and etc. is the applications in the future. Finally, new morphology creation and expansion are expected to be a reality.

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FINITE ELEMENT ANALYSIS ON MULTI-CHAMBER TENSAIRITY-LIKE STRUCTURES FILLED WITH FLUID AND/OR GAS

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Abstract. The concept of Tensairity-structures [6] developed by the company Airlight in Biasca, Switzerland, has been known since 2004. The advantages of the airbeams with a compression element and a spiraled cable are essentially their light weight and that such beams can be used for wide span structures.

To achieve a further weight reduction Pronk et al [8] proposed to replace the compression element by an additional slim chamber filled with water. Experiments with these multi-chamber beams with and without cables showed a stiffer behavior in bending tests compared to only air filled beams.

In the current contribution different tests like - primarily - the bending of multi-chamber beams will be simulated with finite elements. Explicit and implicit finite element simulations with LS-DYNA [7] and FEAP-MeKa [11] respectively will be performed with a special focus on the interaction of structural deformations and the gas/fluid filling in combination with the cables contacting the membranes. The specific features have been implemented in the above codes.

The fluid and/or gas filling is replaced by an energetically equivalent load and corresponding stiffness matrix contributions to simulate quasi-static fluid-structure interaction taking the effect of the deformations of the chambers on the fluid/gas filling into account. This approach has already been introduced for fluid-structure interaction problems with large deformations and stability analysis in [1], [2]. For the implicit simulation the cables will be added using special solid-beam finite elements [5]. A new curve-to-surface contact algorithm is developed to model the contact interaction between cables and the deformable shell structure.

1

1 INTRODUCTION

Structures filled with fluid and/or gas combined with cables can be found in various fields, for example in balloons, airbags, pipeline buoyancy or water load test weights, airbeams and waterbeams. In all of these examples the filling proceeds quasi-statically, thus the fluid can be described by an energetically equivalent loading. The algorithms based on the theory of quasi-static fluid-structure interaction are implemented in LS-Dyna [7] for explicit and in FEAP-MeKa [11] for implicit finite element simulations and have already been presented for many examples, see [1], [2], [7]. Cables can be modeled by special solid-beam finite elements [5] and a curve-to-surface contact algorithm. The combination of both is applied to airbeams and waterbeams, which consist of one or more fluid filled chambers wrapped with cables.

2 QUASI-STATIC FLUID-STRUCTURE INTERACTION

During the filling of airbeams and waterbeams dynamic behavior is not expected because of a slow, quasi-static filling process. As a result the effect of gas or water inside a chamber can be described by an energetically equivalent pressure load vector and also the standard water loading is quasi static. Both are also leading to corresponding striffness contributions. An important aspect is that the water/gas loading is defined via given initial volumes and gas or fluid deformation relations. This is predominantly important for structures undergoing large deformations or in stability prone problems.

Of course this idea can not only be applied to airbeams and waterbeams. Other structures including quasi static fluid structure interaction are e.g. hydro-forming, buoyancy simulation of ships and inflatable dams.

In the following subsections all three different load-cases (pure gas loading, incompressible fluid loading and incompressible fluid with compressible gas loading) are discussed. A complete derivation and further load cases considering also compressible fluid can be found in [3] and [9].

2.1 Gas

In case of gas loading the whole internal boundary of a gas filled chamber is loaded with a constant gas pressure p^{g} .

As shown in Figure 1 p^g is the gas pressure, $\partial \Omega^g$ the boundary and \mathbf{n}^g the normal directed towards the outside of the wetted structure. The virtual work term resulting in a load vector due to gas loading, which has to be added to an arbitraray structural load vector, reads as

$$\delta \Pi^g_{ext} = \int_{\Omega^g} p^g \mathbf{n}^g \cdot \delta \mathbf{u}^g \mathrm{d}\Omega^g \tag{1}$$

with the variation of the displacement $\delta \mathbf{u}^{g}$.



Figure 1: Pressure in a gas filled chamber

After each time step the gas pressure has to be updated considering a simple gas law with the adiabatic exponent κ and the gas volume v^g :

$$p^g = p^g_{old} - \kappa p^g_{old} \frac{v^g - v^g_{old}}{v^g_{old}}.$$
(2)

The pressure is hereby as in the following load cases controlled by volume modifications.

2.2 Incompressible Fluid

Without an additional gas loading there is a free fluid surface and the following load case is achieved considering only incompressible fluid as the compressibility of the fluid plays no role in the rather soft structure considered in this contribution.



Figure 2: Hydrostatic pressure in a partially filled chamber

With the gravity vector \mathbf{g} , the outward directed normal of the wetted surface \mathbf{n}^{f} , the density of the fluid ρ and the coordinate to the fluid surface \mathbf{x}^{o} the pressure at water level is computed by

$$p^{o} = \rho \mathbf{g} \cdot \mathbf{x}^{o} \tag{3}$$

and the pressure at any point of the wetted surface is

$$p^{x} = \rho \mathbf{g} \cdot \mathbf{x} \qquad \mathbf{x} = \mathbf{0} \dots \mathbf{x}^{o}. \tag{4}$$

This leads to the virtual work of a load vector consisting of p^x and p^o :

$$\delta \Pi_{ext}^f = \int_{\Omega^f} (p^x - p^o) \mathbf{n}^f \cdot \delta \mathbf{u} \, \mathrm{d}\Omega^f.$$
(5)

2.3 Incompressible Fluid combined with Gas

The last presented load case is the combination of the two previous load cases.



Figure 3: Hydrostatic pressure of a fluid and gas filled chamber

The virtual work of the load vector is a combination of equation (1) and (5):

$$\delta\Pi^{f}_{ext} + \delta\Pi^{g}_{ext} = \int_{\Omega^{f}} (p^{g} + p^{x} - p^{o}) \mathbf{n}^{f} \cdot \delta \mathbf{u} \, \mathrm{d}\Omega^{f} + \int_{\Omega^{g}} p^{g} \mathbf{n}^{g} \cdot \delta \mathbf{u} \, \mathrm{d}\Omega^{g} \tag{6}$$

The update of the gas pressure is performed in analogy to equation (2) in the case of only gas loading.

The last two cases and an extension to multiple chambers have been implemented into LS-DYNA following a previous implementation in FEAP-MeKa [11], [10], [3]. For the further theoretical background and its implications for implicit algorithms we refer to [10], [3], [1].

3 CABLES AND CONTACT

Several developments are required to obtain a robust contact algorithm for cable and shell interactions: robust shell finite elements, robust cable finite elements and contact algorithms for the cable-shell interaction. Shell finite element technology is highly developed and a wide range of various formulations is available. Here we are selecting the solid-shell finite element with only displacement degrees of freedoms. For the cable we take a special solid-beam finite element with exact representation of an elliptical cross-section, see in [5].

3.1 SOLID-BEAM FINITE ELEMENT

The solid-beam element is constructed as follows, see Fig. 4: a) the mid-line of the beam is taken, first, with e.g. linear approximation. It is defined then by nodes 1 and 2; b) the left cross-section is defined to be elliptic with the reference main axes defined by nodes 1-3 and 1-4; c) the right cross-section is defined to be elliptical with the reference main axes defined by nodes 2-5 and 2-6. This element allows an iso-geometrical description of the elliptic cross-section, in- and out of plane deformations of this cross-section together with tension type rotations of the cross-section along the mid-line and bending. the element can be easily extended into an iso-geometric spline-element along the mid-line.



Figure 4: "Solid-Beam" with elliptical cross-sections defined by 6 nodes. Definition of local variables.

A special set of shape functions defining iso-geometrically an elliptical shape of the cross-section is exploited. The mid-line can be also represented by e.g. NURBS or other spline functions following the iso-geometric approach.

3.2 CURVE-TO-SURFACE CONTACT ALGORITHM

The kinematics of the curve-to-surface contact element is dually defined, first, in the Serret-Frenet coordinate system following the curve-to-curve contact algorithm developed in [5], and, second, in the surface coordinate system following the covariant description in [4]. This leads in the case of linear approximations for both shells and ropes to the following 6-nodes curve-to-surface (CTS) contact element, see Fig. 5, which is constructed as follows: i) the shell surface (in this case linearly approximated) is covered by the master contact segments with nodes 1, 2, 3, 4; ii) for the contact definition the mid-line represented by nodes 5, 6 of the solid-beam element is taken; iii) contact kinematics is defined in the local coordinate system attached to the master segment with coordinate vectors $\rho_1, \rho_2, \mathbf{n}$, see more in [5], however, assuming representing the coordinate system attached to the



Figure 5: Kinematics of the Curve-To-Surface contact element.

line.

$$\mathbf{r}_{s}(\eta,\xi^{1},\xi^{2}) = \boldsymbol{\rho}(\xi^{1},\xi^{2}) + \mathbf{n}(\xi^{1},\xi^{2})\xi^{3},$$
(7)

where ξ^1, ξ^2 are convective coordinates for the master segment and η is a convective coordinate for the mid-line of the slave solid-beam element. iv) penetration ξ^3 is computed assuming the thickness of the rope as between the distance between the integration point on the mid-line S and its projection on the surface C subtracting the radius of the rope R. It leads in the case of the circular cross section with radius R to the following expression:

$$\xi^3 = (\mathbf{r}_s - \boldsymbol{\rho}) \cdot \mathbf{n} - R. \tag{8}$$

v) The contact integral is computed numerically using the integration formulae of Lobatto or Gauss type depending on modeling purposes. The part responsible for normal contact only is represented by

$$\delta W = \int_{5-6} N \delta \xi^3 ds_{5-6} \tag{9}$$

vi) in the case of the penalty regularization the normal force is computed as $N = \varepsilon_N \xi^3$ with ε_N as a penalty parameter

4 AIRBEAM

Tensairity-structures have been developed by Airlight in 2004 [6]. These light-weight structures consist of a gas filled fabric structure with a compression element and a spiraled cable. The inflation process for airbeams and most of the loading proceeds quasi-statically, so that the theory in section 2 can be applied without any restriction.

For the finite-element simulation the fabric structure is designed like a cylinder with radius of 0, 23m and a length of 2m and was model with shell elements, see Figure 6. The thickness of the material is 1mm. The width of the compression element is 0, 1m with a thickness of 0, 01m and consists of volume elements. In all simulations gravity was added.



Figure 6: Initial configuration for airbeam simulation without cables

In the finite element simulations the airbeams were filled with gas varying the gas pressure from 1 bar to 40 bar. After the inflation process the center line of the compression element was loaded in transverse direction with a linearly increasing force F. The deformation of the airbeam after inflation caused by the force F was measured.



Figure 7: Initial configuration for airbeam simulation without cables

As expected the displacement decreases with a higher gas pressure. Especially the deformations of an airbeam filled with atmospheric pressure of 1 bar are very large. Further investigations with cables - currently underway - should show less deformations even for the low pressure cases.

5 WATERBEAM

The concept of waterbeams, see [8], is a further development of the airbeam. Instead of a stable compression element an additional membrane chamber is included. This chamber is as long as the waterbeam itself, but the radius is much smaller, see chamber 2 in Figure 8 and can be filled with gas or fluid. The larger chamber 1 is filled with gas. A vertical continuous membrane divides chamber 1 to accommodate the shear force. On the left and right end of the waterbeam the beam is fixed to two round steel plates.



Figure 8: Cross-section of waterbeam

The radius of chamber 2 is 3cm and the larger radius of chamber 1 is 23cm. In total the waterbeam has a length of 2m which includes the steel plates. The thickness of the membrane was again set to 1mm. In the finite element simulation the membrane chambers are discretized with 50000 shell elements, and the cables are discretized with 500 beam elements in LS-Dyna. Youngs modulus of the membrane material is $7, 6 \cdot 10^9 Pa$, density $1kg/m^3$.



Figure 9: Initial configuration for waterbeam simulation

In a first simulation the waterbeam was loaded only with fluid to verify the multi-chamber approach in the finite element simulation. While chamber 1 is always filled with a gas

pressure of 1 bar, the second chamber is once filled with gas (gas pressure 1 bar) and in a second simulation filled with fluid (water height 0, 23m).



Figure 10: Gas and Fluid volume for chamber 1 and chamber 2 with different types of fluid loading

Both the fluid and the gas loading are applied until a maximum at time 0,01s. In case of the water filling in chamber 2 the water is added a little bit later than the gas filling of chamber 1. This results in a slightly higher gas volume in chamber 1 and lower fluid volume in chamber 2.

In further simulations -currently underway - the waterbeam will be transversely loaded with a force F in the middle of chamber 2 and the deformations will be measured. A result analogously as in Figure 7 is expected.

First simulations with cables using LS-Dyna show the filling process and the contact with cables during the filling process, see Figure 11.



Figure 11: Waterbeam with cables during filling process showing the displacement

6 CONCLUSIONS

Light-weight structures like airbeams are already used for bridges, for example in the Ski resort Val-Cenis. The newer concept of waterbeams developed by [8] has been simulated to compare the deformation to the deformations of a airbeam with identical loading.

For both concepts quasi-static fluid-structure interaction can be applied. The cable was simulated as a beam structure. In FEAP-MeKa, [11], a newly developed cable formulation could be used while in LS-Dyna a standard beam formulation was used to simulate the cables. In a first step the airbeam structure was simulated without cables. Deformations due to a force F increase with an decreasing gas pressure. For the example of a waterbeam multi-chamber fluid-structure interaction has been used. The results of displacement simulations with a force F as well as the comparison to the experimental results in [8] will be shown at the Structural Membranes 2011 conference.

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DIRECT AREA MINIMIZATION THROUGH DYNAMIC RELAXATION

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Summary. Minimal surfaces, characterized by the property of a minimal area within a fixed boundary, offer an interesting design option for membrane structures, since they are uniquely defined and provide economy of material and more regular fabric patterns. Analytical solution for the non-linear equation governing area minimization may be rather difficult for complex boundaries, leaving numerical solution as the only general way to tackle with the problem. In this paper we show that the dynamic relaxation method offers an interesting alternative to solve the area minimization problem, first interpreted as a nonlinear equilibrium problem, then replaced by a pseudo-dynamic analysis, where fictitious masses and damping matrices are arbitrarily chosen to control the stability of the time integration process.

1 INTRODUCTION

The study of minimal surfaces is important both from theoretical and practical points of view. Minimal surfaces are characterized by the property of a minimal area within a fixed boundary. They are also the solution geometry for a membrane constringed to that same boundary, and under an isotropic and uniform plane stress field^{1,2,3}. These properties render minimal surfaces an interesting design option for membrane structures, since they are uniquely defined, for a given boundary, and provide economy of material and more regular fabric patterns^{4,5,6,7,8}.

Minimal surfaces have also attracted the interest of scientists since the times of Lagrange (who solved the problem for some surfaces of the type z = f(x, y)) and Euler (who proved that minimal surfaces have zero-mean curvature everywhere, and therefore are either plane or anticlastic). Euler was also the first to find the *catenoid* (the minimum surface bounded by two parallel, co-axial rings), which remains one of the few analytical solutions available to this class of problems. In the nineteenth century, the Belgian physicist Joseph Plateau showed that analogue solutions to area minimization problems could be produced by dipping wire frameworks into a bath of soap solution.

2 AREA MINIMIZATION WITH FIXED BOUNDARIES

For the moment, we restrain ourselves to surfaces bounded by closed curves *C* embedded into Euclidian tridimensional space R^3 and spanned by a vector field $\mathbf{x} = \mathbf{x}(\theta_1, \theta_2)$, where θ_1 and θ_2 are continuous and monotonous parameters, as sketched in Figure 1.



Figure 1: A tridimensional surface bounded by a closed curve *C* and spanned by a vector field $\mathbf{x} = \mathbf{x}(\theta_1, \theta_2)$ (adapted from reference⁸)

At every point $P \in S$, we define vectors $\mathbf{g}_{\alpha} = \frac{\partial \mathbf{x}}{\partial \theta_{\alpha}}$, $\alpha = 1, 2$, tangent to the surface. A unit vector, normal to the surface S at P, is given by $\mathbf{g}_3 = \mathbf{g}_1 \times \mathbf{g}_2 / \|\mathbf{g}_1 \times \mathbf{g}_2\|$. The total area of any such a surface is given by

$$A = \int_{S} dA = \int_{S} \|\boldsymbol{g}_{1} \times \boldsymbol{g}_{2}\| d\theta_{1} d\theta_{2}$$
(1)

We seek a surface S^* , spanned by a vector field x^* , such that its area A^* is a minimum. In other words, for any perturbation field δu around x^* , compatible with C, there must hold

$$\delta A^* = \frac{\partial A}{\partial \boldsymbol{x}}\Big|_{\boldsymbol{x}^*} \,\delta \boldsymbol{u} = 0, \quad \forall \,\delta \boldsymbol{u} \,. \tag{2}$$

Thus, the necessary 1^{st} order condition for a configuration x^* to be minimal is given by the non-linear equation

$$\frac{\partial A}{\partial \mathbf{x}}\Big|_{\mathbf{x}^*} = 0, \qquad (3)$$

with the equality restriction $(\mathbf{x}_P - \overline{\mathbf{x}}_P) = \mathbf{0}$, $\forall P \in C$, where $\overline{\mathbf{x}}_P$ is a vector function spanning the prescribed coordinates at $P \in C$.

The field of global coordinates x spanning a generic configuration S can also be decomposed according to $x = x^r + u$, where x^r spans an initial given configuration and u is a displacement vector field. Now, since x^r is constant, and derivatives can be taken indistinctly with respect to global coordinates x or to displacements u, the solution to the area minimization problem consists in finding the configuration u^* such that

$$p(\boldsymbol{u}^*) = \frac{\partial A}{\partial \boldsymbol{u}}\Big|_{\boldsymbol{u}^*} = \mathbf{0}, \qquad (4)$$

where we define the generalized internal load vector $p(u) = \frac{\partial A}{\partial u}$.

Analytical solution of the nonlinear Eq. (3) or Eq. (4) may be rather difficult for complex boundary geometries, leaving numerical solution as the only general way to tackle with the problem.

2.1 Discretization

In order to numerically solve Eq. (4), it is necessary to replace the continuous fields \mathbf{x}^r , \mathbf{u} and \mathbf{p} by some convenient algebraic approximation. Faceted surfaces, although not globally differentiable, offer a convenient alternative for the numerical estimative of the total area of smooth surfaces, improving numerical precision as the number of facets is increased. In this paper, we choose to work with flat triangular facets (the simplest possible choice), laid onto a mesh of n nodes, whose coordinates are collected in a *global position* vector $\mathbf{x} = \begin{bmatrix} \mathbf{x}_1^T & \mathbf{x}_2^T & \dots & \mathbf{x}_n^T \end{bmatrix}^T$, where $\mathbf{x}_i = \begin{bmatrix} \mathbf{x}_i \end{bmatrix}$ stores the Cartesian coordinates of the *i*th node of the mesh. Nodal displacement can also be grouped in a *global displacement vector* $\mathbf{u} = \begin{bmatrix} \mathbf{u}_1^T & \mathbf{u}_2^T & \dots & \mathbf{u}_n^T \end{bmatrix}^T_{3n \times 1}$, where $\mathbf{u}_i = \begin{bmatrix} \mathbf{u}_i \end{bmatrix}$ stores the Cartesian components of the displacement of the *i*th node. Note the double transpositions present in these definitions, used simply to avoid a column-wise notation.

Figure 2 shows the basic geometric quantities required for the definition of a generic triangular facet (an 'element') of index *e*. Facet nodes and edges are numbered with edges facing nodes of same number. We extract the element nodal coordinates and displacements from the global position and displacement vectors according to $\mathbf{x}^e = \mathbf{C}^e \mathbf{x}$, and $\mathbf{u}^e = \mathbf{C}^e \mathbf{u}$, where \mathbf{C}^e is the order $9 \times 3n$ Boolean incidence matrix of that element, which correlates the local node numbers $\{1,2,3\}$ with the global numbers $\{i, j, k\}$ such that $\mathbf{C}^e_{1i} = \mathbf{C}^e_{2j} = \mathbf{C}^e_{3j} = \mathbf{I}_3$ and $\mathbf{C}^e_{1m} = \mathbf{C}^e_{2m} = \mathbf{C}^e_{3m} = \mathbf{0}, \ m \notin \{i, j, k\}$, where $\mathbf{0}$ and \mathbf{I}_3 are, respectively, the null and identity matrices of order three. Of course, these definitions are merely formal, and computer implementation avoids the multiplicity of zero multiplications they contain.

The lengths of the edges of a generic triangular facet can then be computed by $\ell_i^e = \|\mathbf{I}_i^e\| = \|\mathbf{x}_k^e - \mathbf{x}_j^e\|$, with indexes i, j, k = 1, 2, 3 in cyclic permutation. Unit vectors parallel to the element edges are denoted by $\mathbf{v}_i^e = \mathbf{I}_i^e / \|\mathbf{I}_i\|$. Now, an element area vector is defined as $\mathbf{a}^e = \frac{1}{2} (\mathbf{I}_1^e \times \mathbf{I}_2^e)$, and the element scalar area is given by $A^e = \|\mathbf{a}^e\|$, whilst $\mathbf{n}^e = \mathbf{a}^e / A^e$ is an unit vector, normal to the plane of the facet. A coherent node numbering for all the elements provides an oriented surface.



Figure 2: A triangular facet.

The total area of a smooth surface discretized by a mesh of n_e triangular facets is then, approximately,

$$A = \sum_{e=1}^{n_e} A^e = \sum_{e=1}^{n_e} \frac{1}{2} \left\| \left(\mathbf{l}_1^e \times \mathbf{l}_2^e \right) \right\|$$
(5)

2.2 Newton's Method

Although in this paper we are concerned with solving the area minimization via dynamic relaxation, for the sake of comparison we now remind that the more popular method for the numerical solution of nonlinear systems is Newton's Method, in which the solution u^* is sought starting from an initial estimative u_0 and iterating the recurrence formula

$$\boldsymbol{u}_{i+1} = \boldsymbol{u}_i - \boldsymbol{H}_i^{-1} \boldsymbol{p}_i, \qquad (6)$$

where we define the *Hessian tensor*, $H = \frac{\partial p}{\partial u} = \frac{\partial^2 A}{\partial u^2}$.

Now, introducing in (6) the area approximation (5), we obtain an *approximate generalized internal load* vector as

$$\boldsymbol{p} = \sum_{e=1}^{n_e} \mathbf{C}^{e^T} \mathbf{p}^e \tag{7}$$

where we define the *element internal load vector*, $\mathbf{p}^e = \frac{\partial A^e}{\partial \mathbf{u}^e}$.

By its turn, the Hessian tensor is approximated by

$$\boldsymbol{H} \stackrel{\text{\tiny resc}}{\to} \sum_{e=1}^{ne} \mathbf{C}^{eT} \mathbf{H}^{e} \mathbf{C}^{e} , \qquad (8)$$

where we define the *element Hessian matrix*, $\mathbf{H}^{e} = \frac{\partial \mathbf{p}^{e}}{\partial \mathbf{u}^{e}}$.

Deriving the area of a facet with respect to its displacements \mathbf{u}^{e} , after some algebra, there results for the element internal load vector:

$$\mathbf{p}^e = -\frac{1}{2} \mathbf{\Lambda}^e \mathbf{n}^e \tag{9}$$

where $\Lambda^{e} = \begin{bmatrix} \Lambda_{1}^{eT} & \Lambda_{2}^{eT} & \Lambda_{3}^{eT} \end{bmatrix}^{T}$, with $\Lambda_{k}^{e} = skew(\mathbf{l}_{k}^{e})$.

Again deriving (9) with respect to displacements \mathbf{u}^{e} , one gets the element Hessian matrix:

$$\mathbf{H}^{e} = \frac{1}{4A^{e}} \left(\mathbf{\Lambda}^{e} \mathbf{\Gamma}^{e} \mathbf{\Lambda}^{eT} + 2 \mathbf{\Psi}^{e} \right)$$
(10)

where $\Gamma^e = \mathbf{I}_3 - \mathbf{n}^e \mathbf{n}^{e^T}$ and $\Psi^e = skew(\Omega^e)$, and where $\Omega^e = skew(a^e)$. It is seen that Ψ^e and therefore \mathbf{H}^e are both symmetric matrices.

However, it is intuitive to realize that the area of any smooth surface, of fixed boundary, is indifferent to deformations involving infinitesimal displacements tangent to the surface itself. In the case of a curved surface divided into a finite number of plane triangular facets, this property is not exact, but anyway, for every given mesh topology, there exists generally an infinity of possible nodal configurations, approximating the same smooth surface. This reflects in the fact that the Hessian matrix (8) becomes more and more ill-conditioned, as long as the solution is approached, and as long as the mesh is refined. In practice, this characteristic overrules the direct solution of (4) by means of pure Newton's method.

On the other hand, deformations involving displacement transversal to surface are in general capable to alter its area. Thus a way to circumvent this problem is the imposition that the nodal displacements have always a component transversal to the current configuration. A particular case, where this restriction is naturally inserted is given by surfaces described by functions $z = \hat{z}(x, y)$, which reduce the area minimization problem to a scalar degree of freedom at every node, as studied in reference⁷. In the general tridimensional case, however, even this restriction degrades as a solution is approached.

Because of these restrictions, it is usual in the problem of area minimization to replace Newton's Method by other algorithms that avoid the exact inversion of the Hessian matrix, such as conjugate gradient methods (as done in references^{7,8}), or the BFGS algorithm (as done in reference⁹), which are capable to converge to one of the infinite solutions that exist for any given mesh topology.

2.3 The Dynamic Relaxation Method

The dynamic relaxation method (DRM) offers another interesting alternative to solve complicated nonlinear equilibrium problems, replacing the static equilibrium problem by a pseudo-dynamic analysis, where fictitious masses and damping matrices are arbitrarily chosen to control the stability of the time integration process.

Thus, instead of solving (4), we may follow the damped vibrations of the dynamic system

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\ddot{\mathbf{u}} + \mathbf{p}(\mathbf{u}) = \mathbf{0} \tag{11}$$

until it comes to a rest, at a solution of Eq. (4). Usually, damping coefficients close to the system's critical damping are chosen, in order to speed up the convergence to the static equilibrium configuration.

Although the dynamic relaxation method shows no advantage for small to medium sized problems, whenever Newton's Method shows good convergence, there may be considerable economy for very large problems. The rationale is that, as long as the computational costs for Newton's method grows with the square of the number of degrees of freedom, and the cost of

DRM grows linearly, there must be a particular number above which Newton's costs becomes larger than DRM costs.

However, this idea cannot be plainly taken, since, when discretization of a given structure is refined, a critical time-step which governs the numerical stability of the system is also reduced, and thus more steps are required for the system to rest.

Nevertheless, since in the DRM the mass and damping matrices are fictitious, they may be adjusted to keep the time-increments small enough to guarantee stability, but as large as possible to reduce the number of steps required for convergence to the static solution.

2.4 Kinetic damping

Several strategies have been devised along the years to define proper damping matrix for the DRM, but we choose here to circumvent the problem altogether, adopting the process of *kinetic damping*, first proposed in reference¹⁰, whereby the undamped movement of the system, governed by

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{p}(\mathbf{u}) = \mathbf{0},\tag{12}$$

is followed until a maximum of the total kinetic energy is reached, when all the velocity components are cancelled, keeping the current geometry. The pseudo-dynamic analysis is then restarted until new kinetic energy maxima (usually smaller than the precedent ones) are found, and all velocities are zeroed once again. The process is repeated until all kinetic energy is dissipated, thus reaching the static equilibrium configuration. The transient of the system's kinetic energy provides a visual criterion for convergence.

2.5 Central differences

In-depth discussions on the relative performance of the several finite-difference schemes available to solve Eq. (12) can be found in references^{11,12,13,14,15}. Experience has shown that a convenient choice is offered by the *central difference method*, which yields an explicit time-integration scheme, when the mass matrix is diagonal, rendering very fast the calculation of every time-step.

In this paper we have adopted a particular brand of the central-difference scheme, progressing from time t_k to time t_{k+1} according to

$$\dot{\mathbf{u}}_{k+\frac{1}{2}} = \dot{\mathbf{u}}_{k-\frac{1}{2}} + \Delta t_k \mathbf{M}^{-1} \mathbf{p}_k \tag{13}$$

$$\mathbf{u}_{k+1} = \mathbf{u}_k + \dot{\mathbf{u}}_{k+\frac{1}{2}} \Delta t_{k+\frac{1}{2}}, \qquad (14)$$

where $\Delta t_{k+\frac{1}{2}} = (\Delta t_k + \Delta t_{k+1}) / 2$.

Then we update the geometry according to $\mathbf{x}_{k+1} = \mathbf{x}_0 + \mathbf{u}_{k+1}$. Although more memory is required to store both \mathbf{x}_0 and \mathbf{u}_{k+1} , it has been observed that this scheme is less sensitive to round-off errors.

2.6 Numerical stability

The central difference method is only conditionally stable, and time increments must be kept sufficiently small. It can be shown that for linear multiple degrees of freedom (MDOF) systems, numerical stability is guaranteed by

$$\Delta t \le \frac{2}{\omega_{\max}},\tag{15}$$

where ω_{max} is the largest natural frequency of the system, rigorously obtained from the solution of the global eigenvalue problem

$$\det\left(\mathbf{H} - \omega^2 \mathbf{M}\right) = 0 \tag{16}$$

where, in the case of a linear system, \mathbf{H}_0 is a constant Hessian matrix. Proofs for this result, first stated in reference¹⁶, can be found in reference¹¹ to reference¹⁵.

Assembling of the global Hessian matrix is, however, a sheer contradiction with the spirit of DRM, one of the main advantages of it being the possibility of working only with global vectors.

Besides, definition of a global Δt may be quite non-economical, when the mesh is nonuniform, for instance when surfaces presents sharp variations in curvature, because it can be also shown that an upper limit approximation for the maximum frequency of the system is given by the *maximum maximorum* of the element frequencies, *i.e.*,

$$\omega_{\max} \le \max_{e=1}^{n_e} \left\{ \omega_{e,\max} \right\}$$
(17)

where $\omega_{e,\max}$ is the maximum natural frequency of element *e* (see references^{14,16}). Therefore, the smallest element determines the maximum allowable time-step.

Fortunately, Eq. (17) provides also a way to compute an upper bound for the time-step without the necessity of assembling the global stiffness matrix and –even more relevant– it also allows a *mass tuning procedure*, whereby the fictitious element nodal masses are adjusted in such a way that all the elements comply to a prescribed value Δt^* for the time increment, thus overcoming the limitations associated to non-uniform meshes.

A quite general and efficient mass tuning algorithm has been developed by the first author of this paper, a thoroughly discussion of which is deferred to a forthcoming paper.

3 FLEXIBLE BOUNDARIES

Area minimization with flexible boundaries requires the specification of an additional constrain, otherwise the problem becomes unbounded. Indeed, applying the so-called *soap film analogy*, which states that the area of membrane under a uniform isotropic stress field is minimal^{1,2,3}, it is perhaps easier to understand that it is impossible to have a minimal area with free boundaries, since stresses transversal to the membrane boundaries would be zero. Thus cables are always required, to equilibrate stresses along a membrane's boundary.

Here, however, we restrict the problem to purely geometric quantities, and an ingenious way to do so is to redefine the problem as a volume minimization, as done originally in reference⁹. We thus add a thickness $h(\mathbf{x})$ to every point of the surface *S*, and we considered the surface to be bounded by flexible lines along its boundary ∂S , each of point of this lines endowed with an a cross-section area $A_{\phi}(\mathbf{x}_{P})$. We also consider that the surface is restrained at some points $\mathbf{x}_{P_{i}}$, enough to avoid rigid body motions. The total volume of the system is given by

$$V = \int_{S} h(\mathbf{x}) dA + \int_{\partial S} A_{\phi}(\mathbf{x}) df. \qquad (18)$$

Now, analogously to (3), the necessary 1^{st} order condition for a configuration x^* to present a minimum volume is given by the non-linear equation

$$\frac{\partial V}{\partial \mathbf{x}}\Big|_{\mathbf{x}^*} = 0, \qquad (19)$$

with the equality restriction, $(\mathbf{x}_{P_i} - \overline{\mathbf{x}}_{P_i}) = \mathbf{0}$, $i = 1, ..., n_P$, where $\overline{\mathbf{x}}_{P_i}$ are prescribed coordinates at the n_P fixed points.

3.1 Face-volume elements

Assuming a *facet-volume* discretization for the surface, together with *line-volume* elements along its borders,

$$V = \sum_{e=1}^{n_e} V^e \tag{20}$$

Also assuming a constant thickness inside a triangular facet, its unbalanced load vector is proportional to the quantity derived before, Eq. (9). Thus, in this case,

$$\mathbf{p}^e = -\frac{h^e}{2} \Lambda^e \mathbf{n}^e \tag{21}$$

3.2 Line-volume elements

We further assume that the surface boundary is divided into straight line segments connecting end nodes *i* and *j* and endowed with an uniform cross-section area A_{ϕ}^{e} , as shown in Figure 3. The current volume of such line-volume element is given by $V^{e} = A_{\phi}^{e} \ell^{e}$, where $\ell^{e} = \|\mathbf{x}_{2}^{e} - \mathbf{x}_{1}^{e}\|$ is the current element length.



Figure 3: A line-volume element

The line-volume element internal load vector is thus given by

$$\mathbf{p}^{e} = \frac{\partial V^{e}}{\partial \mathbf{u}^{e}} = A^{e}_{\phi} \frac{\partial \ell^{e}}{\partial \mathbf{u}^{e}} \,. \tag{22}$$

After some algebra, we obtain

$$\mathbf{p}^{e} = A_{\phi}^{e} \begin{bmatrix} -\mathbf{v}^{e} \\ \mathbf{v}^{e} \end{bmatrix}, \qquad (23)$$

where $\mathbf{v}^e = (\mathbf{x}_2^e - \mathbf{x}_1^e) / \|\mathbf{x}_2^e - \mathbf{x}_1^e\|$ is the unit vector connecting the end nodes of the line element. Once again, \mathbf{p}^e is added to the global internal load vector according to (7), now with $\mathbf{C}_{1i}^e = \mathbf{C}_{2j}^e = \mathbf{I}_3$ and $\mathbf{C}_{1m}^e = \mathbf{C}_{2m}^e = \mathbf{0}$, $m \neq i, m \neq j$.

4 SOME BENCHMARKS

4.1 A Catenoid

Figure 4 shows a catenoid surface whose generatrix is given by $y(z) = a \cosh(z/a)$. The area of such surface is $A = 2\pi a (h + (a/2) \operatorname{senh}(2h/a))$. We consider the catenoid delimited by two coaxial rings of radius 5.0m, distant 6.0m from each other, for which h = 3.0m, a - 3.725355 mand A = 174.991064 m².

Table 1 shows the relative errors for three different approximations. In all cases, an initial cylindrical geometry connecting upper and lower rings was assumed, and a dynamic relaxation analysis was performed until the kinetic energy the model was damped out, which occurred after about 50 time-steps, for each model. We remark that faceting introduces an intrinsic error in area estimative, which is not related to the precision of the solution method.



Figure 4: A catenoid surface and three different levels of discretization

Mesh	n _e	n_n	Area [m ²]	error
1	72	48	173.5658	8.5×10^{-3}
2	288	168	174.6246	2.1×10^{-3}
3	1152	624	174.9799	6.4×10 ⁻⁵

Table 1: Area estimative for three different discretizations

4.2 A hyperbolic paraboloid with fixed borders

As a second benchmark we consider a hyperbolic paraboloid described by z(x, y) = axy, as depicted in Figure 5(a). Taking a = 0.1m, $-5m \le x \le 5$ m, $-5m \le y \le 5$ m, the area of this surface can be calculated, to any required precision, by

$$A = \int_{S} \left(1 + \left(\frac{\partial z}{\partial x} \right)^{2} + \left(\frac{\partial z}{\partial y} \right)^{2} \right)^{\frac{1}{2}} dA = \int_{-5}^{5} \int_{-5}^{5} \left(1 + \left(0.1y \right)^{2} + \left(0.1x \right)^{2} \right)^{\frac{1}{2}} dx \, dy \approx 107.90370 \,\mathrm{m}^{2} \, .$$

A mesh with 1200 triangular facets and 645 nodes was adopted for the numerical solution. Figure 5(b) shows the initial mesh geometry, purposely far from the minimal configuration sought. Figure 5(c) shows the final configuration, with area $A=107.9235m^2$. About 200 timesteps were required to damp out the kinetic energy of the model.



Figure 5: A hyperbolic paraboloid with fixed boundaries: (a) geometric parameters; (b) initial geometry; (c) final geometry.

4.3 A hyperbolic paraboloid with flexible borders

As a final example, we consider a hyperbolic paraboloid with flexible boundaries, taking the same mesh and initial geometry used in previous example. Only the displacements of the vertices are restrained, and a series of line-volume elements is arrange along the borders. Figure 6 shows the resulting geometries for $A_{\phi} = 50$, $A_{\phi} = 20$ and $A_{\phi} = 10$, according to Eq. (22). About 250 time-steps are required to damp out the model's kinetic energy. Element distortion increases considerably as the borders' flexibility is increased, indicating that initial meshes laid onto geometries too far from solution may degrade considerably.



Figure 5: Some hyperbolic paraboloids with flexible boundaries: (a) $A_{\phi} = 50$; (b) $A_{\phi} = 20$; (c) $A_{\phi} = 10$.

4 CONCLUSIONS

- The Dynamic Relaxation Method (DRM) offers an interesting alternative to solve the area minimization problem, first interpreted as a nonlinear equilibrium problem, then replaced by a pseudo-dynamic analysis, where fictitious masses and damping matrices are arbitrarily chosen to control the stability of the time integration process;
- A discussion on the algorithms adopted for stability is postponed to a future paper, but we believe that the examples herein presented encourage the analyst to consider the application of DRM as a general method to solve nonlinear problems, not necessarily of mechanical nature.

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Finding Minimal and Non-Minimal Surfaces through the Natural Force Density Method

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Key words: Natural Force Density Method, Membrane Structures, Shape finding, minimal surfaces; non-minimal surfaces.

Summary. This paper discusses the Natural Force Density Method, an extension of the well known Force Density Method for the shape finding of continuous membrane structures, which preserves the linearity of the original method, overcoming the need for regular meshes. The method is capable of providing viable membrane configurations, comprising the membrane shape and its associates stress field in a single iteration. Besides, if the NFDM is applied iteratively, it is capable of converging to a configuration under a uniform and isotropic plane stress field. This means that a minimal surface for a membrane can be achieved through a succession of viable configurations, in such a way that the process can be stopped at any iteration, and the result assumed as good. The NFDM can also be employed to the shape finding of non-minimal surfaces. In such cases, however, there is no guarantee that a prescribed, non-isotropic stress field can be achieved through iterations. The paper presents several examples of application of the NFDM to the shape finding of minimal and non-minimal membrane surfaces.

1 INTRODUCTION

Design and analysis of membrane structures constitute an integrate process, including procedures for shape finding, patterning and load analysis. The Finite Element Method is a versatile way to pose this overall process, directly providing, besides a viable shape, also a map of the stresses to which the structure is subjected. It is also adequate to determine the behavior of the structure under design loads, as well as to transfer data to the patterning routines. On the other hand, procedures based on the FEM or in other forms of structural analysis result in nonlinear analyses, and require specification of a convenient initial geometry, load steps and boundary conditions, which are not always known from start.

An alternative method for finding viable configurations, which avoids the problems related to nonlinear analysis, is given by the *force density method*, which was first proposed in the context of cable nets^{[1],[2],[3]}. The method is routinely applied to shape finding of membrane surfaces, replacing the membrane by an equivalent cable network, which must be as regular as possible (otherwise is may become quite dubious which force densities should be prescribed to achieve a desired shape).

This paper discusses an extension of the force density method for the shape finding of

continuous membrane structures, which preserves the linearity of the original method. The new NFDM was first suggested in 2006 by Pauletti^[4], based on the natural approach introduced by Argyris for the Finite Element Method^[5]. Afterwards, Pauletti and Pimenta^[6] presented a more rigorous foundation for the method, recognizing that the imposition of natural force densities (NFD) is equivalent to the imposition of 2nd Piola-Kirchhoff (PK2) stresses to a reference mesh, a property first recognized by Bletzinger and Ramm^[7] for the original force density method.

2 FORMULATION OF THE NATURAL FORCE DENSITY METHOD

Consider a three-node plane triangular finite element shown in Figure 1. Let ℓ_i^0 , ℓ_i^r and ℓ_i , i = 1, 2, 3, be the element side lengths at an undeformed, a reference and an equilibrium configurations, respectively. We define three "natural deformations" along the sides of the element, according to $\varepsilon_i = (\ell_i - \ell_i^0)(\ell_i^0)^{-1}$, and collect them in a vector of natural deformations $\varepsilon_n = [\varepsilon_1 \quad \varepsilon_2 \quad \varepsilon_3]^T$. There exists a linear relationship between ε_n and the linear Green strains $\varepsilon_n = \mathbf{T}\varepsilon$, from which we can define a vector of natural stresses $\sigma_n = \mathbf{T}^{-T}\sigma$, where σ is the vector of Cauchy stresses acting on the element. It can be shown that σ_n and ε_n are energetically conjugate.



Figure 1. (a) Unit vectors \mathbf{v}_i , i = 1, 2, 3, along the element edges; (b) internal nodal forces \mathbf{p}_i , decomposed into natural forces $N_i \mathbf{v}_i$; (c) determination of natural force N_3 .

We also define three "natural forces" N_i acting along the sides of the element, according to $N_i = V \ell_i^{-1} \sigma_i$, where V is the volume of the element, and collect them into a *natural force vector* $\mathbf{p}_n = \begin{bmatrix} N_1 & N_2 & N_3 \end{bmatrix}^T$. Furthermore, we define the *vector of the natural force densities* according to $\mathbf{n} = \begin{bmatrix} n_1 & n_2 & n_3 \end{bmatrix}^T = \mathcal{L}^{-1} \mathbf{p}_n = V \mathcal{L}^{-2} \mathbf{T}^{-T} \boldsymbol{\sigma}$, where $\mathcal{L} = \text{diag} \{ \ell_1 & \ell_2 & \ell_3 \}$.

Thereafter, we show that, for a prescribed natural force densities vector **n**, there is a linear relationship between the natural force vector \mathbf{p}_n and element nodal coordinates $\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 & \mathbf{x}_2 & \mathbf{x}_3 \end{bmatrix}^T$, according to $\mathbf{P}_n = \mathbf{k}_n \mathbf{x}$, where \mathbf{x}_i , i = 1, 2, 3, are the position vectors of the

element nodes at the equilibrium configuration and \mathbf{k}_n is a constant symmetric *element* stiffness matrix, given by

$$\mathbf{k}_{n} = \begin{vmatrix} (n_{2} + n_{3})\mathbf{I} & -n_{3}\mathbf{I} & -n_{2}\mathbf{I} \\ -n_{3}\mathbf{I} & (n_{1} + n_{3})\mathbf{I} & -n_{1}\mathbf{I} \\ -n_{2}\mathbf{I} & -n_{1}\mathbf{I} & (n_{1} + n_{2})\mathbf{I} \end{vmatrix}$$
(1)

After assembling the load and stiffness contributions of all elements, we arrive to a linear problem at the structural level, which is completely independent of any reference configuration.

However, instead of prescribing directly some natural force densities vector **n** for each element, it may be more convenient calculate them from stresses $\boldsymbol{\sigma}_r$ defined in a reference configuration, according to $\mathbf{n}_r = V^r \mathcal{L}_r^2 \mathbf{T}_r^{-T} \boldsymbol{\sigma}_r$. It can be shown that $\boldsymbol{\sigma}_r$ corresponds to the 2nd P-K stresses associated to the final Cauchy stresses, calculated at the equilibrium configuration according to $\boldsymbol{\sigma} = (V^{-1} \mathcal{L}^2 \mathbf{T}) \mathbf{n}_r$. A thorough derivation of the formulation outlined above is given in references ^[4] and ^[6].

If the NFDM is applied iteratively, always re-imposing a constant, uniform and isotropic 2^{nd} P-K stress field, the method will converge to a configuration under a uniform, isotropic Cauchy stress field. This means that a minimal surface for a membrane can be achieved through a succession of viable configurations, in such a way that the process can be stopped at any iteration, and the result assumed as good. This is a clear advantage, if compared to Newton's Method, which may also converge to a minimal solution, but through a series of unfeasible, non-equilibrium configurations.

Moreover, the NFDM can also be applied to the shape finding of non-minimal membrane surfaces through the imposition of non-isotropic PK2 stress fields. In this case, however, even though a viable shape can still be obtained at every linear step, there is no guarantee that an arbitrary prescribed, non-isotropic Cauchy stress field can be achieved through iterations. Furthermore, since geometry varies during iterations, definition of principal stress directions becomes more complicate.

3 APPLICATIONS

3.1 Linear NFDM, isotropic initial stress field

As a first application of the linear NFDM, consider the transformation of the same square reference mesh into different surfaces, in a single NFDM step. The first row of Figure 2 shows the reference mesh transformed into different shapes, simply prescribing displacements to some selected nodes, along with a uniform isotropic PK2 stress fields on the membrane and uniform normal loads on the border cables. The resulting Cauchy stress fields at the equilibrium configurations are no longer uniform. This is fully coherent with the original FDM, which also has no control over the normal forces acting on cables, at the equilibrium configuration.

Since any minimal membrane surface is intrinsically associated to a uniform and isotropic

Cauchy stress field, clearly none of the equilibrium shapes shown in Figure 2 is minimal, although stress gradients are quite restricted to the membrane vertices, thus actually none of these shapes is too far from the corresponding minimal shapes.

Although the original mesh geometry is basically irrelevant, the *topological genus* of the surface has to be respected. Thus, in order the produce a conoidal surface, a hole must be cut into the original mesh, as shown in the Figure 2(d). Moreover, while the original FDM requires a two-directional layout of linear FDM elements, as regular as possible, as shown in Figure 3(a), the NFDM is capable to deal with irregular meshes, as the one shown in Figure 3(b). Figure 3(c) shows how an isotropic stress field $\hat{\sigma}_0 = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}^T$ defined onto a rectangle-triangle is converted into an equivalent natural force density $\mathbf{n}_0 = \frac{1}{2} \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}^T$.



Figure 2: 1st row: viable configurations generated through the imposition of different sets of nodal displacements to the same plane squared reference mesh, with and without a hole; 2nd row: 1st principal Cauchy stress at the viable configurations.



Figure 3: (a) a regular, two-directional layout of FDM line elements; (b) a generic mesh of NFDM surface elements; (c) isotropic stress field $\hat{\sigma}_0$ defined onto a triangule and its equivalent NFD vector \mathbf{n}_0 .

3.2 Linear NFDM, non-isotropic initial stress field

A broader class of shapes can be achieved prescribing non-isotropic PK2 stress fields to the reference configuration. In the case of the original FDM, a two-directional layout of line elements very conveniently provides two directions with respect to which different force densities can be prescribed (for instance $n_x \neq n_y$, in Figure 3(a)). On the other hand, in the case of the NFDM, it is necessary to define a convenient director plane Π , whose intersection with the surface Ω^e of a given element define the direction of one of the principal PK2 stresses acting onto the element.

Figure 4(a) shows how a horizontal plane may serve as director for a straight conoid, whilst a vertical plane adequately define a principal direction for every element of the *hypar* shown in Figure 4(b). Defining a unit vector $\vec{n} \perp \Pi$, and $\langle \hat{i}, \hat{j}, \hat{k} \rangle$ the unit vectors of the local coordinate system, with $\hat{k} \perp \Omega^e$, the principal stress directions are given by unit vectors $\hat{i}' = \hat{k} \times \vec{n} / ||\hat{k} \times \vec{n}||$, and $\hat{j}' = \hat{k} \times \hat{i}'$, which are rotated with respect to the element local coordinate system by an angle $\theta = \arcsin((\hat{i}' \times \hat{i}) \cdot \hat{k})$.



Figure 4: Definition of principal stress directions onto a NFD element requires conveneint director planes Π .



Figure 5: 11st row: non-minimal *hypar* surfaces, for different initial PK2 principal stress ratios; 2nd row: final 1^{st} principal Cauchy stresses; 3rd row: final σ_I / σ_{II} stress ratios.
The first row of Figure 5 shows different hypers generated by the imposition of a nonisotropic PK2 initial stress field with uniform initial mean stress $\bar{\sigma}_0 = \frac{1}{2} (\sigma_I^0 + \sigma_{II}^0)$ an uniform initial stress ratio $\sigma_I^0 / \sigma_{II}^0$ onto an originally flat squared mesh, with director plane Π aligned with one of the square diagonals. All resulting geometries where obtained in a single iteration, thus the final 1st principal stresses and final σ_I / σ_{II} ratio vary over the surface, as can be seen in the second and third row of Figure 5, respectively.

3.3 Finding minimal surfaces with the Iterative NFDM

As a third example, inspired by a physical experiment illustrated by Isenberg^[8], consider a helicoidal soap film, shown in Figure 6(a). The same previous square reference mesh is deformed such that sides S1 and S2 are transformed into small radial segments (see Figure 6(b/e)). Side S2 is displaced transversally to the reference plane. Side S3 is deformed into a helix. Side S4 is constrained to slip over the vertical axis. Figure 6(b) shows the initial square reference mesh and the resulting geometry, associated to a Cauchy stress field with quite high stress concentration close to borders S2, S3 and S4. Subsequent iterations do not alter the geometry significantly, but do smooth the stress field. After the 10th iteration, a practically uniform, isotropic Cauchy stress field is achieved, with the 1st principal Cauchy stress σ_1 ranging from 1.005 to 1.063 (Figure 6(d)). Thus, the minimal surface associated with the prescribed boundary is in practice obtained. first principal stress : min 1 max 1.0644



Figure 6: A helicoidal soap film

Next, we consider the generation of a minimal Costa's surface^[9], starting from a nonminimal, non-smooth one (topologically, there is no distinction between them) and repeatedly imposing an isotropic PK2 stress field $\hat{\boldsymbol{\sigma}}_0 = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}^T$. In the 1st row, Figure 7(a) shows a non-minimal Costa's surface connecting three fixed circular rings. Figs. 7(b/c) show the geometry obtained after the 1st and 6th iteration of the NFDM. It is seen that the 1st iteration of the NFDM already provides a fair approximation to the minimal surface. At the 2nd row, Figs. 7(d/e/f) show the σ_I fields resulting after the 1st iteration (1.0288 $\leq \sigma_I \leq$ 1.8086), the 2nd

iteration $(1.0015 \le \sigma_I \le 1.0594)$ and the 6th iteration $(1.0001 \le \sigma_I \le 1.0124)$. It is seen that after the 2nd iteration the σ_I field has already smoothed out any stress concentrations. It is also seen that geometry converges much faster than the imposed stress field, and, for practical purposes, the analysis could be stopped after a single iteration, or a couple of them, since there is no point in performing several iterations chasing a result (the imposed stress ratio) which is known *a priori*.



Figure 7: Numerical model of Costa's Surface

Figure 8(a) shows a physical realization of Costa's surface, exhibited at the atrium of the Civil Engineering building of the Polytechnic School of the University of São Paulo. Figure 8(b) shows the patterning used to produce the physical model.



Figure 8: Physical model of Costa's Surface and corresponding fabric patterning

3.4 Finding non-minimal surfaces with the Iterative NFDM

As a final example, Figure 9 compares a minimal conoid (stress ratio $\sigma_r / \sigma_{\theta} = 1$ over the whole surface) to a non-minimal conoid ($\sigma_r / \sigma_{\theta} = 3$, arbitrary imposed over the whole surface). Both geometries were obtained after 10 NFDM iterations, required for convergence of the stress ratios. Results compare very well with analytical solutions, as shown in reference ^[10]. Once again, geometry converges much faster than stresses and, for practical purposes, the analysis could be stopped after a couple iterations.



Figure 9. Comparison between minimal and non-minimal conoids

12 CONCLUSIONS

- The Natural Force Density Method is a convenient extension of the FDM for the shape finding of continuous membrane structures, which preserves the linearity of the original method. It is particularly adequate to deal with the general non-structured, irregular meshes provided by automatic mesh generators;
- The method provides quite convenient viable configurations, comprising both a viable geometry and the associated viable stress, field in a single iteration;
- Although the analyst has no absolute control over the final stress field, if a uniform isotropic

stress field is prescribed, the resulting geometry does not differ too much from a minimal surface;

- Besides, if a uniform isotropic stress field is repeatedly prescribed, the method quickly converges to the geometry of the minimal surface associated to the given boundary;
- Non-minimal shapes are also easily generated, through the imposition of non-isotropic stress fields at a reference configuration. This can be accomplished in a single linear step, without control of the final stress field, or again through iterations, repeatedly prescribing a given non-isotropic stress field;
- In this last case, however, even though a viable shape can still be obtained at every linear step, there is no guarantee that an arbitrary prescribed, non-isotropic Cauchy stress field can be achieved through iterations;
- It is worth to point out that, as the original FDM, the Natural Force Density method is an un-material method, simply providing a viable configuration, regardless of material properties. It is a method intended solely for shape finding.
- As far as load analysis is concerned, up to date the author does not know any good reason to supersede a proper nonlinear structural analysis by any sort of adapted force density method. But, of course, this statement is far from conclusive.

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ENERGY SAVING DESIGN OF MEMBRANE BUILDING ENVELOPES

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1 INTRODUCTION

Besides glass, a variety of other translucent and transparent materials are just as highly attractive to architects: plastics, perforated metal plate and meshing, but maybe most of all membrane materials which can also withstand structural loads. Earlier applications of textile materials have served the purpose to keep off the sun, wind, rain and snow while offering the advantage of enormous span widths and a great variety of shapes. The development of high performance membrane and foil materials on the basis of fluoropolymers, e.g. translucent membrane material such as PTFE-(poly tetraflouroethylene) coated glass fibres or transparent foils made of a copolymer of ethylene and tetrafluoroethylene (ETFE) were milestones in the search for appropriate materials for the building envelope.

The variety of projects that offer vastly different type and scale shows the enormous potential of these high-tech, high performance building materials which in its primordial form are among the oldest of mankind. Their predecessors, animal skins, were used to construct the very first type of building envelopes, namely tents. Since those days, building has become a global challenge. Usually building structures are highly inflexible but long-lasting and they account for the largest share of global primary energy consumption. It is obvious that the building sector has to develop international strategies and adequate local solutions to deal with this situation.

Principally, building envelopes as facades or roofs are the separating and filtering layers between outside and inside, between nature and adapted spaces occupied by people. In historic terms, the primary reason for creating this effective barrier between interior and exterior was the desire for protection against a hostile outside world and adverse weather conditions. Various other requirements and aspects have been added to these protective functions: light transmission, an adequate air exchange rate, a visual relationship with the surroundings, aesthetic and meaningful appearance etc. Accurate knowledge of all these targets is crucial to the success of the design as they have a direct influence on the construction. They determine the amount of energy and materials required for construction and operation in the long term. In this context, transparent and translucent materials play an important role for the building envelope as they not only allow light to pass through but also energy. [1]-[6]

2 INNOVATIONS

In the last few decades, rapid developments in material production types (e.g. laminates) and surface refinement of membrane materials (e.g. coatings) have been constant stimuli for innovation. As a result, modern membrane technology is a key factor for intelligent, flexible building shells, complementing and enriching today's range of traditional building materials (Fig. 1). [7]





2.1 Second skin façades

The Centre for Gerontology, a spiral building in the South of Germany, houses a shopping area on the ground floor and provides office space on the upper floors (Fig. 2). A special characteristic is the horizontal walkway arranged outside of the standard post and rail facade which forms the thermal barrier. The walkway is protected from the weather by a secondary skin. The complex geometry, the creative ideas of the architect and the economical conditions have been a special challenge and led to the implementation of a highly transparent membrane facade with high visibility between the inside and the outside due to its much reduced substructure. Moreover, because of this 'climate envelope', an energy saving intermediate temperature range is created as a buffer, which can be ventilated naturally by controllable, glazed flaps in the base and ceiling area. This secondary skin has a surface area of approximately 1550 m2 and was constructed by the Hightex Group as a facade with a prestressed single layer ETFE membrane with a specially developed fixing system using lightweight clamping extrusions. This was the first implementation of this type of facade featuring a second skin made of single layer stressed ETFE membrane anywhere in the world.



Figure 2: Second skin façade of the Centre for Gerontology, Bad Tölz (Source: Jan Cremers)

Printing the transparent membrane with a silver dot fritting pattern serves as light scatter and sun protection. The flouropolymer-plastic ETFE used, which until then was mainly used for pneumatically pre-stressed cushion structures (comp. Fig. 4+5), has a range of outstanding properties which predestinates it for building envelopes:

• The life expectancy is far beyond 20 years if the material is used according to specifications.

• The ETFE-membrane is flame retardant (B1) according to DIN 4102 and other international standards. Tests have shown that, due to the low mass of the membrane (which is only between 0.08 and 0.25 mm thick, with a density of approx. 1750 kg/m3); there is minimal danger of any material failing down in the event of fire. [11]

• The ETFE membrane is self-cleaning due to its chemical composition, and will therefore retain its high translucency throughout the entirety of its life. Any accumulated dirt is washed off by normal rain if the shape and the connection details are designed correctly.

• The material is maintenance-free. However, inspections are recommended in order to find any defects (for example damage caused by mechanical impact of sharp objects) and to identify and repair such damage as early as possible. It is also recommended that the perimeter clamping system and the primary structure are regularly inspected.

• The translucency of the ETFE membrane is approximately 95 % depending on the foil thickness, with scattered light at a proportion of 12 % and direct light at a proportion of 88 %. Compared to open air environment, the dangerous UV-B and UV-C radiation (which causes burning and is carcinogenic) is considerably reduced by filtration (comp. Fig. 3).

• ETFE membranes can be 100% recycled. Additionally, this membrane system is extremely light (about 1/40 of glass). The ETFE system is unmixed and therefore separable.

• In order to reduce solar gain or to achieve specific designs while maintaining the transparency, two dimensional patterns can be printed on the membrane.

• Because of the zero risk of breakage, unlike glass, no constructive limits have to be considered when used as overhead glazing.

The outstanding properties of this membrane material ensure a constant high-quality appearance lasting over decades.



Figure 3: Solar Transmission of different envelope materials (Source: Jan Cremers/ ZAE-Bayern)



Figure 4: Slovenská Sporiteľňa Bank Headquarters, Bratislava, Slovakia (Source: Hightex)



Figure 5: Building physics of a pneumatic cushion structure (Source: Jan Cremers [5])

2.2 A modular approach to membrane and foil facades

Most projects incorporating textile constructions are prototypes and have an extremely high share of innovative aspects, which have to be solved and also impose a certain risk to the designer and the executing companies. Therefore it looks promising to closely look into the options of following a modular approach. Most of the activities are still in an R&D phase, however, a first important building has been realised: For the Training Centre for the Bavarian Mountain Rescue in Bad Tölz a modular facade has been developed together with the architect Herzog+Partner which comprises of approx. 400 similar steel frames with a single layer of pre-stressed ETFE foil (Fig. 6).



Figure 6: Training Centre for the Bavarian Mountain Rescue in Bad Tölz, Herzog+Partner (Source: Jan Cremers)

2.3 Flexible photovoltaics integrated in translucent PTFE- and transparent EFTEmembrane structures: 'PV Flexibles'

Hightex is working together with its sister company SolarNext on significant innovations to improve building with advanced membrane material. Among them are new 'PV Flexibles' that are applied on translucent membrane material or fully integrated in transparent foil structures (Fig. 7+8). The technology being developed is flexible amorphous silicon thin film PV embedded into flouropolymer foils to be used on PTFE membranes and ETFE foils. These complex laminates can be joined to larger sheets or applied in membrane material and be used on single layer roofs or facades. They can also be used to replace for example the top-layer in pneumatic cushions. [8]

PV Flexibles do not only provide electricity - in an appropriate application in transparent or translucent areas it might also provide necessary shading which reduces the solar heat gains in the building and thereby helps to minimise cooling loads and energy demand in summer. This synergy effect is very important because it principally helps to reduce the balance of system cost for the photovoltaic application. In a report, the International Energy Agency gives an estimation of the building-integrated photovoltaic potential of 23 billion square meters. This would be equivalent to approx. 1000 GWp at a low average efficiency of 5%.



Figure 7 and 8: PV Flexibles, flexible a-Si-PV integrated in ETFE foils (functional mock-up built in 2010 [left side] and PV ETFE cushion built in Rimsting in 2007 [right])

Up to now solutions for the integration of photovoltaic in free spanning foil and membrane structures have not been available, although these structures are predestined for the use of large scale photovoltaic applications (shopping malls, stadium roofs, airports etc.). PV Flexibles allow addressing market segments of the building industry which are not accessible to rigid and heavy solar modules in principle. The basic PV cell material is very thin (only approx. 51 μ m) and lightweight. Therefore, it is predestined for the use in mobile applications. But as it is fully flexible at the same time, it is also an appropriate option for the application on membrane constructions. [9]

PV Flexibles can be directly integrated in ETFE and PTFE membranes for the generation of solar energy. First applications have been executed successfully in the South of Germany allready in 2007 and since than are currently monitored with regard to their output performance (Fig. 7+8).

2.4 Functional coatings for membranes

The development of functional coatings on membrane material has a special impact also. In the past this has led to the development of low-E-coated and translucent PTFE-Glass fabric (emissivity less than 40%) which has been applied for the first time by for the new Suvarnabhumi Airport in Bangkok, Thailand which was opened at the end of 2006. [10]

The development of transparent selective and low-emissivity functional layers on ETFE film consequently has been the next step to allow accurate control of the energy relevant features of the material. The first project to make use of this newly developed type of material will be the large shopping mall "Dolce Vita Tejo" near Lisbon in Portugal with a roof area of approx. 40,000 m2 (Fig. 9+10).



Figure 9 and 10: Principle of roof cushion solution for Dolce Vita Tejo, Portugal (Source: Transsolar, Stuttgart)

The cushions are very with dimensions of 10 m x 10 m very large and made of three layers. Here, the transparent, selective low-E-coatings together with the specific north-shed-like geometry of the foil cushions help to realize the client's wish to have as much light as possible but also to reduce the solar-gains at the same time: Customers shall feel like being outside but in an environment of highest climate comfort (Fig. 11+12).



Figure 11 and 12: Dolce Vita Tejo, desgined by Promotorio Architects (Source: FG+SG)

3 DESIGN PROCESS

The variety of new technologies developed in the field of foil and membrane construction and materials are definitely expanding and enriching architectural design options to realize advanced technical solutions and new shapes. However, a solid background of know-how and experience is needed to derive full advantage of the innovative and intriguing offers. As an architect or designer you can only feel comfortable with technologies of which you have at least a basic understanding. This actually poses a great challenge to the educational system for architecture but also to the membrane industry, which is a comparable small sector. At the end, every new product and technology has to be introduced to the market and made known to the architects and designers, which needs resources for marketing activities and promotion. Also, it requires a great deal of pre-acquisitional activities of direct consulting to planners in early design stages to enable the development of functional and technical sound and also economical solutions. Therefore, it will be a long (but still very promising) road to follow until the technologies described here will be commonly used in the building sector and become something that could be called a 'standard'.

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4 ABOUT HIGHTEX

Hightex Group is a specialist provider of large area architectural membranes for roofing and façade structures. The membranes are typically used in roofs and façades for sporting stadia and arenas; airport terminals; train stations; shopping malls and other buildings. This type of structure is a competitive alternative to glass as it is lighter and safer as well as being flexible to create complex shapes and it can span larger areas. Hightex uses environmentally friendly materials and is focussed on innovative technology and coatings, which help to reduce a building's energy costs. Hightex, one of only a very few international companies to design and install these structures worldwide, has been involved in the construction of a number of very high profile buildings including The Cape Town Stadium and Soccer City Stadium in Johannesburg, both for use in the FIFA 2010 competition, the Wimbledon Centre Court retractable roof, the roof of the Suvarnabhumi International Airport in Bangkok and the grandstand roof at Ascot Race Course.

Recent projects include Ansih Kapoor's "Leviathan" for Monumenta (Grand Palais, Paris 5-2011), Stadium «Olimpijskyj» in Kiev/Ukraine, National Stadium Warsaw/Poland and the retractable roof of the famous BC Place Stadium in Vancouver/Canada.

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NUMERICAL INVESTIGATION OF THE STRUCTURAL BEHAVIOUR OF A DEPLOYABLE TENSAIRITY BEAM

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Key words: Tensairity, deployable structures, inflatable structures

Abstract. This paper investigates numerically the load bearing behaviour of a deployable Tensairity beam. More precise, it studies the influence of the cables that connect the upper and lower strut of the deployable Tensairity beam on its load bearing behaviour. Finite element analysis shows that these cables are pretensioned when the airbeam is inflated. When both diagonal and vertical cables are present, only the vertical cables become tensioned. These tensioned cables are able to take compressive forces, by the same amount as their initial pretension. This has as result that these cables avoid the hinges to deflect under compression. Or in other words, the pretensioned cables 'block' the hinges. Once the external load has reached the value whereby the value of the pretension becomes zero in at least one cable, the hinge is not blocked or supported anymore by this cable. The hinge will experience larger displacements and the stiffness of the deployable Tensairity beam decreases.

1 INTRODUCTION

Inflatable structures have been used by engineers and architects for several decades. These structures offer lightweight solutions and provide several unique features, such as collapsibility, translucency and a minimal transport and storage volume. In spite of these exceptional properties, one of the major drawbacks of inflatable structures is their limited load bearing capacity. This is overcome by combining the inflatable structure with cables and struts, which results in the structural principle called Tensairity.

1.1 Tensairity structures

Tensairity is a synergetic combination of struts, cables and an inflated membrane (by low pressurized air), as illustrated in figure 1. The tension and compression elements are physically separated by the air inflated beam, which – when inflated – pretensions the tension element and stabilizes the compression element against buckling.



Figure 1: The basic cylindrical Tensairity beam [1].

A Tensairity structure has most of the properties of a simple air-inflated beam, but can bear several times more load [1]. This makes Tensairity structures very suitable for temporary and mobile applications, where lightweight solutions that can be compacted to a small volume are a requirement. However, the standard Tensairity structure cannot be compacted without being disassembled. By replacing the standard compression and tension element with a mechanism, a deployable Tensairity structure is achieved that needs - besides changing the internal pressure of the airbeam - no additional handlings to compact or erect the structure.

1.2 Foldable truss system

A promising concept for a deployable Tensairity structure has been developed by Luchsinger [2], inspired by the foldable trusses of Santiago Calatrava. Calatrava developed in 1981 in his PhD-dissertation 'Zur Faltbarkeit von Fachwerken' ('On the folding of trusses') novel deployable structures by introducing hinges in trusses and by investigating their kinematics [3]. One of his deployable structures is a conventional truss where the horizontal tension and compression bars of each triangle are divided in two and reconnected with an intermediate hinge (figure 2). This way, the truss becomes a mechanism. To stabilize the system in deployed configuration, Calatrava applied vertical bars and a locking mechanism at the intermediate hinges.



Figure 2: Foldable truss by Calatrava [3].

Luchsinger adjusted the system for applying it in a Tensairity structure by replacing the vertical bars with (pre-tensioned) cables, as illustrated in figure 3 [2]. The diagonals can be included or excluded and materialized as struts or cables. The linear compression and tension elements, resp. on the upper and lower side, are in the deployable Tensairity structure continuously attached with the hull, and this way, the truss is stable when the air beam is fully inflated. The structure can be folded and unfolded without disassembling, as illustrated in figure 4.



Figure 3: Foldable truss for Tensairity beam by Luchsinger et al. [2].



Figure 4: The deployment sequence of the foldable truss [2].

2 PHYSICAL AND NUMERICAL MODEL

A physical model (with its simplifications and approximations) is applied in this section for a basic understanding of the effect of interactions between load, pressure, membrane, compression element and cables. Conclusions derived from this simplified model are verified by means of a numerical model.

As mentioned, the investigated deployable Tensairity beam is constituted of an airbeam, an upper and lower strut and cables connecting the hinges of upper and lower strut. Figure 5 illustrates a longitudinal and sectional view of the structure that will be investigated in this paper.



Figure 5: A longitudinal and sectional view of the deployable Tensairity beam.

2.1 Inflation

When *inflating* the airbeam, the overpressure pushes the upper and lower struts outwards and the hull tends to become a circle. This action will be counterbalanced by the cables that connect the compression and tension element. As a result, the cables become tensioned and experience thus a tensile force. The value of this force can easily be calculated.

For inflated beams, the radial membrane tension $n_{radial} \left[\frac{N}{m}\right]$ is the product of the radius of the hull and the internal overpressure: $n_{radial} = p \times R$. As a result, the cable force F in one cable equals

$$F = 2 \times p \times R \times l \times sin(\alpha) \tag{1}$$

with α being the angle between the membrane (tangential) and the horizontal (indicated in figure 5) and *l* the distance between two adjacent cables. This normal force F[N] can also be called the pretension in the cable due to inflation (F_{pre}) .

The deployable Tensairity beam with vertical and diagonal cables connecting upper and lower strut (as shown in figure 5) is investigated by means of finite element calculations in ANSYS. The finite element model, illustrated in figure 6, is inflated with an internal overpressure of 100 mbar ($p = 10 \frac{kN}{m^2}$). The beam is isostatically supported and has a length of 2 m. The distance between adjacent vertical cables (length l) measures 0.333 m. The airbeam has a radius R of 0.127 m, a height h of 0.25 m between upper and lower strut and an angle α between the hull and the horizontal of 10°.



Figure 6: The deployable Tensairity beam is also investigated numerically. The airbeam is only modeled until the first and last cable for reasons of convergence.

From the figure can be seen that no end caps are modeled: the membrane is not fully closed at the ends and only modeled until the first cable. Otherwise, the hull interfered with the struts and convergence was an issue. It is the scope of further studies to ameliorate this finite element model. This approximation has as result that the cables closest to the ends (cable 1 and 5) experience half of the calculated and expected pretension $(\frac{1}{2}F_{pre})$. The cable pretension derived from the numerical calculations is 149 N for the middle cables and 74,5 N for cables 1 and 5. All forces introduced by inflation are taken by the vertical cables. The diagonals are not pretensioned under inflation because of their angulated position.

The pretension in the vertical cables is also calculated with equation 1. With using the same parameters as in the finite element model, one obtains the value of 147 N, which is a good approximation of the numerical value.

2.2 Loading

The cable pretension is decreased by *loading* the deployable Tensairity beam (downwards). When the amount of external load taken by one cable is equal to the pretension in this cable (F_{pre}) , it becomes slack. This means that the cable has from that point on zero stiffness and cannot support any additional compressive loading. As a consequence, the cable does not contribute anymore to the structural behaviour and one can expect the stiffness of the Tensairity beam to change at the value whereby the cables become slack.

The deployable Tensairity beam from figure 5 is loaded with a point load in each upper hinge. If F_{ext} is the total amount applied load, then in each hinge a load of $\frac{1}{5}F_{ext}$ is applied. From standard trusses, one knows that the first verticals (cable 1 and 5) experience a compression force of $\frac{1}{2}F_{ext}$. This means that these cables become slack when $F_{pre} = \frac{1}{2}F_{ext}$ or $F_{ext} = 2F_{pre}$. Since the cables 1 and 5 are tensioned with a normal force of 74,5 N, the maximal load this deployable Tensairity beam can bear before changing stiffness is thus 149 N.

This is also investigated by means of finite element calculations on the model presented in figure 6. The displacement of all upper hinges is noted and the average value in relation to the applied load is illustrated in figure 7. From the curve can clearly be seen that the stiffness changes at a total load of approximately 150 N, which corresponds with the analytical derived value. The graph on the right in figure 7 shows the same curves, but with a scaled x-axis to illustrate the stiffness of the airbeam and deployable Tensairity beam after 150 N. Figure 8, plotting the tension in the cables throughout loading, shows that cables 1 and 5 indeed reach zero tension at this value. The graph also shows that the diagonal cables are tensioned under loading, as is also the case for a truss with the same configuration of diagonals. This holds true until cable 1 and 5 become slack. From that point on, the tension force in the diagonals decreases.

The deployable Tensairity beam has also been investigated numerically under various pressures. Figure 9 shows the load-displacement graph of the case under 50, 100 and 200 mbar. As long as all cables are pretensioned, all curves have the same stiffness.



Figure 7: Left: Average displacement of the upper strut in relation to the applied load. (pressure is 100 mbar, five point loads in upper hinges). (Numerical results). Right: same curves, but the x-axis is scaled to show the stiffness of the airbeam and deployable Tensairity beam.



Figure 8: The tension in the cables in relation to the applied load. (Numerical results).



Figure 9: The load-displacement graph of the deployable Tensairity beam under 50, 100 and 200 mbar (loaded with five point loads in the hinges) (Numerical results).

Because the pretension in the cable is dependent of the internal pressure (equation 1), the case with the lowest internal pressure experiences as first a slack cable and thus another stiffness.

When all cables are pretensioned and thus able to take compressive forces, the deployable Tensairity beam has the same stiffness as a truss (with the same configuration of diagonals and with the same sections). This can be seen in figure 7. Once a cable does not contribute anymore to the structural behaviour, the bar structure becomes a 'mechanism'. This is illustrated in figure 10. However, the deployable Tensairity beam does not collapse immediately since it is still supported by the airbeam. This is why the stiffness of the beam is similar to the stiffness of an airbeam after the first cables are slack (figure 7).



Figure 10: Once a cable does not contribute anymore to the structural behaviour, the bar structure becomes a 'mechanism'.

3 CONCLUSIONS

The influence of cables on the structural behaviour of the deployable Tensairity beam is investigated in this paper. Finite element simulations confirm the physical model and show a relation between the structure's load-displacement behaviour and the contribution of pretensioned cables.

The cables connecting upper and lower strut that are pretensioned at inflation are able to take compressive forces, by the same amount as their initial pretension. This has as result that these cables avoid the hinges to deflect under compression. Or in other words, the pretensioned cables 'block' the hinges and the structure's stiffness is similar to the stiffness of a truss. Once the external load has reached the value whereby the value of the pretension becomes zero in at least one cable, the hinge is not blocked or supported anymore by this cable. The hinge will experience larger displacements and the stiffness of the deployable Tensairity beam decreases.

Further research will focus on the validation of these numerical findings by means of experimental investigations on a two meter prototype. Also, a more detailed numerical model will be developed and implemented. In addition, several cable configurations will be investigated.

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SHEAR DEFORMATIONS IN INFLATED CYLINDRICAL BEAMS: AN OLD MODEL REVISITED SALVATORE S. LIGARÒ^{*}, RICCARDO BARSOTTI[†]

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Key words: Inflatable structures, Cylindrical beams, Wrinkling, Shear deformations, Nonlinear analysis.

Summary. The paper considers the effects of the shear deformations on the loaddisplacement response of pressurized thin-walled cylindrical beams of circular shape, subject to lateral loads. In order to schematize the nonlinear behavior of partly wrinkled beams under simultaneous bending and shear, use is made of some classical structural models which account for the inability of the wall material of sustaining compressive stresses. A particular attention is posed on the correct determination of the shear stiffness within the wrinkled zones of the beam. The system of non linear equations that govern the equilibrium of the inflated beams after the onset of the post-critical phase, when wrinkling of the cross-sections still remains small or moderate, is suitable to be numerically solved by standard incrementaliterative algorithms.

1 INTRODUCTION

A number of efficient structural and continuum models are currently available to describe the mechanical response of pressurized beams in bending until their final collapse. Obviously, each approach, structural or continuum, has strengths and weaknesses since their validity fields as well as their goals are often very different. Structural models^{1, 2} are more common and easier to use compared to the continuous models (either analitical³ or numerical⁴) but cannot provide those local information that very often are necessary and that are obtained more quickly with the continuous models. By contrast, analyses that use continuous models are always slow and problematic since they require a more precise and detailed description of the mechanical problem. For example, it is easy to assign a concentrated lateral load in a generic section of the beam while it is extremely difficult to specify an equivalent load condition to the continuous model. Things are even more difficult when describing the constraints. In many cases the results offered by the two models for the same mechanical problem may differ unexpectedly and to be hard to compare each other. Fortunately, a third opportunity is offered by the bridging models^{5, 6}, definable also as advanced structural models, by which it is possible to get the required local information, usually the averaged values of some quantities, without suffering the complications of the continuum models.

With reference to the cylindrical inflatable beams of circular cross-section in bending, the founder of such models is undoubtedly the one proposed by Comer and Levy⁵ to study the mechanical response of a cantilever beam subjected to a concentrated or a uniformly distributed lateral load, along the wrinkled phase which precedes the collapse. Because of its simplicity and capability of giving the correct response in many practical situations, this model has been intensively used in the following by many other authors^{7, 8} as a starting point in order to incorporate some typical properties of the materials used to realize the cylindrical wall, in particular, anisotropy or, very often, other special non-linear constitutive laws characterizing the tissues composing the structural membranes.

A point of the above model that, in our opinion, has not yet been sufficiently considered so far concerns the effects of the shear deformations, usually disregarded, on the loaddisplacements response of the partly wrinkled inflated beams. Although this simplification be legitimate when considering the ordinary slender beams in bending, this argument is no more valid for the inflated beams: in fact, because of their peculiar small bending stiffness, to make acceptable the values of the lateral displacements under possible transversal loads of appreciable magnitude, they need using sizes of the cross-sections no longer negligible compared to their span. In addition, this topic becomes particularly important during the wrinkled phase since, contrary to the bending stiffness which decreases slowly for increasing wrinkling, the shear stiffness of the wrinkled cross-sections diminishes very rapidly.

For this reason, in this paper, we analyze the kinematical effects of the shear deformations arising in inflated cylindrical beams, partly wrinkled, subjected to bending, focusing on the still reagents parts of their cross-sections. The system of nonlinear equations governing the equilibrium of the inflated beams subject to simultaneous bending and shear appears suitable to be numerically solved by standard incremental-iterative algorithms, so that the mechanical response of the beams during the loading phase that follows the onset of wrinkling can be accurately monitored.

2 THE MECHANICAL MODEL AND THE STATE "0"

We consider a cylindrical thin-walled beam of circular shape, subjected to an internal pressure p. Let r be the radius of its mean surface and t the wall thickness. This may be the real value in the case of a very thin shell or an equivalent fictitious one if the wall is made of a structural tissue. In any case, we admit the thickness be so small that the wall does not possesses any bending or torsional stiffness, so that it may sustain only membrane states of stress; moreover, we admit that the material is sufficiently soft when it is contracted, so that no compressive stress may be engendered, but at the same time, it is stiff enough in tension to avoid appreciable variations of the radius r.

With reference to a rectangular coordinate system (O, x, y, z), in the initial configuration, chosen as reference, the axis of the cylinder lies along the z-axis. The origin of the reference system is placed at the centroid of one of its bases.

For what concerns the kinematical aspects, we assume that the hypotheses of small displacements/rotations and small deformations hold; moreover, we assume that during the considered loading phase the internal pressure be sufficient to maintains circular the shape of the cross-sections. Finally, the usual Navier-Bernoulli hypothesis on the plane sections holds for all the cross-sections of the cylinder, whether they belong to taut or partly wrinkled regions. With regard to the constitutive law, we assume the material be linear elastic, homogeneous and isotropic when it is subject to elongations. In case of contraction, instead, the material does not react in any way.

Before the application of any external load, the inflated beam lies in its "0" state. Here, if we admit the effects of the dead load may be neglected, the state of stress within the wall is uniform and characterized by the principal stresses

$$\sigma_1 = \sigma_c = p r/t \text{ and } \sigma_2 = \sigma_l = p r/2t,$$
 (1)

where σ_c denotes the circumferential stress and σ_l the axial one. The distribution of the axial stress is represented in Fig. 1a.

3 THE UNWRINKLED PHASE

For small or moderate values of the bending moment M, the neutral axis n-n does not intersect the cross-section, so the state of stress appears as in Figure 1b.



Figure 1: Stress distributions: a) at the state 0, b) during the unwrinkled phase, c) at the onset of wrinkling.

Because of the Navier-Bernoulli hypothesis, the normal stress at point $P(r,\theta)$ is

$$\sigma_z(\theta) = \sigma_e (1 + \cos \theta)/2 + \sigma_i (1 - \cos \theta)/2 \quad \text{for} \quad 0 \le \theta \le \pi ,$$
⁽²⁾

where θ denotes the angular position of the point P and σ_i and σ_e denote the unknown normal stresses at the intrados and at the extrados of the cylinder, respectively. In the absence of axial force, N = 0, the equilibrium of the beam along the *z*-axis

$$2\int_0^{\pi} \sigma_z(\theta) r t d\theta - p \pi r^2 = 0, \qquad (3)$$

gives the first static equivalence relation

$$\sigma_i + \sigma_e = p \ r/t , \qquad (4)$$

while the rotational equilibrium about the x-axis

$$2\int_0^n \sigma_z(\theta) r^2 \cos(\pi - \theta) t \, d\theta = M , \qquad (5)$$

leads to the second static equivalence relation

$$(\sigma_i - \sigma_e) \pi r^2 t/2 = M .$$
⁽⁶⁾

Putting together (4) and (6), we have

$$\sigma_i = \frac{p r}{2 t} + \frac{M}{\pi r^2 t} \quad \text{and} \quad \sigma_e = \frac{p r}{2 t} - \frac{M}{\pi r^2 t}.$$
(7)

At the onset of wrinkling $\sigma_e = 0$, thus $M = M^w = p \pi r^3 / 2$ and $\sigma_i = \sigma^w = p r / t$.

As long as $\sigma_i \leq \sigma^w$ the cross-section remains in the active state (taut), its moment of inertia is $J_x = \pi r^3 t$ and the following linear constitutive law holds between the local elastic curvature k_x^E and the bending moment *M*

$$k_x^E = \frac{1}{R_x} = \frac{\varepsilon_e - \varepsilon_i}{2r} = \frac{\sigma_e - \sigma_i}{2Er} = -\frac{M}{EJ_x} , \qquad (8)$$

where *E* is the Young's modulus of the material.

The Jourawski formula gives the expressions for the shear stress $\tau_z(\theta)$ and the corresponding shearing strain $\gamma_z(\theta)$ at point $P(r,\theta)$

$$\tau_{z}(\theta) = \frac{T_{y} \sin \theta}{\pi r t}, \qquad \gamma_{z}(\theta) = \frac{\tau_{z}(\theta)}{G} = \frac{T_{y} \sin \theta}{G \pi r t}, \qquad (9)$$

where G is the shear modulus of the material. Finally, by means of the Clapeyron theorem, we derive the sought expression for the characteristic shearing strain of the cross-section

$$\gamma_{y} = \frac{T_{y}}{G \pi r t} = \frac{2 T_{y}}{G A},\tag{10}$$

where $A = 2 \pi r t$ is the area of the complete section and $\chi_y = 2$ is the shear factor.

4 THE WRINKLED PHASE

When $M > M^w$, the neutral axis n - n intersects the cross-section, so the mechanical behavior of the cylindrical beam changes considerably; in fact, since the membrane is unable to sustain compressive stresses, from a structural point of view this is equivalent to admit a loss of resisting material. Thus, for increasing M, the active zone of the cross section reduces

progressively to that represented by a thicker line in Figure 2. We denote with $2\theta_0$ the angular amplitude of the wrinkled zone.



Figure 2: stress distributions along the wrinkled phase.

Since $\sigma_e = 0$ and $\sigma_i > \sigma^w$, the normal stress at point $P(r, \theta)$ now becomes

$$\sigma_z(\theta, \theta_0) = \frac{\cos \theta_0 - \cos \theta}{1 + \cos \theta_0} \,\sigma_i\,,\tag{11}$$

and the equilibrium along z-axis gives the first relation of static equivalence

$$\sigma_i(\theta_0) = \frac{p \pi r}{2t} \frac{1 + \cos \theta_0}{\sin \theta_0 + (\pi - \theta_0) \cos \theta_0}.$$
(12)

Putting together (11) and (12) we have

$$\sigma_{z}(\theta,\theta_{0}) = \frac{p \pi r}{2t} \frac{\cos \theta_{0} - \cos \theta}{\sin \theta_{0} + (\pi - \theta_{0}) \cos \theta_{0}}.$$
(13)



Figure 3: Law of variation of $\sigma_i(\theta_0)$.

It is interesting to consider the law of variation of $\sigma_i(\theta_0)/\sigma^w$ illustrated in the Figure 3. We observe how, just after the onset of wrinkling, *i.e.*, when the values of θ_0 are still small or moderate, $0 \le \theta_0 < \pi/4$, $\sigma_i(\theta_0)$ remains almost constant, $\sigma_i(\theta_0) \cong \sigma^w = p r/t$; conversely, when θ_0 approaches π , $\sigma_i(\theta_0)$ is not limited. In other words, the global equilibrium imposes locally the presence of a concentrated force.

The rotational equilibrium about the x-axis

$$2\int_{\theta_0}^{\pi} \sigma_z(\theta, \theta_0) r^2 \cos(\pi - \theta) t \, d\theta = M , \qquad (14)$$

leads to the second static equivalence relation

$$\sigma_i(\theta_0) = \frac{M}{r^2 t} \frac{1 + \cos \theta_0}{\pi - \theta_0 + \sin \theta_0 \cos \theta_0}, \qquad (15)$$

which, by means of (12), furnishes

$$M = \frac{p \pi r^3}{2} \frac{\pi - \theta_0 + \sin \theta_0 \cos \theta_0}{\sin \theta_0 + (\pi - \theta_0) \cos \theta_0}.$$
 (16)

Once *M* is assigned, this equation gives the sought value of $\theta_0(M)$.

If we take the limit of the above expression for $\theta_0 \to \pi$, we obtain

$$M^{u} = \lim_{\theta_{0} \to \pi} M(\theta_{0}) = p \pi r^{3} = 2M^{w}, \qquad (17)$$

the *ultimate bending moment* that an inflated cylindrical beam of radius r subject to internal pressure p may sustain if the material of the membrane were able to resist at the intrados to the concentrated axial force $T_{\text{max}} = p \pi r^2$.

Since $p \pi r^3 / 2 = M^w$, from (16) we obtain

$$\frac{M}{M^{w}} = \frac{\pi - \theta_{0} + \sin \theta_{0} \cos \theta_{0}}{\sin \theta_{0} + (\pi - \theta_{0}) \cos \theta_{0}} = m(\theta_{0}), \qquad (18)$$

an expression that permits to recover $M(\theta_0)$ once the angular amplitude $2\theta_0$ of the wrinkle zone is known. A graph of $m(\theta_0) = M / M^w$ is given in the next Figure 4.

We notice how a good approximation for $m(\theta_0)$ within the interval $(0, \pi)$ is given by the simpler function

$$\frac{M}{M^{w}} \cong \mu(\theta_0) = \frac{3 - \cos \theta_0}{2}, \qquad (19)$$

from which we obtain the value

$$\theta_0 = \arccos(3 - 2M/M^w), \qquad (20)$$



of the angle which individualizes the position of neutral axis n - n.

Figure 4: Law of variation of $M(\theta_0)/M^w$.

By analogy with (8), the local curvature of a wrinkled section is

$$k_x^w = \frac{1}{R^w} = -\frac{\varepsilon_i}{r\left(1 + \cos\theta_0\right)} = -\frac{\sigma_i}{E r\left(1 + \cos\theta_0\right)} = -\frac{M}{E J_x} \frac{\pi}{\pi - \theta_0 + \sin\theta_0 \cos\theta_0},$$
(21)

from which we obtain the expression of the fictitious moment of inertia of a wrinkled section

$$J^{w}(\theta_{0}) = J_{x} \frac{\pi - \theta_{0} + \sin \theta_{0} \cos \theta_{0}}{\pi}, \qquad (22)$$

able to simultaneously account for the two different static schemes that a wrinkled section uses to resist to an assigned bending moment: the first one is that of a couple deriving from two eccentric axial forces, and the second, of minor importance on a quantitative basis, of pure bending. In effect, the real moment of inertia of a wrinkled section is

$$J_{x'}(\theta_0) = J_x \left(\frac{\pi - \theta_0 - \sin \theta_0 \cos \theta_0}{\pi} - \frac{2 \sin^2 \theta_0}{\pi (\pi - \theta_0)} \right), \tag{23}$$

where x' is the axis parallel to the x-axis but containing the centroid of the wrinkled section. A plot of both the ratios $J^{w}(\theta_{0})/J_{x}$ and $J_{x'}(\theta_{0})/J_{x}$ is given in the following Figure 5. From this it is evident how the real moment of inertia decreases quite rapidly even for small values of θ_{0} , so that the capacity of a wrinkled cross-section to resist trough pure bending is soon frustrated. This feature is important also with regard to the shear deformations since the shearing force is resisted only through the unwrinkled zone.



Figure 5: Law of variation of the moment of inertia of a wrinkled cross-section: $(J^{w}$ fictitious, $J_{x'}$ effective).

The assessment of the shear deformations within a wrinkled zone is more involved with respect to that of a taut zone since now M and $\theta_0(M)$ both depend on z. As a consequence, the expressions of the shear stress $\tau_z(\theta, \theta_0)$ and the shearing strain $\gamma_z(\theta, \theta_0)$ at point $P(r, \theta)$ as well as for the characteristic shear strain $\gamma_y(\theta_0)$ of the overall cross-section are much more complicated and will be given in an incoming paper. Here, for the sake of simplicity, we consider the frequent case of distributed loads of small magnitude. Within this assumption, since the shear force T_y is small, the bending moment M changes slowly along the z-axis, so we may disregard the variation of the angular amplitude $\theta_0(M)$.

Thus, we may make use once again of the Jourawski formula which gives the following expressions for the shear stress $\tau_z^w(\theta)$ and the corresponding shearing strain $\gamma_z^w(\theta)$ at the point $P(r,\theta)$

$$\tau_{z}^{w}(\theta,\theta_{0}) = \frac{T_{y}}{r t} \frac{(\pi-\theta_{0})\sin\theta - (\pi-\theta)\sin\theta_{0}}{(\pi-\theta_{0})(\pi-\theta_{0}-\sin\theta_{0}\cos\theta_{0}) - 2\sin^{2}\theta_{0}}, \quad \theta_{0} \leq \theta \leq \pi, \quad (24)$$
$$\gamma_{z}^{w}(\theta,\theta_{0}) = \frac{T_{y}}{G r t} \frac{(\pi-\theta_{0})\sin\theta - (\pi-\theta)\sin\theta_{0}}{(\pi-\theta_{0})(\pi-\theta_{0}-\sin\theta_{0}\cos\theta_{0}) - 2\sin^{2}\theta_{0}}.$$

Finally, by means of the Clapeyron theorem, we derive the sought expression for the characteristic shearing strain of a partly wrinkled cross-section

$$\gamma_{y}^{w}(\theta_{0}) = \frac{\chi_{y}(\theta_{0}) T_{y}}{GA}, \qquad (25)$$

where $A = 2 \pi r t$ is still the area of the complete section and $\chi_y(\theta_0)$ is the shear factor of the wrinkled cross-section whose expression is

$$\chi_{y}(\theta_{0}) = \frac{2\pi \left(\left(2(\pi - \theta_{0})^{3} - 12(\pi - \theta) \sin^{2} \theta_{0} + 3(\pi - \theta)^{2} (\pi - \theta - 3\sin \theta_{0} \cos \theta_{0}) \right) \right)}{3(\pi - \theta_{0})^{2} (\pi - \theta_{0} - \sin \theta_{0} \cos \theta_{0} - 2\frac{\sin^{2} \theta_{0}}{\pi - \theta})},$$
(26)

whose graph is represented in the Figure 6.



Figure 6: Law of variation of the shear factor of a wrinkled cross-section.

The graph gives a clear indication of the decreasing rate of the shear stiffness of a pressurized beam for increasing wrinkling.

CONCLUSIONS

In this paper, we analyzed the effects of the shear deformations on the load-displacement response of inflated cylindrical beams in bending. A particular attention was posed on the equilibrium state (stress-strain) within the partly wrinkled zones of the beam.

The analysis was performed by means of some classical structural models which account for the inability of the material of the cylindrical wall of sustaining compressive stresses.

The obtained system of nonlinear equations governing the equilibrium of the inflated beams appears suitable to be numerically solved by standard incremental-iterative algorithms, so the mechanical response of these beams after the onset of wrinkling can be accurately monitored.

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DESIGN TOOLS FOR INFLATABLE STRUCTURES

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Key words: Inflatable Structures, Form Finding Process, Cutting Pattern, Parametric Design, Tensairity[®].

Summary. This paper shows different tools and approaches that can be useful for the definition of the design of pneumatic structures. Some of these tools have been applied for the design of a Tensairity[®] hull.

1 INTRODUCTION

"Form finding" is the process to generate the optimal configuration from the structural and visual point of view due to a given stress distribution and boundary conditions acting on a flexible structure¹.

Several software for form finding are available on the market and each of them present pros and contras. Considering the contra, the most complete and advance software for form finding and non linear behavior calculations require a strong background about finite elements analysis⁸ and a lot of mathematics, too. Moreover, the outcome, most of the time, is not "ready to use" and its result must be carefully checked to avoid errors and misunderstandings. In other words, these software are powerful and can solve complex design but their results can be tricky too and that's the reason why the user should have a certain level of experience to came out with trustable results.

From the usability point of view, most of these software are not user-friendly: time and effort are necessary to interact with their interface. Considering the already large number of software an architect or engineer should know to carry on a project of architecture, a process of integrations of different "specific tools" accessible within the same domain would dramatically reduce the waste of time (for example saving the time of import/export procedures) in the design process and would enlarge the audience of designer ready to apply tensile principles in their projects.

From a technical point of view, some of these software require computers that can offer high performances because solving algorithms are time and CPU consuming. Some firms or design studios, for this reason, use to develop their own software or tools, focused on their specific task. These scripts are lighter than complete software and they are usually dedicated to a specific purpose.

Some commercial firms already offer toolbars that operate within CAD domains such as Autodesk Autocad[®] or McNeel Rhinoceros[®] (Rhino). However, the features offered by those tools are most of the time limited even if they can solve perfectly standard shapes with well known boundary conditions. The positive side of a "friendly interface" and the "lightness" of the tool is counterbalanced by the limitation in forms and restrains they can solve.

A further step on this process would be the integration of different specific tools and form finding "solvers" through the use of an unique and adaptable interface, easy to deal with and to personalize. That's possible, for example, using the plug-in "Grasshopper[®]" (GH) for Rhino. GH is a visual programming language. Programs, called *definitions*, are created by dragging components onto a canvas. The outputs of these components are then connected to the inputs of subsequent components. The GH environment provides an intuitive way to explore designs without having to learn to script². Users can generate their own *definition* and "plugging" the useful *components*. These *components* can collect the geometrical input from the graphic interface of Rhinoceros and compute equilibrium or non linear analysis using the same algorithms developed by the most advanced finite elements software. In other cases, expert users can generate and define their own *components* are far away to be accurate but surfing the net and looking at the developments and growing of the GH's community, it's clear how fast improvements can be done².



Figure 1: Design process with Grasshopper®

In this paper, several case studies about the form finding and the design process for different inflatable shapes, using GH and other plug-ins, will be presented. Special attention will be given to the design and manufacturing of a parametric Tensairity[®] hull.

2 DEFINITION OF SHAPES: TWO APPROACHES

The definition of the shape of an inflatable structure is not an easy task⁵. Not every shapes can be done with the inflatable technology: that's the case, for example, of sharp edges. On the other hand, some envelope, under the load of pressure can't reach, a well distribution of stresses: as a result wrinkles are produced. One of the goals of the designer is to avoid wrinkles in the structure, because wrinkles can't guarantee an effective behavior of the system as a whole and they can generate unpredictable deformation under loading. To generate a pneumatic structure, one should predict how a certain envelope will accommodate itself to reach its equilibrium under a certain inner overpressure level.

Pneumatic structures are form-resisting structures and their shapes follow specific equilibrium rules. Complex shapes can be done connecting and intersecting several basic geometrical shapes. Each of these basic geometrical shapes present circular or portion of circular sections. The definition of the exact position of these circular sections and, sometimes, their deformations due to the interaction with boundary conditions, such as inner or outer struts or cables, is the crucial task to design an inflatable structure⁵. The higher the number of circular sections I can define, the more accurate the prediction of the shape of the pneumatic structure will be. Unfortunately, in most cases, only few circular sections are known in advance, especially if the structure is complex.

That's the reason why two different design approaches for pneumatic structures can be identified. In the first case, the designer should look for the circles or portions of circles that will be generated by the inflation, set them as strict boundary-conditions and generate from them the final shape (figure 2). This approach requires time and extensive geometrical abstraction's capacities. Its results are accurate but complex shapes can hardly be done. In the second case, the designer should look for the envelope and then, using specific scripts, simulate its behavior under the load of inflation (figure 3).



Figure 2: cross sections approach

Figure 3: envelope approach

In this case, geometrical intuition is not required. Any shape can be generated but the accuracy of the results has to be tested. The two approaches are different but complementary: the first one requires a clear idea about the final shape one has in mind; the second one focuses on the flat envelope before the inflation. The first one can be done with simple CAD systems or even by hand; the second one requires specific software and plug-ins (in this paper
the use of GH for Rhino will be considered). As we will see in the conclusions, most of the time, both approaches should be used: firstly the envelope approach will give the raw shape and will help to define the position of the circle sections; secondly, the cross-section approach will generate the final shape with better accuracy. The following section will describe the first approach; section four will investigate the second one.

3 CROSS-SECTIONS APPROACH

Any envelope, under the load of inner pressure, tries to accommodate most of its crosssections in the shape of circles or portions of circles. According to different boundary conditions such as borders connections, inner or outer springs or struts, these sections deform and they become more difficult to be identified.



Figure 4: basic shapes from points and linear axes

Figure 5: basic shapes from curved axes

The simplest pneumatic shape is the sphere: on it, an infinite number of circles with their centers on the center of the sphere can be identified. In a cylinder, infinite circular sections are placed, normal, along the axes of the cylinder itself. In the case of a spindle, the circles vary their radius along the length of the axes (figure 4). If the axes is not a straight line but a generic curve, more complex shapes can be defined. Cylinder-arches or spindle-arches are generated if the axes is an arc. In a similar way, a torus can be defined if circular sections are placed normal to a circle (figure 5).



Figure 6 and figure 7: asymmetrical linear pneumatic element, perspective and front view

Symmetrical and asymmetrical elements can be designed with this approach scaling and rotating circular sections (ax in this case would be a generic curve or a "*spline*") in specific

points as shown in figure 6 and 7.

Starting from these simple considerations and connecting together primitive forms, a large number of more complex shapes can be obtained. The final configuration can be predicted by the simple combination of each primitive form as far as no extra deformations will occur in the connection points.

3.1. INPUTS AND OUTPUTS

In McNeel[®] Rhinoceros[®] environment is rather easy to define geometries and make them interact parametrically using the free plug-in GH. To generate any inflatable element with the cross-sections approach one should be able to define as input: 1) the axes of the element, with a start and end point (or length); 2) a series of circular sections (radius), normal to the axes; 3) other restrains such as inner or outer springs or struts. All these inputs can be drawn in two ways: 1) all dates and geometrical elements (.dxf) can be drawn in Rhinoceros[®] or with any CAD system: all these dates can be then connected to each other parametrically with GH; 2) all inputs can be defined already in GH and then connected to each other. The second approach may take more time in the definition of the boundary conditions but it is way better as far as the final structure will be fully parametric.



Figure 8: changing of parameters

This means that changing any of the parameters, the structure is automatically redefined according to the new given inputs (figure 8).

Outputs of these kind of definitions can be as vary as designer's imagination; anyway, the basic outputs are twofold: 1) the envelope (surface or mesh); 2) dimension of any restrains (cables, struts, springs). The envelope can be split in different parts through cutting lines defined by the designer and ready to be "flattened" using specific tools as shown in section five.

4 ENVELOPE APPROACH

Sometimes, it is not possible to identify in advance circular sections that can be set as fixed boundary conditions for the definition of the final shape of an inflatable. Moreover, in some cases, designers would like to investigate how an envelope would look like under inflation without passing through the construction of hand models. Hand models cost times and effort and their behaviors are strongly dependent on their size, the properties of the materials used, the accuracy of the model itself and the definition of the cutting pattern for the production⁶. That's why the envelope approach would give an answer to those who would like to explore the world of pneumatics starting from the envelope itself. A process of trial and error would guide the designer to reach the final shape and maybe in the near future, it would help designers in controlling the whole inflation process, for example, of a folded pneumatic element. At the moment, inflation processes, for example for applications in space, are studied through complex finite elements analysis⁷. That's definitely not the tool architects or designer could apply for their purposes.

Sometimes, even some simple shapes can hardly be defined using the cross-section approach. A pneumatic pillow generated by the inflation of a flat rectangular peace of membrane, for example, is something that cannot be modeled accurately (especially at the ends) based on the cross-section approach (figure 9). In the case of a cushion with several holes, the identification of circular sections becomes quite difficult or even impossible (figure 10).



Figure 9: inflated pillow from a rectangular mesh



Figure 10: pillow with holes, two configurations

The generation of the process of inflation of the meshes showed in the pictures above has been done through the use of a plug in for GH called Kangaroo Physics[®]. Kangaroo is an add-on for GH/Rhino which embeds the physical behaviour directly in the 3D modeling environment and allows a live-interaction with the model when the simulation is running. It is used for various sorts of optimization, structural analysis, animation and more³. Kangaroo simulates the physical behaviour applying the "particle system". "Particles are objects that have mass, position, and velocity, and respond to forces but have no spatial extent. [...] Despite their simplicity, particles can be made to exhibit a wide range of interesting behaviour. For example, a wide variety of non-rigid structures can be built by connecting particles with simple damped springs"⁴. How Kangaroo applies forces and simulate physical behaviours is not the goal of this paper: further information can be found in the manual³. Connecting Kangaroo physics with any triangular or quadrangular mesh through a *definition* designed with GH, it is possible to simulate how the envelope accommodates itself under the load of internal pressure. The definition can simulate the inflation of any mesh, in real time, with a relatively low consumption of CPU. The definition deforms the mesh applying a

uniform distributed load on both inner surfaces of the mesh keeping the total surface area of the envelope constant. The stiffness of the envelope can be changed: for low stiffness, the mesh behaves like a toy balloon or a soap bubble (figure 11); increasing the level of stiffness, wrinkles appear in the structure (figure 12).



Figure 11: stiffness influence during inflation process



Figure 12: different pressure and stiffness levels

4.1 VALIDATION OF THE RESULTS



Figure 13: flat envelopes before inflation



Figure 14: flat meshes in Rhinoceros

Three different simple shapes have been built to validate the results of the inflation process using GH. A rectangle of size 47 cm*25 cm, a right-angle triangle of size 30cm*25cm and a second rectangle of size 13 cm*16 cm. The latter rectangle has been designed with two welding lines (every 4.5 cm) of 2 cm in length normal to the upper and lower edges between the two layers of the envelope. This design wants to generate deep wrinkles in some controlled points. All three envelopes are planar before inflation (figure 13). The reason why no cutting pattern has been done for these envelopes stems from the desire to check if this design approach could identify and predict if and where wrinkles would appear. The material used for the construction of the mock-ups is a light nylon fabric coated with polyurethane. According to the stiffness of the material, wrinkles would change the configuration and would appear at different pressure levels. As far as the definition in GH is not able yet to take into account material properties, a light membrane has been chosen in order to simplify the influence of the of the material in the deformation of the initial mesh.

Considering the case of the first rectangle (sample 1), the results of the inflation using GH

is quite promising. With an increment of the total surface of less than 1%, the inflated mesh presents, from a quick sight, the same shape of the mock-up (figure 15). Wrinkles' position is also quite accurate: biggest wrinkles appear close to each corner; other smaller wrinkles can be identified along the upper and lower edges (figure 16). Moreover the circumference at central section and the total longitudinal length after inflation have an accuracy of 5%.





Figure 15: mock-up envelope 1



Concerning the case of the right triangle (sample 2), the size of the final envelope is increased by 1%. Looking at the final shape of the mock-up one can clearly see how two deep wrinkles in the edges close to the right angle appear (figure 17).



Figure 17: mock-up envelope 2

Figure 18: mesh envelope 2

Along the hypotenuse, smaller wrinkles are generated, too. On the contrary, considering the inflated mesh (figure 18), wrinkles in the area of the catheti seem quite smooth whereas the ones along the hypotenuse look quite reasonable. Geometrical analysis shows that as in sample one, the main dimensions have an accuracy around 3%.

Investigating the behavior of more complex shapes like the one of sample 3 (figure 19), deep wrinkles are generated starting from the 2 cm welding; some of them go in two directions along the surface. Inflating the mesh, a similar behavior can be noticed. Anyway, the main difference comes for the fact that material and seam properties (i.e. stiffness, friction) are not taken into account at all: That's the reason why a rotation of the upper and lower part of sample two appears in the mesh envelope.



Figure 19: mock-up envelope 3

Figure 20: mesh envelope 3

From the geometrical point of view, the central circumference of the mesh envelope matches the length of the circumference of the mock-up built; on the contrary, the total longitudinal length decreases more than in the mock up case. This discrepancy probably comes again from the fact that no material properties and no interactions derived by the contact of the surface with itself is taken into account.

5 CUTTING PATTERN

The generation of the cutting pattern is the last step of the design process and it influences the manufacturing phase, the performances of the element under inflation and its aesthetical properties. An accurate cutting pattern produces a nice smooth shape once the envelope is inflated. To generate the cutting pattern, a series of cutting lines (that will become seams during the manufacturing process) are required. The arrangement of the cutting lines is the crucial part of an effective cutting pattern. In the case of a simple shape like an inflatable spindle arch, two main arrangements of seams are possible. In the first case, cutting lines are circular sections of the structure (figure 21). In the second case, seams follow the length of the arch, from one end to the other (figure 22).



Figure 21: cross section cutting pattern

Figure 22: longitudinal cutting pattern

The result of the cutting pattern of the first arrangement is a series of strips of different lengths: their lengths is equal to the circumference of the sections at that point. The result of

the cutting pattern of the second arrangements is a series of curved or straight strips. Having as a result straight (or almost straight) seam lines would speed up the process of manufacturing as far welding will be straight too. On the contrary, the longitudinal cutting pattern generates curved and straight panels that have to be welded together.

The final result of an inflated spindle arch manufactured using the cross section cutting pattern is most of the time poor because the final shape will be a combination of straight segments (figure 23). On the contrary, having a longitudinal cutting pattern, the shape will be continuous and smooth (figure 24).



Figure 23: spindle arch, cross section cutting pattern Figure 24: spindle arch, longitudinal cutting pattern

Terraflat® was the plug-in tool used in Rhino for the generation of the cutting pattern.

6 DESIGNING OF TENSAIRITY[®] STRUCTURES

Tensairity[®] are special structures⁸ and they require adequate design tools, too. The design of a Tensairity[®] could take a lot of time and effort especially if one doesn't have a well defined and clear idea of what the final result should look like. In other words, once one design has been made, any change in its geometry (length, height of section, position and number of struts/cables) would require to restart the design process from the very beginning. That's the reason why a parametric design for Tensairity[®] is highly recommended. In the next paragraphs a simple parametric tool for designing Tensairity[®] elements will be presented.



Figure 25: generation of the Tensairity® hull and some of the inputs

Five parameters have been considered crucial for the design of this kind of structure:1) the total length; 2) the height of the section at certain points (usually at 25%, 50 % and 75% of the total length) 3) the curvature of the element itself (to create straight elements or arches); 4) the number of struts or cables involved 5) the position of these struts of cables along the radius of the hull. Combining these parameters, a large number of Tensairity[®] elements can be designed, modified by changing each parameters (simply by changing the input values), adapting it to each specific design case (figure 25).

Output of the tolls are 1) the complete membrane, already divided into parts ready to be flatten with other software like $\text{Terraflat}^{\mathbb{R}}$; 2) length of $\text{Tensairity}^{\mathbb{R}}$ cables and struts), 3) definition of pockets for struts and cables. The accuracy of the tool was tested by comparison with several built prototypes (figure 26 and figure 27).



Figure 26: spindle asymmetric tensairity, version 1

Figure 27: spindle asymmetric tensairity, version 2

7 CONCLUSIONS

Two design methods for inflatable structures have been presented in this paper. The cross section approach is the most used method so far and doesn't require any particular software tool. It has been shown how a parameterization of this approach through software like GH does dramatically improve and speed up the design process. This approach is useful for simple shapes or aggregation of simple elements. The definition of circular or segments of circular cross-sections is the main input. For complex shapes, these cross sections are difficult to be identified thus limiting this approach.

The envelope approach is useful when cross sections cannot be identified clearly in advance. It is also useful if one would like to test how a generic envelop would look like under the inner pressure load without any handmade models. The accuracy of the tools tested is still low and cannot be considered sufficient to be used in architecture. The main problems identified are two: 1) as far as wrinkles depends on the stresses in the membrane, these tools should be able to control the inner pressure too: in GH the value of pressure or overpressure is not accurate at present and it is not related to real pressure; 2) the material properties are not taken into account. Future development of the research should test the accuracy of the mock up built with other materials (i.e. a thicker membrane) and should try to take into account the presence of seams and valves.

However, thanks to a continuously growing open-source on line community improvements in the field are incredibly fast. It is possible that in the near future some of these tools will reach the same level of accuracy as commercial software on the market with the great advantage to be extremely user-friendly and much less CPU consuming. To achieve the best result in designing inflatable structures both approaches are recommended. The cross-section approach can not be used for complex shapes but it can assure high accuracy. On the contrary, the envelope approach could give a raw idea about the shape any envelope would accommodate under inflation and can be extremely useful for complex shapes in which cross circular sections cannot be easily identified.

Cutting pattern generation plays the key role in the manufacturing of inflatable elements. Welding arrangements should be thought carefully to match the need of the construction phase. Specific tools like Terraflat[®] are now under development and work fine for most of the basic shapes. If meshes are too dense or if they are badly arranged, programs would not give an accurate result or, sometimes, cannot give result at all. Anyway, improvements are going fast and they are occurring during the time this paper was written, too.

In case of simple Tensairity^{\mathbb{R}} elements both approaches are effective and give accurate results. If the accuracy needs to be high, the cross-section approach is the most useful for this purpose. The help of the parametric tool of GH results in a dramatic speed up the design process.

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SIMULATION OF INFLATABLE STRUCTURES: TWO PROPOSALS OF DYNAMIC RELAXATION METHODS USABLE WITH ANY TYPE OF MEMBRANE ELEMENTS AND ANY REVERSIBLE BEHAVIOR

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Abstract. This work deals with the numerical study of inflatable fabric structures. As implicit integration schemes can lead to numerical difficulties, such as singular stiffness matrices, explicit schemes are preferred. Since the final objective of this study is to obtain the final shape of a structure, dynamic relaxation (DR) methods are used. These methods permit to obtain the final and stable shape of the inflatable fabric structures without doing so many time increments, which is the case when using a classical explicit integration method. Han and Lee (Computers and structures, 2003, 81, pp. 1677-1688) proposed an extension of the DR method stated by Barnes (Computers and Structures, 1988, pp. 685-695) suitable for triangular elements and elastic behavior. In this work we propose a modification of the method presented by Han and Lee which permits the method to be used with any kind of membrane or volumetric finite elements and any reversible behavior. Also, we propose another formulation based on the one initially proposed by Barnes. Furthermore, these presented methods are adapted to incremental loadings, allowing this way to obtain the pseudo-equilibriums of the intermediate phases. Numerical examples from academic problems (rectangular and circular membranes) show the efficiency and the reliability of proposed methods, with linear elasticity behavior, and also with general hyperelasticity and finite deformation states.

1 INTRODUCTION

The simulation by the FE method of inflatable fabric structures, when a pressure load is applied and an implicit scheme is used, can lead to severe instabilities due to the lack of stiffness in the fabric. Explicit time schemes overcome this difficulty, but they need a huge number of time steps to obtain a realistic stable final shape. This occurs when using natural damping. We can find several examples of this issue in civil engineering: geotechnical problems [1], prestressed coated fabric membranes [2], achitectural structures [3], and space inflatable structures [4]. There have been several solutions proposed [5, 6, 7, 8, 9] by using *dynamic relaxation* methods.

Among the existing dynamic relaxation methods, we are interested on the one proposed by Barnes [10]. He has initially applied it to the calculation of prestressed cable structures and it has been further extended by Han and Lee [5] to be used with triangular elements and a linear elastic behavior. This method combines a kinetic damping (resetting the speed to zero at each kinetic energy peak), often used in form-finding, and an optimization of the mass matrix (proposed by Han and Lee).

One application of the method is thin fabric structures loaded by pressure, which are notably unstable during loading due to the lack of flexion stiffness. The static final form does not have to depend on the inertial forces that act during the transient evolution. Considering this, the right value of the mass is supposed to have no influence on its static final form. In order to quickly reach the stable deformed state, we must first adapt the mass matrix (a correct choice leads to an optimal convergence) and then use kinetic damping.

In this paper, we will present two main formulations. Firstly, we propose an extension of that Barnes-Han-Lee method. Secondly, we propose a general expression based on the works of Barnes for the mass matrix calculus. The basis of this second expression has already been proposed in previous papers (see Underwood [11] or Barnes [10]), but to our knowledge, no systematic studies have been done concerning its applications for simulation of the inflation of unstable structures. Our methods aim to find one solution when one or more solutions exist (there can be several stable final shapes).

2 DYNAMIC RELAXATION METHOD

The dynamic relaxation method proposed by Barnes [10] uses an arbitrary mass term in order to improve the kinetic damping while keeping the numerical stability. Barnes proposes a lumped mass matrix where the elements m_i in the diagonal are:

$$[m_i] = \lambda \frac{\Delta t^2}{2} [k_i] \tag{1}$$

The optimum mass matrix is calculated by adjusting the parameter λ . In the shapefinding process of membrane structures, due to the large variations of the structures, Barnes [12] proposed to choose the largest stiffness term for the calculation of mass term. Han and Lee [5] stated, for CST (constant stress triangle) elements, that the stress k_i at node *i* with *m* members can be approximated as

$$k_{imax} = \sum_{e} \frac{h}{4S_0^e} \left(\frac{E}{1 - \nu^2} + \sigma_x + \sigma_y + \sigma_{xy} \right)$$
(2)

where h is the thickness of the element e; S_0^e is the initial surface of the element e and $\sigma_x, \sigma_y, \sigma_{xy}$ are the components of the stress tensor in an orthonormal basis associated to the surface element. In reference [13], the authors propose to suppress the surface term S_0^e in order to obtain mass dimensions in equation 1. They show, particularly, that in this case the optimal value of the coefficient λ is more stable, what is advantageous when it has to be defined.

2.1 Proposal 1: extension of the formulation of Barnes-Han-Lee

We propose an extension of the previous formulation, on one side to other type of elements and on other side to other material's behavior. The aim is therefore to study the feasability of this extension. Thus, the following expression would replace Han-Lee's [5]:

$$k_{imax} = \sum_{e} \frac{l_e}{4} \left(\alpha K + \beta \mu + \gamma \frac{I_\sigma}{3} + \frac{\theta}{2} \sigma_{mises} \right)$$
(3)

Comparing it with the expression 2, the term $\frac{E}{1-\nu^2}$ can be considered as controlling the shape changing or the element volume changing. It could be replaced by a linear combination of the average compressibility modulus K and shear modulus μ , available for all elastic and hyperelastic laws: $\alpha K + \beta \mu$. Concerning the second part of the equation 2 proposed by Han-Lee, the term $\sigma_x + \sigma_y + \sigma_{xy}$ can be considered as representing the stress state in the material (cumulating the spheric and deviatoric parts). For our proposal, and in order to extent the use of the formula to other geometries than triangular elements, we replace this term by an invariants' combination: $\gamma \frac{I_{\sigma}}{3} + \frac{\theta}{2} \sigma_{mises}$, where $I_{\sigma} = \sigma_k^k$ is the trace of the Cauchy stress tensor and σ_{mises} is the Mises stress. These two quantities are tensor invariants so they could be calculated for any type of element.

The parameters α, β, γ and θ in the expression 3 permit to control the influence of each entity. And finally, l_e represents a geometrical characteristic length, suitable for 2D elements (thickness) and for 3D elements (cubic root of the volume).

2.2 Proposal 2: alternative formulation for the mass matrix

Our second proposal refers to the theoretical elements proposed by the early work of Underwood [11] by using the theorem of Gerschgorin which permits to obtain an upper bound to the eigenvalue "i" of the stiffness matrix "K" of the system:

$$\rho_i \leqslant \sum_j |K_{ij}| \tag{4}$$

The mass matrix is then built to satisfy the stability condition with a unitary time step.

$$m_i = \frac{\lambda}{2} MAX_{k=1}^3 \left(\rho_{3(i-1)+k} \right) \tag{5}$$

Unlike the physical masses, we can expect a variation of the mass matrix built this way during the calculation. Given that on one side we choose the maximum value over the 3 axes (loop over k in 5) and on the other side the stiffness of the initial material behavior is generally more important than during deformation, it has been proved in our simulations that the mass matrix calculated at the beginning was enough to "guide" the whole simulation, i.e. the update of the mass matrix along the calculation of our simulations did not provoke any time gainings.

The method presents as a disadvantage that it needs at least the calculation of one stiffness matrix, what implies the need of being able to calculate the tangent behavior. Generally, at the beginning of the loading process, the evolution is mainly elastic, so a priori the stiffness belonging to the tangent behavior should be enough if we consider that the material tends to soften.

2.3 Incremental scheme and convergence criterion

In the case of an incremental law of behavior, a priori not totally reversible, when the loading leads to big deformation-stress final states, the final-form finding procedure in one step is not correct anymore. The final state depends indeed on the loading path which in the case of DR can be very different to the real path. A solution is to use an incremental loading procedure. Assuming that increments are small enough, the procedure then guarantees a succession of points of static physical equilibrium that allows to be close to the real response of the structure during the loading.

The convergence criterion we use in the calculations is the following, being ε the instruction value:

$$Max\left(\frac{||Residual||_{\infty}}{||Reactions||_{\infty}}, \frac{Kinetic\ Energy}{Internal\ Energy}\right) \leqslant \varepsilon \tag{6}$$

3 NUMERICAL CASE STUDIES

We will present in the following subsections several numerical case studies on the formulae 3 and 5. We use the C++ academic finite elements software Herezh++ [14], and for the meshing and postprocessing, we use the software Gmsh [15]. Calculations are made on an Apple computer (Processor: 2x2.93 GHz Quad-Core Intel Xeon, Memory: 16 Gb 1066 MHz DDR3) with just one processor.

3.1 Membranes: complex meshes

We study here 2D meshes, with triangular and rectangular elements, and with linear and quadratic interpolations. Also, two different qualities of mesh are considered: a grid of 25 × 25 elements and another one of 50 × 50. For the first proposal, we use the parameters $\alpha = 0.9022557$, $\beta = 0.9022557$, $\gamma = 1$, $\theta = 1$. The calculation is carried out in linear elasticity, E = 125MPa and $\nu = 0.41$, which are coherent with the parameters of behavior of a usual thin fabric.

This first numerical test is a classical one, which has already been studied, for example, in reference [6]: the inflation of a rectangular shaped cushion. It consists in two membranes joined at their periphery, with dimensions $500mm \times 500mm \times 0.27mm$. Due to the symmetries, just 1/8 of the cushion is studied. The cushion is loaded with an instantaneous internal pressure of 0.015 MPa. The mesh is constituted of a ruled triangular division, $25 \times 25 \rightarrow 625$ elements. The convergence criterion 6 is set to: $\varepsilon = 1.e - 3$.

For the different tested samples, we use the notation indicated in table 1. Thus, as an example, the notation RL1 means: test made with a mesh with 25×25 Rectangular elements, and using Linear interpolation.

T : Triangular Elements	R: Rectangular Elements
L: Linear Interpolation	Q: Quadratic Interpolation
1: Mesh with 25x25 elements	2: Mesh with 50x50 elements

Table 1:	Notation
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The figure 1 shows an example of inflated membrane. For each geometry, the table 2 shows the obtained results with an optimum λ .



Figure 1: Inflated squared cushion: representation of 1/8 of the cushion, displacement isovalues

It can be observed that convergence is reached in all the cases. Particularly, the quadratic interpolation does not induce a particular difficulty.

3.2 Membrane: circular mesh

Inflation of a circular cushion, with a diameter or 400mm, where the mesh, Figure 2, includes both triangular and quadrilateral linear elements. The other material, geometric, etc, characteristics are identical to the squared cushion's ones, and also the methods.

		Proposal	1	Proposal 2		
Mesh	λ_{opt}	Iterations	Time [s]	λ_{opt}	Iterations	Time [s]
TL1 (2028 dof)	10	546	14,1	0,6	565	13,8
TL2 (7803 dof)	10	923	101,5	0,7	1081	111,8
TQ1 (7803 dof)	13	1128	118,4	0,6	1185	119,8
TQ2 (30603 dof)	14	2158	943,3	0,7	2358	970,1
RL1 (2028 dof)	6	422	23,7	0,5	423	22,6
RL2 (7803 dof)	6	671	150,4	0,6	841	183,9
RQ1 (7803 dof)	10	1015	159,8	0,5	970	148,8
RQ2 (30603 dof)	9	1688	$1085,\!6$	0,5	1889	1552,1

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Table 2: Inflation of 1/8 of cushion in just one loading step, for different meshes



Figure 2: Inflated circular cushion: displacement isovalues

The Table 3 shows that the number of necessary iterations for convergence is coherent with those obtained for squared geometries. The mix of elements does not seem to alter the convergence. The proposal 2 is here more interesting, because even with the same previously used value of $\lambda = 0.6$, which is not the optimum for this case study, we obtain a very good convergence.

3.3 Incremental calculations

We introduced an incremental version of both proposals 1 and 2 in order to use them with incremental laws of behavior. The dynamic relaxation method is used here to find the steady state at the end of each loading step. The method is thus analogous to a classic iterative one, with the difference that it does not need the determination of a tangent evolution; but in return it needs a larger number of iterations.

We observed that the method works for all kind of elements. Figures 3 and 4 are a

		Proposal	! 1	-	Proposal 2)
Mesh	λ_{opt}	Iterations	Time [s]		Iterations	Time [s]
Circular	10	2096	1068,3	$\lambda_{opt} = 0,4$	1322	616,745
$17856 ext{ dof}$				$\lambda = 0,6$	1703	$792,\! 6$

Table 3: Inflation of a half of a circular cushion, with a mix of linear triangular and quadrangular elements

sample. They show the different steps of loading constituting the result of the intermediate pseudo-steady states resulting of the multi-step loading. To improve the clarity of the figures, not all the increments are shown.



Figure 3: Incremental inflation of a cushion, representation of 1/8 of the cushion, 2D membrane mesh

3.4 Complex law of behavior

This last case study is exploratory. It consists in observing the influence of a complex law of behavior, preferably incremental. For that, we consider the inflation of a squared membrane, but now meshed with 3D quadratic hexahedral elements (showing therefore that our proposals work also with 3D elements). The geometric dimensions are $250mm \times 250mm \times 6mm$, the mesh is constituted of a grid of $10 \times 10 \times 1$ and the used λ is 3 (bigger than before, to be sure to overcome nonlinearities). The loading is quasi-static, so the speed effects are negligible. The material is considered an elastomer Vitton where the law is modeled by assembling an additive hyperelastic stress and a stress hysteresis. For more details of the law, see [16].

The calculation converges despite the complex behavior (see figure 4). We observe in the table 4 a number of iterations much higher for the first increment, and then a big regular decreasing of the number of iterations, in opposition to the case of linear elasticity. The reason is that the weak initial stiffness of the material leads to a very big displacement



Figure 4: Incremental inflation of a cushion, representation of 1/8 of the cushion, 3D mesh

Mesh	λ	Inc 1	Inc 2	Inc 3	Inc 4	Inc 5
3969 dof	3	32020	8470	6540	4730	2830
		Inc 6	Inc 7	Inc 8	Inc 9	Inc 10
		2910	970	2590	910	690

 Table 4: Inflation of an elastomeric plate: Needed iterations per loading step

at the first increment. Then, the material rigidifies and the displacements per increment decrease importantly. The observed evolution of the number of necessary iterations in function of the loading step is therefore logical.

4 Conclusions and discussion

We presented two proposed formulae to extend Barnes-Han-Lee's dynamic relaxation method with kinetic damping. Barnes-Han-Lee's method was limited to the particular case of linear triangular elements and elastic behavior. Our proposed formulae allow for applications beyond the original limitations. This is our main contribution.

Furthermore, we have numerically demonstrated several other advantages of our formulae. We showed our proposals are effective for 2D and 3D elements, with linear and quadratic interpolation. We showed the formulae are compatible with an incremental formulation, which minimizes the influence of the loading path. Our exploratory work showed that the method works with a complex incremental law of behavior.

Seen all these results, we present dynamic relaxation with kinetic damping, using the incremental formulation, as an useful alternative to the classic Newton's method in the cases where instabilities are found.

This work covered structural instabilities. In future work, the study will continue with material instabilities.

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Preliminary investigation to Tensairity arches

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Key words: Tensairity, pneumatic structures, arch, inflatable structures

Summary: This paper investigates the load bearing behavior of Tensairity arches. The main goal of the research is to get a basic understanding of Tensairity arches and their feasibility. It analyzes the influence of the width/height ratio and the shape of the arch. Four different test models were constructed and experimentally tested. The experiments were conducted on 2m span scale models supported on two hinges and were symmetrically loaded. The arches have a constant cross section throughout their entire length and have a strut attached to the bottom and top of the airbeam. The experiments were performed with an internal pressure in the airbeam of 200 mbar and no cables were used.

1 INTRODUCTION

The use of inflatable structures for temporary constructions is worldwide known. They give a large advantage in terms of weight, transport/storage volume and set-up which can't be offered by other conventional structures. Pneumatic structures are exceptionally light and easy to set-up. By simply inflating a closed membrane, a structure with a load bearing capacity can be attained. Still, this load bearing capacity is very limited. A further enhancement of these structures is possible by adding cables and slender struts to the pneumatic components. This new structural concept is called "Tensairity".

1.1 Tensairity structures

Tensairity is the synergetic combination of a pneumatic structure and a Tensegrity structure (cable-strut structure). By attaching slender struts and cables to the pneumatic components, a structure with a larger stiffness can be attained. A classic Tensairity beam consists of a slender strut which is attached to the upper side of a pneumatic airbeam under low pressure, in the range of 100-300mbar. Cables are then fixed at the end sides of the slender strut and wrapped around the airbeam. This combination brings forth a lightweight structure which has a comparable load bearing behavior as a traditional truss system (Figure 1).

To guarantee a proper working of Tensairity, the slender strut needs to be firmly fixed to the pneumatic airbeam. This way, the pneumatic airbeam will act as a sort of elastic foundation for the slender strut and thus stabilizes it against buckling¹.



Figure 1: Tensairity beam¹

1.2 Tensairity arches

In the domain of transportable lightweight structures, the concept of Tensairity can certainly find his applications. The main advantages of Tensairity, such as its lightweight, small transport/storage volume and easy set-up by simply inflating the structure, makes Tensairity a feasible solution for transportable structures. However, most recent projects realized with the Tensairity concept are permanent constructions. Good examples are the parking garage in Montreux and the ski bridge in Lanslevillard (Figure 2). In both these projects Tensairity beams are applied².





Figure 2: Parking garage Montreux (right) and Skibridge in Lanslevillard (Left)²

Where the Tensairity beams usually find their application in roofs and bridges, Tensairity arches can find theirs in easy space coverage for a wide range of applications. The arch spans a width which can directly be used as covered space. In the case of tensioned textile structures, the curve of the arch makes the membrane spanned between two arches anti-clastic pretensioned which has an advantage on the load bearing capacity of the membrane.

However, the knowledge about Tensairity arches is still limited³. Parameters such as the width to height ratio, the shape of the arch (circular, parabolic,...), the inner pressure of the airbeam, the number of struts, the section of the Tensairity beam, the use of an inner web, etc has an influence on the structural behavior of the Tensairity arch and needs to be investigated (Figure 3). In this paper, the width to height ratio and the shape of the arch are taken into account in the analysis. Four different models are made, each different in one of the two parameters, and experimentally tested.



Figure 3: Tensairity arch

2 EXPERIMENTAL INVESTIGATION

2.1 Models

2m scale models were tested. Each arch has a section of 10 cm diameter and an inner pressure of 200mbar. At the upper and lower side of the airbeam two slender flexible aluminum struts were attached. Each strut had a section of 25mm*2mm. The tested models were made out of two different layers of membrane. For the inner membrane, an airtight PU foil was used and welded together, with the purpose to keep the structure airtight. The purpose of the outer membrane on the other hand is to take the tension in the membrane and is constructed out of a technical fabric (polyester fibres with a PU coating). The different parts from the cutting pattern were stitched together to create a closed membrane. Also two pockets were provided, one at the upper and lower side of the airbeam to accommodate the struts (Figure 4).



Figure 4: Inner and outer membrane (right) and section of the Tensairity arch (left)

Four different models were tested each different in either the shape of the arch or the width/height ratio (Table 1).

	shape	L/H
Model 1:	Circle	1,5
Model 2:	Parabolic	1,5
Model 3:	Circle	3
Model 4:	Parabolic	3

Table 1: Testmodels

2.2 Testrig

The testrig is dimensioned for arches with a 2m span where the arch is hinged to the supports. There has been chosen to work with an approximation for the loading of the models. The span of the arch (2m) is divided into eight equal segments. The uniform load on these segments is simulated by point loads so that there are 7 load points in total.

The point loads are constructed as follows. A horizontal beam is attached to the upper strut of the arch. This way the load can be applied to the arch without touching the membrane (Figure 5).





Figure 5: Horizontal beams point load

By means of chains small platforms are linked to the separate horizontal beams. On this platform, different weight elements can be placed, which simulates the uniform load on the arch. Chains have been used because these have a minimal elongation under loading.

The deformation of the total arch is registered at the seven loading points, by means of an analog displacement gauge. The device can register a deformation up to 4cm (Figure 6).





Figure 6: Loading of the arch (left) and measurement of the deformation (right)

The arch is hinged to the supports. These hinges can only rotate in the plane of the arch. The upper and lower struts of the Tensairity arch are fixed to these hinges. However, the end of the airbeam can't be too far from the hinge. Otherwise, the small section of the strut - which is situated between the hinge and the airbeam - will buckle (Figure 7).



Figure 7: Hinged support (left) and hinge connected to Tensairity arch (right)

The inner pressure in the airbeam is controlled by a water column, where 200mbar is the same as 204cm difference between the two water surfaces in the bended tube. By connecting one side of the bended tube to the airbeam and the other side open to the atmospheric pressure, the difference in pressure can be checked. The flow of air into the airbeam and thus the inner pressure is controlled by a pressure gauge (Figure 8).



Figure 8: Water column (left) and pressure gauge (right)

Various approximations need to be taken into account when using the mentioned testrig. By manually applying the different weight elements to 7 different points, the arch will be asymmetrically loaded for a short period of time. This can have an influence on the buckling behavior of the arch. Secondly, the deformation is read from an analog displacement gauge and manually noted, which increases the risk of accuracy errors. And lastly, only the vertical deformations can be measured in this method, which decreases the understanding of the overall deformation behavior of the arch.

2.3 Testing and results

2.3.1 Model 1

The first model is a circular arch with a width/length ratio of 1.5. This model has been symmetrical loaded with a maximum load of 80 N/m. The vertical relative displacements are plotted in Table 2. The first loading of 64 N/m is the force that results from the weight of the horizontal beam, the chains and the platform.

Loading of the arch (N/m)	Loading point 1	Loading point 2	Loading point 3	Loading point 4	Loading point 5	Loading point 6	Loading point 7
64	0,00	0,00	0,00	0,00	0,00	0,00	0,00
72	0,40	0,82	1,01	1,18	1,25	0,99	0,35
76	0,51	1,68	2,06	2,36	2,59	2,05	0,81
80	0,42	2,28	2,75	3,04	3,11	2,93	1,17



Table 2: Deformation of model 1symmetrical loaded in %

Figure 9: Global deformation of model 1 under symmetrical loading

After applying a load of 80 N/m the displacement of the arch keeps slowly increasing. There can thus be concluded that the arch had lost his stability and was slowly moving to his buckled state.





Figure 10: Deformation of model 1 under symmetrical loading

The picture illustrates very clearly how the arch will buckle (Figure 10). The middle of the arch is moving inwards while the left and right sides are moving outwards. This until either the left or right side buckles. This could not been seen on the data that was extracted from the vertical displacements in the seven points, where only a downward movement can be seen with a slight difference between the left and right side of the arch (Figure 9).

There can be concluded that Tensairity arches with a circular shape and a width/length ratio of 1.5 isn't suitable for a uniform loading. The model could only withstand a load of 80 N/m.

2.3.2 Model 2

The second model is a parabolic arch with a width/length ratio of 1.5. This model has been symmetrical loaded with a maximum load of 204 N/m. The vertical relative displacements are plotted in Table 3.

Loading of the arch	Loading						
(N/m)	point 1	point 2	point 3	point 4	point 5	point 6	point 7
64	0,00	0,00	0,00	0,00	0,00	0,00	0,00
84	0,11	0,15	0,04	0,07	0,08	0,12	0,08
104	0,13	0,15	0,05	0,10	0,12	0,14	0,15
124	0,18	0,14	0,02	0,13	0,15	0,17	0,21
144	0,18	0,10	0,06	0,18	0,20	0,26	0,33
164	0,16	0,05	0,07	0,24	0,27	0,32	0,39
184	0,09	-0,02	0,09	0,32	0,36	0,41	0,47
204	-0,05	-0,12	0,09	0,43	0,49	0,54	0,61

Table 3: Deformation of model 2 symmetrical loaded in %



Figure 11: Global deformation of model 2 under symmetrical loading

After applying a load of 224 N/m the arch buckles outwards at the left side (Figure 12). This is also visible in Figure 11, where the left side is moving upwards. When analyzing the data in Tabel 3, we see that the arch is slowly moving downwards in all seven points. After a load of 124 N/m the arch is slightly moving upwards at its left side, which means that the arch

deforms into an asymmetrical shape and will be loaded as such until it buckles. This deformation can be a cause of numerous reasons. First, the Tensairity arch is manufactured by hand, thus small differences can occur between the two sides of the arch. There is also the loading manner of the test rig. By manually applying loads to the model, the arch will be asymmetrical loaded for a short time.





Figure 12: Deformation of model 2 under symmetrical loading

2.3.3 Model 3

The third model is a circular arch with a width/length ratio of 3. This model has been symmetrical loaded with a maximum load of 144 N/m. The vertical relative displacements are plotted in Table 4.

Loading of the arch (N/m)	Loading point 1	Loading point 2	Loading point 3	Loading point 4	Loading point 5	Loading point 6	Loading point 7
64	0	0	0	0	0	0	0
84	0,15	0,28	0,13	0,26	0,15	0,33	0,23
104	0,39	0,43	0,29	0,44	0,33	0,43	0,29
124	0,50	0,54	0,45	0,63	0,45	0,44	0,28
144	0,55	0,70	0,69	0,96	0,68	0,43	0,02

Table 4: Deformation of model 3 symmetrical loaded in %



Figure 13: Global deformation of model 3 under symmetrical loading

The arch reaches his maximum deformation at a load of 144 N/m. After this point the arch becomes unstable and slowly moves to a buckled state. Figure 14 illustrates that the arch has a horizontal translation to the right side, which isn't visible in Figure 13. There can be assumed that the arch will buckle at either the left (inwards) or right side (outwards).





Figure 14: Deformation of model 3 under symmetrical loading

2.3.4 Model 4

The fourth model is a parabolic arch with a width/length ratio of 3. This model has been symmetrical loaded with a maximum load of 344 N/m. The vertical relative displacements are plotted in Table 5.

Loading of the	Loading						
arch (N/m)	point 1	point 2	point 3	point 4	point 5	point 6	point 7
64	0,00	0,00	0,00	0,00	0,00	0,00	0,00
84	0,02	0,18	0,15	0,41	0,12	0,56	0,29
104	0,28	0,25	0,25	0,49	0,08	0,56	0,43
124	0,38	0,32	0,30	0,41	0,08	0,58	0,47
144	0,40	0,38	0,38	0,71	0,14	0,62	0,56
164	0,44	0,43	0,43	0,79	0,18	0,64	0,63
184	0,48	0,50	0,51	0,90	0,21	0,68	0,77
204	0,53	0,55	0,57	0,96	0,25	0,68	0,80
224	0,47	0,64	0,76	1,06	0,32	0,60	0,70
244	0,48	0,69	0,82	1,14	0,36	0,61	0,76
264	0,55	0,76	0,89	1,24	0,40	0,56	0,71
284	0,57	1,03	0,98	1,31	0,43	0,54	0,75
304	0,40	0,97	1,12	1,62	0,56	0,41	0,61
324	0,38	1,05	1,40	1,73	0,60	0,37	0,64
344	0,28	1,15	1,70	1,98	0,69	0,29	0,60

Table 5: Deformation of model 4 symmetrical loaded in %



Figure 15: Global deformation of model 4 under symmetrical loading

This model can clearly withstand more loads than the previous models. The arch moves uniformly downward until a loading of 244 N/m has been reached. The right side moves hereafter upwards while the left side continues moving downwards. Thus the arch has reached his instability point. This movement will continue until either the left or right side buckles. The same behavior of the arch is also visible in Figure 16.





Figure 16: Deformation of model 4 under symmetrical loading

2.3.5 Results

Table 6 gives the maximum loads applied to the different models with their respective maximal deformation.

	Maximum load (N/m)	Maximum deformation (mm)	Loading point of max deformation
Model 1	80	40.54	Point 4
Model 2	204	6.14	Point 5
Model 3	144	6.38	Point 4
Model 4	344	13.18	Point 4

Table 6: Maximal deformation and loading

There can be concluded that parabolic shaped arches have a better load bearing behavior under symmetrical uniform loads. This is the same for traditional arched structures. However, the parabolic arch is still very sensitive to asymmetrical differences. There can be assumed that the buckling behavior of a Tensairity arch is dependent differences between the left and right side of the arch.

Model 4 (parabolic shape with a width/height ratio of 3) has the best load bearing behavior under a symmetrical uniform load. This is the same with conventional arched structures.

Still, the overall maximum loads applied to the models are relatively low. This can be explained by the use of flexible slender struts of only 25*2mm section, which influences the stiffness of the Tensairity arch. These flexible straight struts have been used to easily position them in the parabolic or circular shape. Otherwise, the straight struts need to be plastically deformed to position them, which is very time consuming and expensive. The struts will simply get their desired form by fixing them to the airbeam. In future research the difference between a stiffer bended strut needs to be compared with a flexible strut.

The deformations given in Tabel 6 are only the deformations on the Y-axis. However, the experimental test prove that the global deformation and thus also the deformation on the X-axis is important (see Model 1 and 3). In the future, other methods must be considered to measure the global deformation of the Tensairity arch. A feasible method could be Digital Image Correlation.

3 CONCLUSIONS

A structure in the domain of transportable structures is needed which has the following advantages: it needs to be as light as possible, an easy straightforward set-up is advised and the structure must have a compact storage/transport volume. These advantages are all met by the Tensairity principle. However, the knowledge about Tensairity is still limited.

In this paper, Tensairity arches were investigated. Four different models, each different in either the shape of the arch or the width/length ratio, were experimentally tested. Through these tests, the load bearing behaviour of the different arches were investigated.

This paper shows that a parabolic shaped Tensairity arch is the best solution for a symmetrical uniform load. However, these types of arches are sensitive to asymmetrical differences. When the results are compared to traditional arched structures the same conclusions can be made. From the four models, the parabolic arch with a width/length ratio of 3 was the stiffest one.

Further research will compare the results described in this paper with numerical models. Also another measurement tool for the displacement of the arch is advisable, one that also measures the displacement in the horizontal direction, for example Digital Image Correlation. In further research larger test models will be made to decrease the effect of accuracy errors.

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SHEAR AND BENDING STIFFNESSES OF ORTHOTROPIC INFLATABLE TUBES

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Key words: Bending and shear stiffnesses, Inflatable Beam, Orthotropic Fabrics

Abstract. This work concerns the behavior of inflatable beams subjected to bending loads. Some authors have worked on this subject and strength of materials theories have been written. In this paper, we first propose to calculate precisely the radius and length of inflated tubes which are needed to use the strength of materials theory in the case of high pressure. The usual linear equations are not precise enough and it is needed to use a non-linear set of equations. The second part focuses on the strength of materials theory. The assumptions that the straight section remains in a plane after deformation is first considered. Then the shear and bending stiffnesses are identified by the use of the results of a 3D code dedicated to inflatable structures which takes into account the orthotropic behavior of fabrics. Finally, a correction of the moduli is proposed. This correction takes into account the variation of geometry between the natural state, which corresponds to the non inflated beam, and the initial state which corresponds to the inflated beam.

1 INTRODUCTION

Inflatable objects are widely used nowadays to built temporary structures. They are often made of assemblies of elementary parts like panels, tubes, arches or cones. These elements can be calculated with a strength of materials theory. In the case of inflatable structures, three states can be taken into account. The first state is the "natural" state. In this case, there is no pressure in the structure and the fabric has geometrical dimensions that change when the structure is pressurized. After inflation, a new state called "initial" state is reached. It corresponds to the structure when it is just subjected to the internal pressure. The "final" state is obtained when other external loadings are applied. Strength of materials theory is commonly used to predict the behavior of the structure between the initial and the final state. It has been written mainly in the case of tubes, panels and cones. This work concerns the behavior of inflatable tubes and it is important to

quantify correctly the geometrical dimensions of the structures at the initial state. The modern fabrics used on earth are more resistant, and therefore it is possible to consider important pressures inside the beams. In these cases, the linear equations that have been written using the assumptions of little deformations to estimate the length and the radius of inflated tubes just after the first inflation are less precise. This demands then to study the evolution of the radius and the length as function of the pressure in the case of important deformations. This will lead to a non-linear set of equations. Once the radius and the length of the tube are known, it is possible to use strength of materials theory. Bernoulli's kinematic has been used in some $papers^{1,2,3,4}$. Some authors have written strength of materials theories based on Timoshenko's assumptions in the case of inflatable structures ${}^{5}, {}^{6}, {}^{7}, {}^{8}$. The second part of the paper concerns the behavior of tubes submitted to bending loads. First of all, the behavior of the straight section is quantified in order to verify the validity of the assumption that the straight section remains in a plane or not. Afterwards, the paper is focused on the shear and the bending rigidities in the case of orthotropic tubes. The bending and the shear rigidities of inflatable tubes can be obtained following different formulations⁷,⁸. The results are similar because the bending stiffnesses are the same and the shear stiffnesses present a little difference. The use of the results of a 3D code will allow to see which theoretical formulation is the closer to the 3D results. The last part concerns the moduli of the fabrics taken into account in strength of materials theory. In fact, the evolutions of the bending and shear stiffnesses as function of the pressure show that it is important to take into account the changing of observer between the natural and the initial state.

2 RADIUS AND LENGTH IN THE CASE OF HIGH PRESSURE

2.1 The different states

In the case of inflatable structures, two steps of loading are necessary as can be seen on Figure 1. The starting point is called natural state. It corresponds to the geometrical definition of the fabric without any pressure inside the beam. Index \oslash corresponds to the natural state. ℓ_{\bigotimes} is the natural length of the beam and r_{\bigotimes} is its natural radius. During the inflation, some tension is inducted in the walls and the beam is rigidified. This first step of loading ends with the definition of the geometry of the beam called initial or reference state. The index 0 corresponds to the reference state. So, just after inflation, the characteristic dimensions become the initial length ℓ_0 and the initial radius r_0 . The final state (named also current state) of the beam is attended after the second step which corresponds to the application of external loads.



Figure 1: Steps of inflation

2.2 Linear equations used to calculate the radius and the length of inflatable tubes

In some papers, the two dimensions ℓ_0 and r_0 are calculated with the linear formulations obtained from the longitudinal and normal stresses in an inflated tube and by using the Saint Venant Kirchoff behavior law in the case of an orthotropic fabric. This is written with the small deformation assumptions.

Let denote X and Y a local basis where X corresponds to the longitudinal direction of the beam. In this work, the warp direction lines up the longitudinal direction of the beam. X is then parallel to the warp direction of the fabric, and Y is parallel to the weft direction. E_{ℓ} is the Young modulus in the warp direction, E_t is the Young modulus in the weft direction. $\nu_{t\ell}$ and $\nu_{\ell t}$ are the Poisson's ratio.

In the case of the inflation of a tube, the stress and the strain tensors follow the behavior law: $\begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & &$

$$\left\{ \begin{array}{c} \sigma_{XX} \\ \sigma_{YY} \end{array} \right\} = \left[\begin{array}{c} \overline{E_{\ell}} & \nu_{t\ell} \overline{E_{\ell}} \\ \nu_{\ell t} \overline{E_{t}} & \overline{E_{t}} \end{array} \right] \left\{ \begin{array}{c} \varepsilon_{XX} \\ \varepsilon_{YY} \end{array} \right\}$$
(1)

where $\overline{E_{\ell}} = \frac{E_{\ell}}{1 - \nu_{\ell t} \nu_{t\ell}}$ and $\overline{E_t} = \frac{E_t}{1 - \nu_{\ell t} \nu_{t\ell}}$. In the case of an inflated beam, we assume that the Cauchy strains are :

- longitudinal stress: $\sigma_{\ell} = \frac{pR_0}{2}$.
- transversal stress: $\sigma_t = pR_0$

and the longitudinal and transversal strains are uniform: $\varepsilon_{XX} = \frac{\Delta \ell_{\odot}}{\ell_{\odot}} = \frac{\ell_0 - \ell_{\odot}}{\ell_{\odot}}$ and $\varepsilon_{YY} = \frac{\Delta R_{\odot}}{R_{\odot}} = \frac{R_0 - R_{\odot}}{R_{\odot}}$ The use of the behavior law and of the longitudinal and transversal stresses allows us to

The use of the behavior law and of the longitudinal and transversal stresses allows us to obtain the following set of equations:

$$\begin{cases} \frac{pR_{\odot}}{2} = \overline{E_{\ell}}\varepsilon_{XX} + \nu_{t\ell}\overline{E_{\ell}}\varepsilon_{YY}\\ pR_{\odot} = \nu_{\ell t}\overline{E_{t}}\varepsilon_{XX} + \overline{E_{t}}\varepsilon_{YY} \end{cases}$$
(2)

We finally obtain:

$$\ell_0 = \ell_{\odot} + \frac{pR_{\odot}\ell_{\odot}}{2E_{\ell}}(1 - 2\nu_{lt})$$
(3a)

$$R_0 = R_{\odot} + \frac{p R_{\odot}^2}{2E_t} (2 - \nu_{lt})$$
(3b)

2.3 Non linear set of equations used to calculate the radius and length of the initial state

If high pressure are used, the linear results are less close to the results of the 3D code, and it is therefore required to use a Lagrangian formulation. In the case of the inflation of a tube, the Piola Kirchof stress tensor and the Green Lagrange strain tensor follow the behavior law:

$$\left\{ \begin{array}{c} \Sigma^{XX} \\ \Sigma^{YY} \end{array} \right\} = \left[\begin{array}{c} \overline{E_{\ell}} & \nu_{t\ell} \overline{E_{\ell}} \\ \nu_{\ell t} \overline{E_{t}} & \overline{E_{t}} \end{array} \right] \left\{ \begin{array}{c} E_{XX} \\ E_{YY} \end{array} \right\}$$
(4)

where $\overline{E_{\ell}} = \frac{E_{\ell}}{1 - \nu_{\ell t} \nu_{t\ell}}$ and $\overline{E_t} = \frac{E_t}{1 - \nu_{\ell t} \nu_{t\ell}}$.



Figure 2: Steps of inflation

Figure 2 presents a piece of fabric subjected to two tensions and the displacement of a point between the natural and the initial states. ℓ_{\odot} is the length of the rectangular piece of fabric in the natural state and h_{\odot} is its width. If X and Y are the coordinates of a point M in the natural state ($^{\odot}M$), then $X\frac{\ell_0}{\ell_{\odot}}$ and $Y\frac{h_0}{h_{\odot}}$ are the coordinates of the point M in the initial state (^{0}M). The displacements of the points are:

$$\begin{cases} u_X(X,Y) = X \frac{\ell_0 - \ell_{\odot}}{\ell_{\odot}} \\ u_Y(X,Y) = Y \frac{h_0 - h_{\odot}}{h_{\odot}} \end{cases}$$
(5)

Concerning the radius, we assume that $\frac{h_0}{h_{\odot}} = \frac{2\pi r_0}{2\pi r_{\odot}} = \frac{r_0}{r_{\odot}}$. It allows to calculate the displacement gradient tensor and deduce the deformation gradient tensor:

$$\overline{\overline{F}} = \begin{bmatrix} \frac{\ell_0}{\ell_{\oslash}} & 0\\ 0 & \frac{r_0}{r_{\oslash}} 0 \end{bmatrix}$$
(6)

and the jacobian is $J = \frac{r_0 \ell_0}{r_0 \ell_0}$. The Cauchy stresses are $\sigma_\ell = \frac{pR_0}{2}$ and $\sigma_t = pR_0$.

The second Piola-Kirchoff stress tensor is calculated with: $\overline{\overline{\Sigma}} = J\overline{\overline{F}}^{-1}\overline{\overline{\sigma}}\overline{\overline{F}}^{-T}$, consequently:

$$\overline{\overline{\Sigma}} = \begin{bmatrix} \frac{pR_0}{2} \frac{\ell_{\odot}h_0}{\ell_0h_{\odot}} & 0\\ 0 & pR_0 \frac{h_{\odot}\ell_0}{h_0\ell_{\odot}} \end{bmatrix}$$
(7)

The Lagrangian finite strain tensor becomes:

$$\overline{\overline{E}} = \begin{bmatrix} \frac{1}{2} \left(\left(\frac{\ell_0}{\ell_{\odot}} \right)^2 - 1 \right) & 0 \\ 0 & \frac{1}{2} \left(\left(\frac{R_0}{R_{\odot}} \right)^2 - 1 \right) \end{bmatrix}$$
(8)

following the behavior law, and using the relation $\nu_{t\ell}E_{\ell} = \nu_{\ell t}E_t$, it is possible to write

$$\begin{cases} \Sigma^{XX} - \nu_{\ell t} \Sigma^{YY} = E_{\ell} E_{XX} \\ \Sigma^{YY} - \nu_{t\ell} \Sigma^{XX} = E_t E_{YY} \end{cases}$$
(9)

We get then the following set of equations:

$$\begin{pmatrix}
\frac{pR_{\odot}}{2}\frac{\ell_{\odot}}{\ell_{0}}\left(\frac{R_{0}}{R_{\odot}}\right)^{2} - \nu_{\ell t}pR_{\odot}\frac{\ell_{0}}{\ell_{\odot}} = \overline{E_{\ell}}\frac{1}{2}\left(\left(\frac{\ell_{0}}{\ell_{\odot}}\right)^{2} - 1\right) \\
pR_{\odot}\frac{\ell_{0}}{\ell_{\odot}} - \nu_{t\ell}\frac{p}{2}\frac{\ell_{0}}{\ell_{\odot}}\left(\frac{R_{0}}{R_{\odot}}\right)^{2} = \overline{E_{t}}\frac{1}{2}\left(\left(\frac{R_{0}}{R_{\odot}}\right)^{2} - 1\right)$$
(10)

If one writes $\widetilde{X} = \frac{\ell_0}{\ell_{\odot}}$ and $\widetilde{Y} = \frac{R_0}{R_{\odot}}$, the system can be re-written under the following form:

$$\begin{cases} \widetilde{X}^3 + \frac{2\nu_{\ell t}pR_{\odot}}{E_{\ell}}\widetilde{X}^2 - \frac{pR_{\odot}}{E_{\ell}}\widetilde{Y}^2 - \widetilde{X} = 0\\ \widetilde{Y}^2\widetilde{X} - \frac{2pR_{\odot}}{E_t}\widetilde{X}^2 + \frac{p\nu_{\ell t}R_{\odot}}{E_t}\widetilde{Y}^2 - \widetilde{X} = 0 \end{cases}$$
(11)

This finally leads to a non-linear set of equations which can be easily solved.

2.4 Comparisons with the results of the 3D code

The theoretical radius and length are compared to the results of a 3D code. This finite element code is a non-linear program dedicated to the study of membrane structures based on the total Lagrangian formulation. The membrane elements have zero bending stiffness and satisfy the plane stress conditions. They are supposed to have an orthotropic behaviour. Locally, the warp and weft direction allow to define a basis, which is called orthotropic basis. In this local orthotropic basis, the relation between the 2nd Piola - Kirchhoff stress tensor and the Green Lagrange strain tensor is written by using the following parameters: E_{ℓ} , the Young's modulus in the warp direction, E_t , the Young's modulus in the weft direction, $G_{\ell t}$ the in-plane shear modulus, and $\nu_{\ell t}$ and $\nu_{t\ell}$ are the


Figure 3: Radius and length of an inflated tube

Poisson's ratios that respect the following equality: $\frac{\nu_{\ell t}}{E_{\ell}} = \frac{\nu_{t\ell}}{E_t}$.

$$\left\{ \begin{array}{c} \overline{\Sigma}^{11} \\ \overline{\Sigma}^{22} \\ \overline{\Sigma}^{12} \end{array} \right\} = \left[\begin{array}{c} \frac{E_{\ell}}{1 - \nu_{\ell t} \nu_{t \ell}} & \frac{\nu_{t \ell} E_{\ell}}{1 - \nu_{\ell t} \nu_{t \ell}} & 0 \\ \frac{\nu_{\ell t} E_{t}}{1 - \nu_{\ell t} \nu_{t \ell}} & \frac{E_{t}}{1 - \nu_{\ell t} \nu_{t \ell}} & 0 \\ 0 & 0 & G_{\ell t} \end{array} \right] \left\{ \begin{array}{c} \overline{E}_{11} \\ \overline{E}_{22} \\ 2\overline{E}_{12} \end{array} \right\}$$
(12)

The graphics on Figure 3 present the results of the inflation of an inflatable beam. The moduli of the fabrics used here are products of the moduli by the thickness. Therefore, the surface of the straight section is replaced by $S = 2\pi r$ and the second bending momentum is replaced by $I = \pi r^3$. The moduli are $E_{\ell} = E_t = 50000 \ MPa.m$, $G_{\ell t} =$ 12500 MPa.m, $\nu_{\ell t} = 0.08$. The non linear set of equations has been solved here by using the solver of Microsoft Excel. It shows that if the pressure is important the length and the radius are not linear function of the pressure. This is most important in the case of the length. The use of the linear formulation can lead to quite important relative errors. In the case studied here, the relative error for the length is greater than 10%. It is in the range of 2% in the case of the radius. The results of the non-linear set of equations is much closer to the results of the 3D code than the results of the linear formulation, and justify the use of the non linear approach in such cases.

3 BEHAVIOR OF THE STRAIGHT SECTION

Following Timoshenko's assumptions, the straight section is supposed to remain in a plane but not orthogonal to the neutral fibber. Since this phenomenon is all the more important that the beam supports heavy loads, the choice has been done to consider the case of a clamped-simply supported beam subjected to a local load in the middle of the beam. The radius of the beam considered here is 0.3 m and its length is 6 m. The moduli of the fabric are $E_{\ell} = E_t = 160000 Pa.m$, $G_{\ell t} = 16000 Pa.m$ and $\nu_{\ell t} = 0.08$. The inside pressure is 2000 Pa and the applied load is 800 N. In this case, the displacement of the point in the middle of the beam is 0.167 m. The left figure of Figure 4 shows the planes orthogonal to the neutral fibber and the trace of the projection of the points of a straight section in a deformed position. The right figure shows the distances of every points of the straight section to the medium plane of the points of the straight section. Following Saint-Venant principle, the sections considered here are not too close to the points of loading. The assumptions that inflatable tubes follow a Timoshenko's kinematic is then valuable. The Figure 5 presents the evolution of $\frac{dv}{dx} - \theta$ along the beam obtained with the 3D code.



Figure 4: Behavior of the straight section of an inflated beam

4 BENDING STIFFNESS AND SHEAR STIFFNESS

4.1 Analytical stiffnesses of a cantilever inflated beam

Two formulations have recently been written to calculate the deflection of inflatable beams. The first one has been written in the case of inflatable tubes made of isotropic



Figure 5: $\frac{dv}{dx} - \theta$ obtained with the 3D code

material⁷. In this case, the deflection of the beam is:

$$v(x) = \frac{F}{(E+P/S_0)I_0} \left(\frac{x^3}{6} - \frac{\ell_0 x^2}{2}\right) - \frac{Fx}{P+kGS_0}$$
(13a)

$$\theta(x) = \frac{F}{(E + P/S_0)I_0} \left(\frac{x^2}{2} - \ell_0 x\right)$$
(13b)

where E is the Young elasticity modulus and G is the shear elasticity modulus.

The case of orthotropic fabrics has also been written ⁸. In the case of a cantilever inflated beam, the results are:

$$v(x) = \frac{F}{(E_{or} + P/S_0)I_0} \left(\frac{x^3}{6} - \frac{\ell_0 x^2}{2}\right) - \frac{Fx}{P + \frac{1}{2}kG_{or}S_0}$$
(14a)

$$\theta(x) = \frac{F}{(E_{or} + P/S_0)I_0} \left(\frac{x^2}{2} - \ell_0 x\right)$$
(14b)

where $E_{or} = \frac{E_{\ell}}{1 - \nu_{\ell t} \nu_{t\ell}}$ and $G_{or} = G_{\ell t}$. By replacing E by E_{or} and G by G_{or} in the isotropic formulation, it is possible to obtain another formulation to calculate the deflection of the beam in the orthotropic case. The bending parts of the deflection are the same but there exists a difference in the shear part due to a coefficient 1/2. The results of the 3D code will be compared to the different analytical results.

4.2 Identification of the stiffnesses

The shear and the bending stiffnesses are identified by using the Timoshenko's strength of materials theory. In the case of a cantilever beam, the equilibrium equations and the boundary conditions lead to:

$$kGS(\frac{dv}{dx} - \theta) = F$$
 and $EI\frac{d^2\theta}{dx^2} = -F$ (15)

Let's denote R_B the bending rigidity and R_s the shear rigidity.

$$R_s = \frac{F}{\frac{dv}{dx} - \theta} \qquad and \qquad R_B = -\frac{F}{\frac{d^2\theta}{dx^2}} \tag{16}$$

The quantities $\frac{dv}{dx} - \theta$ and $\frac{d^2\theta}{dx^2}$ are determined with the 3D membrane code in the case of a cantilever inflatable beam.

4.3 Comparison between the analytical stiffnesses and the results of the 3D code

Figures 6 and 7 present the results of the comparison between the theoretical stiffnesses and the identified stiffnesses. It is clear that they are non linear function of the pressure. In the case of the Figure 6, the moduli are $E_{\ell} = E_t = 50000 MPa.m$, $G_{\ell t} = 12500 MPa.m$, $\nu_{\ell t} = 0.08$. The curve EI + PI/S is close to the curve of the 3D results for little pressures, but it differs for higher pressures. In the case of the shear rigidity, P + kGS is coincident with the 3D results in the case of little pressures and P + 1/2kGS is inferior. For higher pressures, the both curves differ to the 3D results. The Figure 7 presents analogous results. In this case, the moduli are $E_{\ell} = E_t = 50000 MPa.m$, $G_{\ell t} = 12500 MPa.m$, $\nu_{\ell t} = 0.3$ This has been done to look at the influence of the Poisson's ratio. The same conclusions can be written.

The divergence between the curves decreases by using a correction of the moduli of the fabrics used in the analytical formulations. The strength of materials theory allows to study the behavior of the bended beam between the initial and the final states. The comparisons are made by using the moduli that have been used with the 3D code. These moduli are used in reference to the natural state. The fact that the strength of materials concerns the behavior of the beam between the initial and final states, and not between the natural and final states which is the case of the 3D numerical developpements, demands to take into account the changing of geometry between the two states.

This can be done by using the Piola Kirchoff stress tensor and the Green Lagrange strains tensor and changing the observer between the natural state and the initial state. The use of the behavior law leads finally to the definition of ${}^{\oslash}E_{\ell}$ and ${}^{0}E_{\ell}$. ${}^{\oslash}E_{\ell}$ is the Young modulus in the warp direction used in the 3D simulations. It corresponds to the fabric in the natural state. ${}^{0}E_{\ell}$ is the Young modulus in the warp direction used in the strength of materials theory. It corresponds to the fabric in the initial state. These corrective coefficients are geometrical terms which come from the variation of geometry due to the pressurization.

For example, in the case of the Young modulus, the correction is: ${}^{0}E_{\ell} = \frac{h_{\odot}}{h_{0}} \left(\frac{\ell_{0}}{\ell_{\odot}}\right)^{3} {}^{\oslash}E_{\ell}$.



Figure 6: Theoretical and numerical bending and shear stiffnesses - first fabric



Figure 7: Theoretical and numerical bending and shear stiffnesses - second fabric

The results of the corrections of the moduli can be seen on Figures 6 and 7. The curves $EI + PI/S \ CORR$ and $P + kGS \ CORR$ show the effect of the use of the corrective coefficients on the theoretical stiffnesses. In the case of $\nu_{\ell t} = 0.08$, the shear and bending analytical stiffnesses are coincident to the results of the 3D code. In the case of $\nu_{\ell t} = 0.3$, the curve $P + kGS \ CORR$ lines up with the shear rigidity obtained with the numerical simulation. In the case of the bending stiffness, the use of the corrective coefficient leads to an improvement of the analytical result.

5 CONCLUSIONS

The use of high pressure in inflatable structures require to make clearly the difference between the natural state of the beam and the initial state. This is due to the fact that the initial state is the reference in the case of the use of a strength of materials theory. In this work, the geometrical dimensions of the pressurized beam are established by using a non-linear set of equations. Then the assumptions that the straight section remains in a plane is verified. The bending and shear stiffnesses of different theories are compared to the results of a 3D code dedicated to inflatable structures. Finally, geometrical coefficients allow to correct the theoretical stiffnesses.

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RECENT APPLICATIONS OF FABRIC STRUCTURES IN VENEZUELA

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Summary. The paper presents some examples of fabric structures designed and built by the company Grupo Estran in Venezuela, describing the different phases within the design method: from the first draft and the use of specialized software to develop double curvature surfaces as architectonic forms, to its manufacture and erection. Furthermore, the paper discuses the interaction of this type of structures with its context and its contribution to create and enhance the architectural space.

1. FIXED TEXTILE COVER BLEACHERS OF THE BARQUISIMETO BASEBALL STADIUM. BARQUISIMETO, LARA STATE. VENEZUELA.

1.1 Proposal

In 2009 the state government of Lara began the intervention of the "Antonio Herrera Gutiérrez" baseball stadium (fig. 1)—home of the professional baseball team *Los Cardenales de Lara*—so that it can meet the parameters established in the new agreement between the Major League Baseball and the Venezuelan League of Professional Baseball.



Fig. 1. General view.

The new covered area has an approximate total of 3,360.00 sqm on a single level. There are two wings, each with a rectangular projection of approximately 20 m wide and 85 m long.

Each wing is made up of twelve modules (fig. 2 and 3). The structural modulation matches the existing one in the reinforced concrete structure of the bleachers. Support is located in the upper part of the bleachers from where the metallic arms that cover the bleachers fly towards the front, and the traffic hall towards the back part.

Its shape, has a straight line in the edges and transitions into a sequence of twelve parabolic convex arches in the center. Its maximum height is approximately 9.90 m from the floor of the top hall of the bleachers and the minimum is approx. 3.65 m from the same reference point. The average from the ground is about 17.40 m.



Fig. 2. Left wing, top view.



Fig. 3. Left wing, field view.

The structural module (fig. 4) is made up of a pair of compound pillars built with two structural tubes, \emptyset 194 mm separated by 1300 mm and joined by \emptyset 140 mm separators, forming a staircase, which becomes narrower towards the upper part, where some terminals receive a parabolic 6970 mm span arch, which connects the pillars using four bolts. Bolts were used to connect the structural steel components aiming to reduce assembly time. The arch is built with two tubular \emptyset 140mm components calandared at three points, which are joined by \emptyset 89mm tubes.

From this central structure, two 12 m long trussed struts fly towards the side of the bleachers. The trussed struts are formed by three tubular \emptyset 114 mm joined by flat bar triangles which become sharper towards the ends, where we find the heads that allow them to be connected, on one side with the head of the pillar, and on the other, with the edge reinforcement of the membrane. On the opposite side, over the traffic hall, two 6 m long struts, this time built with a single \emptyset 140 mm tube.

From the faces of the arch, membranes that take the parabolic shape of the arches go out and down until they turn into a straight line at the ends of the struts. The membrane is fixed in a continuous manner along the struts with stainless steel flat bars. The outer edge finishes off in a \emptyset 12 mm reinforced edge, which connects at the ends of the struts.



Fig. 4. Structural module.

Towards the arch, the connection is made in eight points, via U type tensors, and it is with these that the membrane will be tensed.

The Membranes support the struts, but, due to the difference in length of the projectings, we manipulated the curvatures in the aim to balance the efforts transmitted to the arch. This way, the big membrane has a bigger curvature than the small one (fig. 5). To prevent the struts from rising due to the effect of the suction load, steel cables go from the struts to the pillars. These steel cables finish balancing the structural system in the vertical plane.

In the horizontal plane, the forces are not compensated internally in the module. One module balances the following, until reaching the ends, where the circuit ends. At one end, the forces are taken to the concrete structure using a tripod, which receives the cables from the front and hind reinforced edges, and at the other end, it is closed against the concrete structure of the roof of the central bleachers.

The profile of the module has an inclination that mimics the one on the existing concrete roof over the main stand, which allows us to include a draining system for rain water. Between modules, and over the arms, the rain water is collected through a fabric canal, and finally taken to the back end through a stainless steel funnel.



Fig. 5. Bleachers and membrane roof section.

1.2. The membrane

NAIZIL Big Cover type II membrane (blackout) was used. Aside from fulfilling the necessary structural and resistance requirements, it has the Rotofluo W treatment, which provides it with a lifespan expectation of more than 25 years.



Fig. 6 and 7. Real-size prototypes.

Once we obtain the final patterns to cut the membrane and before starting mass production of the membranes, they were verified by building two physical scale models and two real-size prototypes (fig. 6 and 7). The length of the steel cables for the reinforced edges and the performance of the corner plates were also verified.

Each module was sent to the assembly site with most of the steel cables and corner plates in place, to avoid inasmuch as possible manipulation at the site.

It is noteworthy that the same textile material was used to solve the rain water collecting canal. To one of the side ends of each membrane module was welded the strip that served as a canal and to the other a pocket that allowed for stapling, posterior and in site, of the canal strip of the neighboring module. All the modules also had—integrated in the main membrane—the overlap for the final finishing against the arch and the polycarbonate sheets.

1.3. Assembly



Fig. 8. Assembly and structure erection.

The entire structure was designed for its components to be assembled at the site, using bolts. With the help of cranes, the new metallic pillars were placed in their position and fixed to the previously built pedestals. The 12 m struts were passed to the side of the bleachers where they are laid for support on two workbenches. The access of the cranes is through the periphery of the stadium, behind the bleachers. From these positions it was difficult to pass the struts if the arches were placed first. On the other hand, this allowed other membrane connection operations to be carried out, while the arches were being raised and connected to the pillars (fig. 8).

The membranes were connected to the struts using a system consisting of stainless steel flat bars fixed to a bar, which was welded to the struts' upper tubular. This allowed the membrane to adopt different inclinations with respect to its anchorage.



Fig. 9. Assembly and structure erection process.

Once the membrane was placed, the hind ends of the arms are raised to be connected to the node of the pillar, from which they will pivot when taken to their final position. The workbenches facilitate this movement and prevent the struts from banging against the bleachers. The small membranes are assembled together with the short struts in three module

groups, at the parking level, to be later raised, using cranes, to the top part of the hall where they are connected to the pillar node.

At first we thought of raising the arms gradually and spaced out, but due to delays in the construction, the assembly time was reduced to a few days. We chose instead to use two cranes located in the back to raise an entire wing at once (fig. 9).

Each crane allowed to lift simultaneously groups of three struts, leaving the side and internal struts—not connected to the cranes—to be lifted by the other struts, using the reinforced edges, and finishing off manually, using lever chain pullers anchored to the cover structure or the existing concrete structure at the site. The free ends of the arm's steel cables were tied with ropes, which allowed the assembly team—that was waiting at the ridges for the arms to reach their position—to connect them (fig. 10). These and other provisional ties were used during the entire procedure to ensure the stability of the structure and its components while they were being manipulated and taken to their positions.



Fig. 10 and 11. Assembly and structure erection.

The assembly of the components and membranes of the two wings of bleachers was carried out simultaneously and with similar procedures. To lift the covers of the right wing, the strategy used the support of a third crane, which accounted for some time saving.

When the struts were raised to their positions, the membranes hung, showing the designed shape but upside down. While they were being raised, they were tied to prevent damage due to the wind. Using a system of pulleys, they were raised to the center of the arch, from where the rest of the established points were connected.

The mechanism used to introduce tension to the membranes is exclusively through these points, since the connections located at the ends of the arms are fixed and do not allow additional graduation (fig. 11). This procedure was repeated in each membrane. The lower steel cables of the arms were connected and the final tensing of all the modules started to be performed.

Finally, the canals and the overlaps of fabric against the arches were finished and the polycarbonate sheets were placed (fig. 12).



Fig. 12. Right wing bleachers and roof view.



Fig. 13. Stadium roof at night.

2. GROUP OF TEXTILE UMBRELLAS COVERS FOR THE *SABANA GRANDE* BOULEVARD. CARACAS, CAPITAL DISTRICT. VENEZUELA.

2.1. The umbrella.

The comprehensive Project for the Rehabilitation of the *Sabana Grande* Boulevard contemplated the incorporation of new elements for protection against rain, sun, and wind. The area designated for planting new vegetation was compromised by the *Metro de Caracas* subway tunnel, throughout the entire boulevard. Therefore, it was necessary to seek alternatives.

An "Umbrella"-type structure was chosen. This way, and through the interaction of several of them, grouped in different heights, the natural movement of the trees could be mimicked. On the other hand, the translucent membranes filter the sunlight, allowing the projection of a shadow similar to that of treetops.

The set of covers is formed by six inverted Umbrellas, which cover an approximate surface of about 510 sqm (fig. 14). They have an individual surface of 85 sqm per cover. The base geometry stems from an octagonal plan, confined in a 10 m diameter circumference (fig. 15). As for its height, there are three 9 m tall umbrellas and another group of three which are 8 m tall.



Fig. 14. Boulevard cross section.

The support structure for the membrane is made up of a central mast, divided in two segments. The lower part—where the height difference is located—is reinforced with a group of four flat bars that run along the Ø 194 mm tube, finishing off in the collecting dish, near the connection bridle of the upper part. The upper part (Ø 140 mm tube) is where the textile membrane module is developed, shaped as a cone through its connection to the group of eight Ø 4" and 5m long horizontal struts, suspended radially through steel cables. The lower end of the cone of the membrane finishes off in a stainless steel ring, which permits the definite tensing of the surface, when it is anchored to the bottom end of this section of the mast. The decision to divide the mast in two sections, besides being due to practical reasons regarding manufacture, transport, and manipulation, is the result of the study of the procedure for the onsite assembly, as we will see later.



Fig. 15 and 16. The umbrella, section and top view.

The inverted cone, performs like a huge funnel that when it rains, directs the water to a collecting dish located in the bottom segment of the mast. This system allows for the canalization of rain water, through the inside of the masts, to directly enters the boulevard's draining system. The electrical and sound fittings also go inside the pillars for their distribution to the equipment located in the middle of the structure.

2.2. Patterning

The general geometry defines a surface with polar and specular symmetry, which is why it is possible to achieve in the design of a single pattern repeated 16 times (8 of these cuts are mirror cuts) to conform one cover module.



Fig. 17 and 18. Patterns.

In designing the pattern, both the manner to achieve the cut with the least possible waste, as well as the image that can be formed when assembling the cover, are taken into account.

When observing membrane surfaces that are translucent against the light, zones that have a double membrane—overlaps, joints, pockets, reinforcements—appear darker, thus, special attention was paid for this effect to serve the geometry in the cone (fig. 17 and 18).

Before manufacturing the membranes, a physical scale model was built to verify the patterning. The making of the 6 covers took place in a series and included the placement of steel cables and corner plates in the workshop before sending them to the assembly site.

Big Cover—type II—translucent membrane by NAIZIL was the material chosen to make the covers because, aside from fulfilling the necessary structural and resistance requirements, it has the Rotofluo W treatment, which provides it with a lifespan expectation of more than 25 years.

2.3. Assembly



Fig. 19. Assembly and structure erection process.

The umbrella's assembly strategy (fig. 19) was conceived so that upper module of the mast was assembled together with the membrane, temporarily supported on the ground level of the boulevard. So, the pretensed module is raised with the help of a crane and placed over the lower section of the mast that was previously placed in its terminal.

We then proceeded to build the first umbrella. On one side of the corresponding mast, the support tripod was set up for the upper part of the mast, allowing enough space for the maneuver. The steel cable and arms are connected to the upper part of the mast, which lies on the ground. All connections are made using bolts and screws.



Fig. 20 and 21. Assembly and structure erection.

The set is hooked at its upper end by the crane; it is raised and placed in its position, using an assembly tripod for temporary support. The horizontal struts open, taking the package that has the membrane closer to the mast. It is lifted once again, allowing the movement of the membrane from one side to the other. The aim is for the opening of the membrane where the ring is to allow the pillar to pass. The upper part of the mast is lowered to the support of the tripod and the membrane is deployed. The tips of the octagon of the membrane (corner plates) are connected to the heads of the struts.

The ends of the steel cables, which hang from the upper part of the column, are connected with lever chain pullers to the heads of the struts. Slowly, these two elements are brought together, which causes the strut to rise, when it pivots in the node where it meets the mast. A scaffold tower and an additional lever chain puller aids movement, a strut at a time and connects the steel cable to the strut through shackles. The steel cable already has the size defined by the project and has been prepared in the workshop. The membrane is tensed through the central ring, and using four lever chain pullers it is taken to its position. Four chains connect the ring to the mast and the lever chain pullers are taken off.

The crane lifts the pretensed module and takes the upper part of the mast to its place (fig. 20 and 21). There is a telescopic-type connection between the lower and upper part, which is fit using a bolted bridle. The electrical and sound fittings run inside the pillar. Therefore, special attention is paid when making this connection.

In the six umbrellas, the strategy for the assembly was similar, but, due to the experience gained in the assembly of the first ones, times improved. This procedure prevented the need for working at heights, which would have further complicated the assembly and probably would have compromised the deadline.



Fig. 22. Umbrellas at the boulevard.

2.4. Lighting and sound

White, direct light was chosen since it is the most adequate for the presentation of everyday cultural, commercial, and traffic activities. This lighting will be carried out with 7W LED luminaires, which will be installed on a metallic trapezoidal piece, located between the flat bars that form the base of the rain water collecting dish (Fig. 23).



Fig. 23. Direct light luminaries.

To highlight the structures as so to give them an architectural overtone, we chose a system that offers changes in light colors and effects on the surface of the membrane, whereas in the masts, the lighting will be fixed in blue.





Fig. 24 and 25. Umbrellas at night.

To achieve this result, we use 8 LED RGB luminaires, handled by a DMX wireless transitioning controller. These luminaires will be installed on arms specially designed to hold them, which will allow them to be moved closer to the ends of the cover in order to achieve a better coverage (fig. 24 and 25).

To light the masts and the dish we use bidirectional projecting luminaires. Each one has two 3W LED bulbs. Out of these bulbs, 4 will be oriented towards the ground, in order to color the mast, and 4 will be oriented upwards to color the rain water collecting dish.

To this type of monumental lighting we incorporate an integral sound system, which allows the programming of moments of interaction between music and lighting. Two sound tracks were specially produced to alternate between day and night environments in which sounds of nature are conjugated with musical arrangements aimed to help the users of that place relax.

THE DESIGN AND APPLICATION OF LANTERNS IN TENT STRUCTURES JUAN G. OLIVA-SALINAS, ERIC VALDEZ-OLMEDO

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Key words: Lantern, historic building, light, lifting, acrylic, polycarbonate.

Summary This work describes the design and construction of lanterns and their application in tent structures by looking at four projects in chronological order that have been developed by the Structures Laboratory of the Faculty of Architecture at the Universidad Nacional Autónoma de México (UNAM) over the last decade. Through these projects this paper deals with the functional and aesthetic aspects of lanterns, their usefulness in assembly and disassembly of tent structures and the materials used in their construction.

1 INTRODUCTION

Throughout the history of architecture light has been an essential element in the construction of dwelling space. Definitions of architecture like that of Le Corbusier "...architecture is the masterly, correct and magnificent play of masses brought together in light..." or the work of Mexican architect, Luis Barragán, make this clear.

Within textile architecture, light plays a very important role, both in terms of function and appearance. The translucence and opaqueness of plastic membranes fulfils the users' need for light in order to allow them to carry out diverse activities in a certain space. Furthermore, light evokes different feelings in the user through the various atmospheres it creates in the interior.

However, in certain cases additional elements are required to provide greater richness and complement the lighting capacity of the membrane and aid installation of the membrane itself. The Structures Laboratory team of the Faculty of Architecture at the UNAM has incorporated these components, which we may identify with the term "*lanterns*", in various projects for tent structures. In this context, the term "lantern" is used to refer to "a structure on top of a dome or roof having openings or windows to admit light" [1].

1 LANTERN DESIGN

In order to present the variables involved in the design and construction of a *lantern*, the two basic components that it includes should be described: firstly, a structure that ensures stiffness and works as a element for fitting the membrane, as well as the fittings for lifting it to the structure of the entire system, and secondly a transparent or translucent surface over the top of the structure that allows the passage of light. The design process involves the specification of the materials for the structure, such as steel or aluminium with welded or bolted fittings, and for the translucent surface, for which tempered glass, acrylic or polycarbonate panels with UV protection may be used. Any of latter materials, used for constructing the translucent surface, may be finished with two-dimensional shapes that are sandblasted or with the application of self-adhesive plastic film to control solar incidence and generate lighting effects inside the covered space.

The geometric configuration of the *lantern* may vary greatly. For the design of the structure, the shapes found on the plane of a fixed circle of radius or similar curves, such as elliptic curves, are the most appropriate, given that the stresses are more evenly spread over what may be described as a traction ring. Fixed double or triple curves, which can be modelled with NURBS curves (*freeforms*), are a good option providing that they comply with strict criteria in relation to the structural behaviour of the membrane.

As regards the translucent layer, the geometry of the surface must be regarded as a developable surface in the event of making flat panels as in the case of glass. In this regard, the use of acrylic or polycarbonate in the design makes it possible to consider using thermoforming for the construction of the lanterns. Using thermoforming the lanterns can be made with surfaces with synclastic or anticlastic curves in one piece or various pieces, depending on the scale required.

For the structural analysis, in the same way as for other components such as poles, arches or cables, the loads transmitted by the membrane to the lantern must be taken into account, considering dead load, pre-traction and the loads produced by the wind or snow (in regions where this is necessary). It is worth noting that providing for the logistics of installation during the design phase is extremely important. This should include providing for the load and stress conditions to which the structure will be subjected during assembly in to order to design the necessary fixtures.

2 PALACIO DE MINERIA.

Various factors make the roof for the central courtyard of the Palacio de Mineria building an icon of textile architecture in Mexico. The first is the importance of the neoclassical building, constructed by Manuel Tolsa in the nineteenth century. The second is the scale of the roof, which covers an area of approximately 1000 m^2 . The final factor is the function that the roof performs, providing shade and protecting from the rain and thus enabling diverse events organised in the space each year, such as the international book fair, conferences, concerts, exhibitions and even fashion shows. The functionality of the structure, together with its lightness and flexibility enhance this historic building with its foundations in the former Mexico Valley basin.

Right from the preliminary design phase of the tent structure, which was constructed in 2002, there were plans to include a lantern that would create overhead lighting in the courtyard, as well as forming the point in which the eight sections that make up the main cover of the structure would join. In addition, the *lantern* was designed to aid the assembly and disassembly of the roof, given that one of the principles on which the roof was based was that it could be removed, i.e. it had to be possible to assemble and disassemble it within a certain length of time in order to leave the courtyard of the building as it was originally designed. This condition, driven by the design team in order to protect the building, was based on the regulations set by the Mexican Institute of Anthropology and History (INAH), the organisation responsible for the protection and conservation of architectural heritage dating back to the period prior to the twentieth century in Mexico.



Figure 1: Interior view of the tent structure over the central courtyard of the Palacio de Minería building.

The structure of the *lantern* rises 23 m above the floor level of the courtyard, has a diameter of 2.5 m and was designed in structural steel with standard profiles with an electroplating finish created using hot immersion. The translucent surface was constructed with eight acrylic sections that follow the geometry of the cross section of a sphere with a height of 0.70 m. The traction ring is stiffened with two sets of two perpendicular cables, in order to avoid any deformation during the installation process. In this position, there are two articulated sheaves around which the cable that is used to lift the membrane towards the two cable poles passes. There are a further two sheaves at the highest point, which direct the cable towards the two independent electric hand winches, from the east to west side of the building. During the installation process, temporary structures were placed in position to protect the balustrade of the building.



Figure 2: Lantern and lifting system

Over the course of nine years of use, it was not necessary to remove the structure. However, at the end of this period, the building authorities decided to change the membrane of the roof without changing the original design, given that, although the structure was still in good mechanical condition, its appearance had been strongly affected by the level of pollution in Mexico City and to the lack of regular maintenance. In 2011, with the technology now available on the Mexican market, the design has been improved. For the lantern, in particular, the dimensions have been maintained. The improvements to the design essentially consisted in the replacement of the lifting sheave with two fixed cables with a parallel electrically controlled disassembly and assembly system, which ensures even lifting, and the replacement of the translucent surface with a thermoformed acrylic surface made up of four sections. The specification for the latter was originally solid polycarbonate. However, conditions for thermoforming this material are very precise as regards the level of humidity in the environment and the initial test for its manufacture was negative. Hence the specification was changed to highly impact-resistant acrylic.

Lastly, with the replacement of this roof, it was proved that the removable system works perfectly and the qualities of the structural system do not affect the building in any way.





Figure 3: Detail of the lantern and construction of the replacement lantern

2 TELEVISION AZTECA.

Television Azteca, located in the south of Mexico City, is the second largest open television network in Mexico. In 2004, it requested a tent structure to cover an area of approximately 700 m^2 located between its recording studios in order to create a space that would be protected from the sun and rain, which actors, presenters, technicians, administrative staff and the other users within the company could use to move from one area to another.

Based on previous experience with the roof of the central courtyard of the Palacio de Minería building and in developing the initial design for the roof of Mexico City's International Benito Juárez Airport, a structure was designed in the form of a stretched sheet deformed by an ogive shape *lantern*, creating a conical surface. The design is made up of five *lanterns* distributed along the walkway, where they create alternating light and shade, which changes over the course of the day according to the position of the sun, creating spatial sequences that contribute to breaking up the linearity of the walkway.



Figure 4: Image of the initial design and a view of the interior.

The fixed geometrical point of the structure is the intersection of the two circular arcs that make up the ogive. The structure was constructed in structural steel with a series of cross profiles to increase its stiffness and enable the fittings to be positioned where the cables for its installation were to be held. The translucent surface is a conoid, which makes up of a polygon of flat plates of tempered glass. The structure measures 4 m at its widest axis, 1.90 m at its narrowest axis and rises 8 m above the finished floor level.



Figure 5: Lantern made with structural steel and tempered glass.

As in the case of the central courtyard of the Palacio de Minería building, the *lanterns* were designed to work as a lifting system during installation. The difference in this case is that the structure was not strictly required to be removable, although this is nevertheless an inherent characteristic of the structure. It is worth noting that during the construction process, the problem arose of bad quality welding of the aluminium. This was owing to the fact that in Mexico there is little experience in working with this kind of material. As a result, it was necessary to reinforce the joints of the structure with bolted plates.



Figure 6: Installation and detail of fittings for assembly

3 PALACIO UNIVERSUM MUSEUM

In 2006, a renovation project was developed for the old Government House building in Oaxaca city. The building is one of great importance. In its day it was the office of Benito

Juárez, Outstanding Patriot of the Americas and it now houses what is known as the Palacio Universum Museum. Based on the prior experience of installing a removable tent structure on a historic building gained at the Palacio de Minería, it was decided that this system was the most suitable for covering the three courtyards, which cover an area of approximately 900 m². The previous layout was followed, with lanterns at the centre of each courtyard providing overhead lighting throughout the day.



Figure 7: View of an interior with the lighting effect in the courtyard and detail of a secondary lantern.

In the rectangular central courtyard, a lantern with an elliptic form was installed, measuring 8 m at its widest axis and 4 m at its narrowest axis. In each of the two neighbouring courtyards a lantern was installed with a diameter of 3 m. In all three cases, the lanterns have a height of 0.90 m and rise 16 m above the floor level of the building. The structure was constructed in steel with an electroplating finish created using hot immersion and metal profiles and cables to provide perpendicular reinforcement. The translucent surface follows the same pattern as the lanterns constructed for Televisión Azteca. However, by contrast to the latter, they were made with solid polycarbonate plates, making the lanterns lighter.



Figure 8: View from above and detail of the lantern in the central courtyard.

As in the previous examples, the lanterns incorporate fittings for the installation of the structure. In this case, provisional lifting cables were installed, directed towards the top of the poles of the supporting structure. There they pass around a series of sheaves to change direction towards a mechanical winch located on the shaft of the pole itself. Once the *lantern* was in position, the cables were replaced with other permanent cables. The challenge in this project was to complete the design and construction phases in just two months. This was successfully achieved.



Figure 9: Installation and detail of the lifting system.

4 CENTRO UNIVERSITARIO DE TEATRO

In 2011, the UNAM's Centro Universitario de Teatro (university theatre centre), asked the

Structures Laboratory at the university to design a permanent structure to cover an area of approximately 400 m^2 between two buildings. The aim of the project was to create a space that would be protected from the sun and rain in which to organise activities connected with classical and contemporary theatre performances. The two buildings in question are of differing heights, allowing the creation of a covered area on the roof of one of the buildings, which may be used as a cafeteria or restaurant.



Figure 10: Elevation showing the layout of the roof and the system of lanterns with floating poles.

The design is made up of two membranes joined in a cone-like shape, with two raised *lanterns* with a system of floating poles measuring 8 m in length. An ellipse measuring 5 m at its broadest axis and 1.50 m in its narrowest axis holds the membrane. The translucent surface has been developed as a segment of an intersected cone with a vertical plane, measuring 8 m at the broadest axis and 2.20 m at the narrowest axis and with a height of 0.40 m. Using these dimensions, which exceed the size of the structure itself, ensures that water cannot get in. The lanterns are designed to have a steel structure with an electroplating finish created using hot immersion. The floating poles will be of variable diameters and have a traction system. The translucent surface will be made of solid polycarbonate. The two elements rise 15 m above the level of the courtyard, making up a lifting system with a series of additional cables that ensure the system remains in equilibrium.

CONCLUSIONS

- The use of *lanterns* in addition to the basic components applied in the design and construction of tent structures, such as poles, arches, reinforcement plates and traction fittings is entirely appropriate given that they provide greater quality to the dwelling space. Furthermore, roof lanterns have become an extremely useful element in completing the installation process of this kind of structure in a satisfactory manner.

- Over the last decade, several projects have been developed on varying scales, applying different materials in the design and construction of lanterns, like steel, aluminium, tempered glass, acrylic and polycarbonate. In certain cases, the results were not satisfactory; however, this has contributed to enhancing the design and construction of new projects.

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TENSEGRITY RING FOR A SPORTS ARENA FORMFINDING & TESTING

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Key words: Tensegrity ring, formfinding, diamond pattern, double layer, pretension, wind tunnel, pressure coefficient.

Abstract. The current prototype's key contribution to the field of light-weight structures is that it is the first time that a pure tensegrity ring has been used in place of a compression ring.

This design features a cladding structure for a sports arena, which consist of a ring-shaped outer section and a central roof structure. The ring-shaped outer section of the stadium consists of a tensegrity structure, which uses textile membranes in a place of conventional tension cables to bear the tensile forces occurring between the pressure elements. The supporting framework and spatial enclosure therefore become one an extension to the tension integrity principle. The central area of the roof is covered over by a Geiger dome, which in turn is a specific version of the tensegrity principle.

1 FORMFINDING



Figure 1: Tensegrity ring diameter Ø 40 cm. (above) - Tensegrity ring diameter Ø 100 cm. (below) - Models scale 1:100

The ring structure is made up of a continuum of ten upper-level paraboloids and ten lowerlevel paraboloids with a diameter of 40 cm. Formfinding for the ring structure is generated by means of a diamond-shaped membrane pattern pieces (rhombus = major axis 11.5 cm, minor axis 4 cm) formed by two layers of twenty bars (L=20 cm), which are arranged either in an oblique or a diagonal position. The bars are connected to the end points of the membrane as shown in Fig 2 and then to the adjacent membrane piece at the corresponding end point.

This procedure is repeated for all adjacent membrane pieces, while at the same time, the upper section is interlaced with the lower section creating one continuous ring structure when the last two bars are put into place.

A dome is created by combining the above ring structure with a "roof" consisting of one central mast (L=9 cm) and ten minor masts (L=6.5 cm) placed in a circular form held in place by the tension of the membrane itself. The membrane balances the system and joins the dome with the tensegrity ring. The final structure is a dome free of any internal supports.



Figure 2: Photos showing the components and final structure as well as a diagram of the tensegrity ring construction method to created a dome in pure tensegrity Ø 40 cm. - Model scale 1:100

2 TESTING

The proposed structure was tested using several methodologies:

Software testing (Wintess), physical testing (wind tunnel), qualitative analysis (physical model).

2.1 Software testing

The proposed structure was first tested against wind and snow loads via WinTess [1] software. Testing demonstrated the following:

- During extreme external wind conditions of 170 km/h, the maximum inward (horizontal) displacement of the bar free nodes is 100 cm, which was decreased by 20% after being reinforced. (The bar free nodes are located on the side of the structure between the upper and lower levels of the tensegrity ring. They are not directly connected to the upper dome or to the foundation nodes).
- It allowed the structural elements (membrane, tubes, and cables) to be analyzed and optimized for dimensional stability.



Figure 3: Structure in equilibrium when wind up-to 170 km/h is applied. Note that the computer model is shown with both the membrane and cables, which prevent the structure from moving in the real world.

- The exterior tubes are placed surrounding the ring so that they continue in the direction of the forces coming from the top membrane dome. The pretension cables increase the stiffness of the structure and contribute to support and balancing of the system.
- The structure is closed, in equilibrium, and able to support its own weight.
- The large displacements must be countered by the use of external tubes and cables if the structure is to be built in the real world.
- During external snow loads of 50 kg/m, a maximum (vertical) displacement of 60 cm is found in the minor dome masts and the maximum reaction in the foundation nodes is 22 tons.

2.2 Physical testing

The proposed structure was then tested in a wind tunnel at UPC in Barcelona using a rigid model made by a three dimension printer.

Due to the nature of tensile-textile construction (lightweight structures), the ability of the structure to withstand external loads relative to weight of the structure itself is much greater that of conventional construction [2]. It is important to note, though, that small changes in wind pressure or snow loads can have a major impact on the size and shape of the structural elements and the deformations that occur. For this reason, it is necessary to understand the pressure and suction coefficients that impact the structure: vertical force (lift coefficient) and horizontal force (drag coefficient).



Figure 4: Vertical force (lift), horizontal force (drag)

Wind tunnel	Lift & Drag Coefficients			
The wind tunnel is open, and works by aspiration	The value of the global lift coefficient			
(Eiffel style); that is, undisturbed air is accelerated	obtained from the experimental			
through a nozzle, due to pressure difference and sent	measurements was Cl ~0.86. Local			
to the model; thus the flow profile is laminar.	measurements of the lift coefficients,			
However, the model size is 0.17 m, and free stream	determined in small holes on the model			
speed was ranged between 5 and 20 m/s, that is	surface, reach values up to 1.5. Given the			
Reynolds numbers from $5x10^{4}$ and $2x10^{5}$, which is	size of the wind tunnel testing section (40			
a fully turbulent regime, as corresponds to the real	cm x 40 cm cross section) and the diameter			
size building.	of the model (17 cm) the statistical error is			
The model is rigid, made in a plastic material, while	estimated to be approximately 15%.			
the real size cover is flexible. The forces measured on	Where Fvertical is the vertical component			
the scale model have been scaled to the real size	of the force acting on the model, r is the air			
building assuming that it acts as a rigid body, due to	density and v is the free stream speed.			
the beams that support the building in tension.	$C = \frac{2F_{vertical}}{(1)}$			
The wind tunnel tests were used to determine the lift	$C_1 = \frac{1}{\pi \cdot (7.5 \text{ cm})^2 \cdot \rho \cdot v^2}$			
(vertical force) and drag coefficients (horizontal				
force). Drag coefficient was used in WinTess to	Testing showed that there is a -0.3 global			
calculate the structure to wind up-to 170 km/h.	Cp pressure drag coefficient (suction)			
The second se	Local pressure coefficients show a			
- 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	significant dispersion. This negative Cp is			
	the result of the very aerodynamic convex			
and	forms, which allow the wind to pass by			
	freely.			
	2F (2)			
	$C_d = \frac{2T_{drag}}{2}$			
	$\pi \cdot S_e \cdot \rho \cdot v^2$			
	Where Se is the elevation surface (model			
	0,008718 m ²)			

Table 1: Wind tunnel testing - Lift coefficient & drag coefficient.

2.3 Qualitative analysis

A 100 cm diameter model was built to perform qualitative testing. To make the larger prototype, there were two options: First, the quantity of bars would have to be increased proportionally to the elasticity of the membrane used. Second, if the length of the bars is increased, the ring diameter will be larger. A larger prototype was assembled using 20 struts (Length=50 cm) in a double layer resulting in an overall diameter of 100 cm. The ratio of length of the bars to the overall diameter is 1:2.



Figure 5: Model scale Ø 100 cm. An 80 cm-diameter fan was used to simulate a 70km/h wind.

In the first sample, we can observe the original geometry. In the second, we see the final geometry that was tested and the largest displacement was observed at point #3, which was 5cm.



Figure 6: Comparative analysis – Wintess software.

In the second sample, the model was tested with WinTess. It is important to note that for the qualitative analysis, the prototype was only built with membrane, while in WinTess analysis, the structure was built with the addition of cables. For this reason no displacement was observed in point #3 of the final geometry. The qualitative-analysis-optimized structure is shown along with the software-optimized structure. The results show that they are very similar, which validates the testing methods and model.

3 RESULTS

After wind-tunnel testing (pressure coefficients) and qualitative analysis, we found that there was an overload of forces on the model and we had to re-optimize the structural elements using WinTess software to construct a model of a structure for the real world.

Structural characteristics of the 40 m WinTess real-world model elements:

- Membrane: Ferrari Fluotop T2 1302 Prestress 0.8 % = 32,6 daN/5cm = 652 kg/m Resistance Rk = 800/700 daN/5cm = 16000,0 kg/m - Safety factor (5) = Rd = Rk / 5 = 160 daN/5cm = 3200 kg/m
- Border cables (Boltrope): WS-2 (36mm) Galv \emptyset 36 Section 855 mm2 Elasticity modulus 1.635 t/cm2 = 163,5 kN/mm2 Q= 125,46 t = 1.254,6 kN
- External cables (Guyrope): WS-2 (36mm) Galv Ø 36 Section 855 mm2 Elasticity modulus 1.635 t/cm2 = 163,5 kN/mm2 Q= 125,46 t = 1.254,6 kN
- **Ring tubes:** L=20 m Ø 400-10_S235 Section 122,522 cm2 Elasticity modulus 2.100 t/cm2 = 210 kN/mm2 Density 7,85 t/m3 = 78,5 kN/m3
- Dome central mast: L=9 m \emptyset 110-5_S235 Section 16,493 cm2 Elasticity modulus 2.100 t/cm2 = 210 kN/mm2 Density 7,85 t/m3 = 78,5 kN/m3
- Dome minor masts: L=6,5 m Ø 90-4_S235 Section 10,807 cm2 Elasticity modulus 2.100 t/cm2 = 210 kN/mm2 Density 7,85 t/m3 = 78,5 kN/m3
- External tubes: L= 8 m \emptyset 250-8_S235 Section 60,821 cm2 Elasticity modulus 2.100 t/cm2 = 210 kN/mm2 Density 7,85 t/m3 = 78,5 kN/m3



Figure 7: Model-elements structural characteristics. Total weight = 69.023 kg - Weight/m2 = 58 kg - Maximum reaction in foundation nodes 24 ton.

					Structure with only cables		
					self weight		
TENSEGRITY STRUCTURE DATA SUMMARY				without	reinforcement -	external tubes	
TENSEC	SRITY ST	RUCTUREL	DATA SUMM	IARY	reinforcement	without ext.	and cables
					Display	tupes	es (mm)
Tensearity	Foundation	nodes - lower by	979		Displa	cement Average	
Ring	Lower bar f	ree nodes	313		950	903	107
rui ğ	Upper bar free nodes				941	900	115
	Lipper bar r	odes connected	to the unner do	me	349	352	178
Tensearity	Central may	st top node	to the upper do		164	422	344
Dome	Central mas	st low node			164	422	340
D'onne	Upper nodes of the minor masts		asts supported v	with 5 cables	345	322	163
	Lower nodes of the minor masts supported with		with 5 cables	345	320	147	
	Upper nodes of the minor masts supported with 4 cables		2533	275	103		
	Lower nodes of the minor masts supported with 4 cables			2500	287	118	
Auxiliar	xiliar Lower nodes of the external tubes supported with 2 cables			-		. 0	
elements Upper nodes of the external tubes supported with 2 cables			-		36		
					5	Structure Resul	ts
		To	tal weight of stru	ucture (kg)	27064	27558	29350
		St	Structure weight per m ²		23	23	25
		Lo	Loads (tons)		-		
		Fo	Foundational Reactions (tons)		5	6	5 5
		Ex	External-tube reactions (tons)		-		. 2
		Ex	External-cable reactions (tons)		-		. 2
		Av	erage ratio of th	e tubes	0,5 a 0,05	0,5 a 0,05	0,5 a 0,05
		Av	erage ratio of th	e cables	0,5 a 0,05	0,7 a 0,05	0,5 a 0,05
		Me	embrane ratio		-	· · · ·	
					SI	tructural Eleme	nts
		Me	embrane Ferrari	Floutop	-		
	Case 5 mor	re efficient Bo	rder cables		Inox Ø 24	Inox @ 24	Inex @ 24
	minor displa	acements Ri	ng tubes (L=20m	1)	Ø 300-8_S235	Ø 300-8_S235	Ø 300-8_S235
		Ce	entral masts (L=9	9m)	Ø 110-5_8235	Ø 110-5_\$235	Ø 90-4_S235
	Case 9 less	s efficient Mi	Minor masts (L=6,5m)		Ø 70-4_S235	Ø 70-4_S235	Ø 70-4_S235
	major displa	acements Ex	External cables (guyrope)		-		Inex ∅ 12
		Ex	External tubes (L=8m)				Ø 150-5_\$235
					case 1	case 2	case 3
		Cánua trans a	little and a sector sector of	and anhian	Ctourst und	have and law a	and some set
		Structure w	vith membrane	and cables	Structure wit	hout cables - m	embrane only
		Structure w self weight	vith membrane wind 170 km/h	and cables snow 50kg/m2	Structure wit self weight	hout cables - m wind 170km/h	embrane only snow 50kg/m2
		Structure w self weight	wind 170 km/h	and cables snow 50kg/m2	Structure wit self weight	hout cables - m wind 170km/h	embrane only snow 50kg/m2
		Structure w self weight	wind 170 km/h with reinforce	and cables snow 50kg/m2 ments, external	Structure wit self weight tubes, and ex	hout cables - m wind 170km/h ternal cables	embrane only snow 50kg/m2
		Structure w self weight	vith membrane wind 170 km/h with reinforce	and cables snow 50kg/m2 ments, external Displacement A	Structure wit self weight tubes, and ex	hout cables - m wind 170km/h ternal cables	embrane only snow 50kg/m2
		Structure w self weight	vith membrane wind 170 km/h with reinforce 0	and cables snow 50kg/m2 ments, external Displacement A 0	Structure wit self weight tubes, and ex verages (mm 0	hout cables - m wind 170km/h ternal cables	embrane only snow 50kg/m2 0
		Structure w self weight 0 140	vith membrane wind 170 km/h with reinforce 0 179	and cables snow 50kg/m2 ments, external Displacement A 0 109	Structure wit self weight tubes, and ex verages (mm 0 328	hout cables - m wind 170km/h ternal cables) 0 573	embrane only snow 50kg/m2 0 1547
		Structure w self weight 0 140 142	vith membrane wind 170 km/h with reinforce 0 179 198	and cables snow 50kg/m2 ments, external Displacement A 0 109 133	Structure wit self weight tubes, and ex verages (mm 0 328 285	hout cables - m wind 170km/h ternal cables) 0 573 725	embrane only snow 50kg/m2 0 1547 1420
		Structure w self weight 0 140 142 149	vith membrane wind 170 km/h with reinforce 0 179 198 73	and cables snow 50kg/m2 ments, external Displacement A 0 109 133 277	Structure wit self weight tubes, and ex verages (mm 0 328 285 464	hout cables - m wind 170km/h ternal cables) 0 573 725 127	embrane only snow 50kg/m2 0 1547 1420 2504
		Structure w self weight 0 140 142 149 172	vith membrane wind 170 km/h with reinforce 0 179 198 73 54	and cables snow 50kg/m2 ments, external Displacement A 0 109 133 277 690	Structure wit self weight tubes, and ex verages (mm 0 328 285 464 349	hout cables - m wind 170km/h ternal cables) 0 573 725 127 151	embrane only snow 50kg/m2 0 1547 1420 2504 4109
		Structure w self weight 0 140 142 149 172 172	vith membrane wind 170 km/h with reinforce 0 179 198 73 54 54	and cables snow 50kg/m2 ments, external Displacement A 0 109 133 277 690 689	Structure wit self weight tubes, and ex tubes, and	hout cables - m wind 170km/h ternal cables) 0 573 725 127 151 105	embrane only snow 50kg/m2 0 1547 1420 2504 4109 4106
		Structure w self weight 0 140 142 149 172 172 289	ith membrane wind 170 km/h with reinforce 0 179 198 73 54 54 217	and cables snow 50kg/m2 ments, external Displacement A 0 109 133 277 690 689 729	Structure wit self weight tubes, and ex verages (mm 0 328 285 464 349 356 575	hout cables - m wind 170km/h ternal cables) 0 573 725 127 151 105 158	embrane only snow 50kg/m2 0 1547 1420 2504 4109 4106 3923
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		Structure w self weight 0 140 142 149 172 289 216 254 183 0 72 57655	ith membrane wind 170 km/h with reinforce 0 0 179 198 73 54 54 217 147 147 182 147 0 24	and cables snow 50kg/m2 ments, external Displacement A 0 109 133 277 690 689 729 651 759 656 0 23 Structure 69024	Structure witt self weight tubes, and ex- 0 328 285 464 349 356 575 576 1302 1320 0 71 Results	hout cables - m wind 170km/h ternal cables) 0 573 725 127 151 105 158 141 450 478 0 23	embrane only snow 50kg/m2 0 1547 1420 2504 4109 4106 3923 3901 4217 4184 0 192 48865
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Table 2: A comparison of the results between structure with only cables, structure with only membrane and structure with both membrane and cables.
In the analysis (Table 2) we can compare the different results of the model with different options through the WinTess software. The nodes displacements in the tensegrity ring and the dome, the weight of the structure, reactions, dimension of the structural elements. All tested under loads of wind, self weight and snow take in account the pressure coefficient. The comparison between structure with only cables, structure with only membrane, and structure with both membrane and cables, which demonstrated major efficiency in the structure tested, to wind 170 km/h (minor displacements) that the structure tested to snow 50 kg/m, (less efficiency, major displacements). After the analysis the proposed structure's aerodynamic and load-bearing features would be helpful if building in an area frequented by high winds and in areas with little-to-no snow.

4 CONCLUSIONS

This methodology allows these conclusions:

- After doing the pertinent calculations, a tensegrity ring is proposed with a central dome, using diamond-shaped membrane patterns with twenty struts in a double layer, to cover a 40m diameter sports arena, which has a surface of 1.200m2 and can be occupied by approximately 626 people.
- Small changes in wind pressure or snow loads can have a major impact on the size and shape of the structural elements and the deformations that occur. Our initial testing demonstrated that it was necessary to re-optimize the structural elements to build a real-world structure.
- The uniqueness of these structures is that, even though they are auto-balanced for external loads such as wind and snow, it is sometimes necessary to increase the stiffness of their structural elements and/or reinforce them with external tubes and cables to prevent a collapse due to extraordinary conditions. To do so, we had to reinforce the membrane (Ferrari Fluotop T2 1302), the ring tubes (Ø 400-10_S235) and external tubes (Ø 250-8_S235).
- For the real-world structure that was tested with simulation software, the minor displacements observed during 170km/h wind was 200mm and the major displacements observed with 50 kg/m2 snow loads was 600mm. Therefore, due to the high displacements observed during snow loading and the small displacements observed during wind loading, the proposed structure's aerodynamic and load-bearing features would be helpful if building in an area frequented by high winds but would not be optimal for use in areas that experience heavy snow.
- This study is the first step in the process to construct a real-world building. To build the real-world structure, additional testing such as a dynamic analysis would need to be performed.

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SHADE DESIGN IN SPAIN: HOW TO PROTECT AGAINST HEAT AND UV-RADIATION

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Key words: Cool shade, shade design, thermal comfort, UV-protection, low-e, solar factor

Summary: Shade design is a new discipline that was originally developed in Australia and New Zealand, where high rates of skin cancer have been a key factor driving innovation. It combines climatic comfort under shading structures with ultraviolet (UV) protection. Some modern fabrics heat up rapidly when exposed to strong sunlight and offer poor protection against UV-radiation. Precise information about the properties of shading materials is scarce and norms used in solar protection are not adapted to the field of tension membrane structures.

1 INTRODUCTION

In southern Europe shade is the basic requirement for thermal comfort in the exterior. Natural as well as built shade permit outdoor living in a Mediterranean climate during most of the year. Alleys, arcades, pergolas and of course *toldos* and textile canopies protect the human being from direct and diffuse solar radiation. Indeed, textile shading has a 2000 year history in Spain, producing highlights such as the *Roman Velum*, the *Andalusian Toldo* and the *Catalan Envelat*. Surprisingly, this rich tradition has not generated a solid line of academic research. Very little information and nearly no literature is to be found about the climatic comfort and the UV-protection characteristics of a horizontal membrane structure. For example, an analysis of existing shading canopies shows that the main criterion for colour selection is purely aesthetical. Usually it fits into an overall design scheme or just repeats the colours of an associated commercial logo. And if the logo happens to be black, the shading material might just follow suit. This raises a number of basic questions which have not been properly answered: How big is the difference in comfort underneath two differently coloured fabrics? Do brighter colours really lead to a higher degree of climatic comfort than darker ones? Does the selection of material or coating make a difference?

2 WARM SHADE

Warm shade makes sense in a cold climate. It allows UV-radiation filtering and provides

heat through a combination of transmitted and absorbed solar energy. In a Mediterranean climate, warm shade should therefore be avoided with the possible exception of the winter months.



Figure 1: Photo of a bicolour acrylic fabric and Figure 2: Infrared image of the same fabric showing clearly the difference in temperature between the dark blue and the white stripes. The latter ones are much cooler.

Dark colours absorb most of the solar radiation leading to higher temperatures and to a higher heat emission from the fabric. Following the Stefan-Boltzmann law, the total heat flux in watts produced by thermal radiation depends on the surface temperature raised to the power of four and can be calculated according to following formula:

$$P = \varepsilon_i \times A \times \sigma \times T_i^4 \tag{1}$$

[ε_i] is the emissivity of the underside surface of the membrane panel, [A] is the total surface area, [σ] is the Stefan-Boltzmann constant [5.6697×10⁻⁸W/m² K⁴] and [T_i^4] is the underside surface temperature in Kelvin raised to the power of four

The uncritical application of the *solar factor* (g-value) suggests that dark colours combined with low solar transmission automatically lead to a higher degree of climatic comfort. This false assumption might explain why manufacturers and designers favour dark colours for acrylic canopies. A good example of this mindset is the following comment made by a the Austrian shade designer Gerald Wurz that appeared in the leading German web data base dedicated to construction, expertise and built objects, www.baunetzwissen.de: "*It is untrue that climatic comfort is higher underneath a bright fabric than underneath a dark one, says the designer. The (African) Touaregs...already knew this for centuries*"[1]. The author ignores that dark pigments were traditionally the only way to protect textile fibres from rapid deterioration under UV-radiation. The same mechanism protects the human body: a dark skin absorbs more solar energy in order to avoid a deeper penetration into the epidermis and a subsequent DNA damage. In both cases a higher level of protection is achieved at the price of a lower level of climatic comfort.

3 COOL SHADE

The tightening of building codes for isolation in northern Europe has led investigators to focus generally on the subject of membrane enclosures. However, the specific problem of open shading structures in southern countries has not been well investigated. The recently presented thesis *Cool Shade Tents* [2] tries to fill this gap and expand knowledge of comfort and protection under light-weight membranes. Cool shade is suitable when temperature and UV-radiation levels are high. Materials should block both of them in order to improve summer cooling and UV-protection. The thermo-dynamic behaviour of 20 different materials has been investigated and extensive outdoor testing been carried out under Spanish summer conditions. The aim was a comparative assessment and classification of the selection.

4 THERMAL HEAT TRANSFER IN TEXTILE MEMBRANES

Shading materials are exposed to strong solar radiation, which includes UV, visible light (VIS) and near infrared radiation (NIR). They also receive a spectrum of mid-infrared radiation (MIR) that corresponds to the level of heat emission of the surrounding terrestrial objects. The three mechanisms of thermal heat transfer on earth are *conduction, convection* and *radiation*. Convection and conduction are slow mechanisms and need either a medium of thermal transport or a direct contact between objects. Only electromagnetic radiation from $0,1\mu$ m to 1000µm conveys thermal energy in forms of photons even through the vacuum of cosmic space.



Figure 3: Hemispheric emission power of the sun (5780K, blue line) and a terrestrial object radiating at +20°C (293K, red line). The blue and red ordinate axes relate to the correspondingly coloured lines. The abscissa axis shows the wavelength in micrometre (μ m) and follows a logarithmic scale. The sun radiates mostly in the visible region (VIS) at a wavelength of 0,38-0,78 μ m and reaches its maximum in the green spectrum. Terrestrial objects radiate at a wavelength of 3,0-50 μ m in the mid-infrared spectrum (MIR). Their maximum emission power lies in the range 8-13 μ m and can be captured by an infrared camera. NIR refers to the near-infrared range 0,78-3,0 μ m and FIR to the far-infrared spectrum from 50-1000 μ m.

Textile membranes have a weight of around 1kg/m² and a thickness of less than a millimetre. Their skin-like quality makes them highly responsive to any fluctuation in received energy. Traditional concepts such as thermal capacity or conductivity cannot explain their thermal behaviour, which is entirely dominated by radiation and convection. Any change of temperature on one side is immediately passed to the other. The sun radiates from the photosphere at a temperature of around 5780K, which leads to a fairly constant solar radiation at the entry of our atmosphere of about 1360W/m². We tend to forget that without the filter of our atmosphere the temperature under bright sunshine would rise to 130°C, as is the case on the moon.

5 IN SITU MEASUREMENTS

As part of the research undertaken for the thesis Cool Shade Tents, a horizontal shading device was constructed in 2009 to quantify the thermo-dynamic behaviour of twelve different shading samples. A data logger saved the temperatures measured by nine PT-100 thermocouples every two minutes, as well as the maximum and minimum values occurring over an interval of 30 minutes. The results were double checked by the parallel use of a professional infrared camera. The camera also made it possible to include three natural shading materials that could otherwise not have been monitored. Measuring temperatures of a thin, skin-like membrane is a delicate procedure. Thermocouples always add mass to the studied membrane and an open mesh lets sunrays pass directly to them right through the openings. Both cases lead to erroneous results. On the contrary, infrared cameras permit temperature measurement without contact, although they do encounter problems with lowemissivity (low-e) treatments and heat reflective surfaces. Using a combination of the two measurement methods, however, produces accurate results. Special care was taken to limit the influence of convection on the comparative study by choosing only days with conditions of bright sunshine, no clouds and little air movement. The total solar radiant flux was recorded with the help of an *albedometer*. It captured the direct and diffuse parts of the solar radiation from above as well as the reflected parts from below. Knowing the solar optical properties of the different materials, it was then possible to calculate the transmitted and reflected parts of the solar flux. It was also necessary to collect the external environmental data such as air temperature, humidity and wind speed in order to correctly assess the results.



Figure 4 and Figure 5: Photo and infrared images of a horizontal shading device with 12 materials exposed to direct sunlight on top of a high rise apartment block in downtown Barcelona. As part of the research, surface temperatures were measured by means of PT 100 thermocouples and a professional infrared camera.

6 SOLAR OPTICAL PROPERTIES OF TEXTILE MATERIALS AND LOW EMISSIVITY TREATMENTS

Thermal optical properties are a function of three parameters: *reflectance*, *transmittance*, and *absorptance*. These describe the ratio of the reflected, transmitted and absorbed radiant flux to the total radiant flux. According to Kirchoff's law of thermal radiation, which applies to black and grey bodies, the sum of these three parameters is equal to one within any specified waveband of the electromagnetic spectrum.



Figure 6: Solar optical properties of a low-e coated polyvinylchloride (PVC)/polyester 1002 fabric membrane measured in the laboratory of Ferrari SA in France. The values of reflection and transmission are directly measured whereas the absorbed part of the thermal energy has to be calculated using Kirchhoff's law. The upper white side of the membrane should point to the sky while the low-e side (treated with nanoparticles of aluminium) should be directed to the earth. The red line shows the reflection of the white side ("côté blanc") and the black line the reflection of the low-e treated surface ("côté Lowe") in a wavelength from 250 to 2500nm. The red line has an optimum of reflection in the visible spectrum (VIS) whereas the black line shows a fairly constant reflection rate between 60-70%. The green line shows the value of solar transmission.

White fabric reflects most of the solar radiation in the visible spectrum (VIS), somewhat less in the near-infrared range (NIR) and absorbs nearly all radiation in the mid infrared range (MIR). A horizontal white membrane therefore combines a high reflection in the short wave spectrum with a maximum loss of heat in the mid-infrared range. This is a great advantage for cool shade tents during summer, but poses serious problems for membrane enclosures in winter.

Today it is possible to manufacture a fabric with specific values of reflectance, transmittance and absorptance in the different wave bands of VIS, NIR and MIR. Low-

emissivity coatings made out of aluminium, silver or *tinox* (titan-nitrit-oxyd) help to reflect thermal radiation in the mid-infrared spectrum. It is important that the coating is applied on the reverse side to the sun, because a white colour reflects much better in the visible spectrum than a metalized surface. *Tinox* coatings are typically used for solar collectors: their dark colour (titan) together with a selective low-e capacity in the mid-infrared range acts as a heat trap for solar energy. The application of this technique on dark acrylic fabric (commercially called *Cold Black*) leads unquestionably to warm rather than cool shade.

7 MEASURED RADIANT FLUX OF HORIZONTAL MEMBRANES COMPARED TO CALCULATED SOLAR FACTOR (G-VALUE)

The following diagram shows the balance of received and emitted radiation (assuming air movement is limited to natural convection). Four main sources of radiant power or radiant flux in Watt need to be considered underneath a horizontal shading structure:

- 1. The reflected solar albedo [Φalbedo] radiation (VIS and NIR)
- 2. The transmitted part $[\Phi_T]$ of global solar radiation (VIS and NIR)
- 3. The radiant flux $[\Phi E]$ of the fabric (MIR)
- 4. The reflected infrared radiation $[\Phi_{back}]$ of the surrounding objects (MIR)



Figure 7: Diagram reproduced from the thesis *Cool Shade Tents* showing four sources of radiant flux underneath a horizontal membrane. $[\Phi T]$ is the reflected and $[\Phi A]$ the absorbed part of the global solar radiation. [T] is the temperature of the underside of the membrane. The temperature of the sky $[T_{sky}]$ typically reaches -20°C at noon during the summer months in Barcelona. [Tbackground] is the resulting mean temperature of the surrounding terrestrial objects.

The Solar Factor (g-value, European norm *EN 410:1998*) consists of two parts, which together express the permeability of a vertical glass pane to diffuse solar radiation on a cloudy day. However, it is a norm that is based on the specific case of a highly transparent and vertical piece of glass, and incorporates only two of the four radiant sources treated in figure 7:

- 1. The transmitted part $[\Phi T]$ of the global solar radiation (VIS and NIR)
- 2. The secondary part [qi] of the absorbed solar heat flux (MIR)



Figure 8: Diagram of the g-value (European Norm *EN 410:1998*) applied to vertical double glazing with a high degree of transparency in the visible range. R stands for reflection, T for transmission and A for absorption.

The incoming solar flux multiplied by the g-value calculates the amount of solar energy entering a confined space. This gives an erroneous result in the case of an open and horizontally disposed shading device, mainly because it totally neglects the heat radiation of the membrane itself.

The norm *EN 410:1998* is based on the assumption that a transparent glass pane never heats up to a degree where the radiant flux exceeds the transmitted part of the solar energy. However, this is simply not the case for most shading structures. Neither does the norm consider additional heat sources such as albedo radiation or the heat flux of surrounding terrestrial objects. The application of the norm leads directly to the mistaken preference of dark coloured fabrics in textile canopies.



Figure 9: Measurements taken in Barcelona on August 21, 2009 at local summer time (14:00 local time is 12:00 in coordinated universal time). The blue line shows the global solar radiation and the green line the albedo radiation from below the horizontal cotton membrane. The violet line results from the multiplication of the calculated g-value of 0,12 with the measured global solar radiation. The red line shows the total heat flux for the lower side based on the four sources of radiant power explained in figure 7. It becomes obvious, that the red and the violet line have little to do with each other.

8 UV-PROTECTON UNDER SHADING STRUCTURES

The spectrum for ultraviolet radiation ranges from 1-380nm. Extreme UV (EUV), vacuum UV (VUV) and UV-C-radiation do not pass the barrier of the atmosphere. UV-B in the wavelength from 280-315nm is therefore the most energetic and potentially dangerous radiation reaching the earth. UV-A radiation from 315-380nm is equally harmful for the human being because it penetrates deeper into the epidermis. The international Agency for Research on Cancer (IARC), part of the World Health Organization (WHO), determined in 1992 that both types of ultraviolet radiation can produce different forms of skin cancer. In order to enjoy the positive effects of solar radiation and limit the negative ones, dermatologists recommend providing shade in summer during the critical time frame of two hours before and two hours after solar noon. Children and adolescents are in particular need of protection as their skin is still not fully developed. Both groups pass lots of time outdoors. Australia and New Zealand suffer the highest rates of skin cancer in the world because the effect of strong solar radiation of the austral summer is multiplied by a huge ozone hole, and the skin of descendants of white Europeans is typically not prepared for the solar radiation of the antipodes. With great success local authorities have converted shade design into a public health issue. In 1996 both countries introduced the ultraviolet protection factor (UPF) as a

UPF Range	UVR protection category	Effective UVR transmission, %	UPF Ratings
15 to 24	Good protection	6.7 to 4.2	15, 20
25 to 39	Very Good protection	4.1 to 2.6	25, 30, 35
40 to 50, 50+	Excellent protection	≤2.5	40, 45, 50, 50+

classification system of textile fabrics that are in direct contact with the skin. UPF CLASSIFICATION SYSTEM

Table 1: The ultraviolet protection factor (UPF) classifies [3] the transmission of ultraviolet radiation: In order to achieve a "good protection", a minimum of 93,3% of the incoming UV-radiation has to be reflected or absorbed. The classification was developed originally for clothing. The values for a horizontal shading structure should be generally much lower because of the effect of scattered and reflected UV-radiation.

An excellent protection (40, 50, 50+) means an effective ultraviolet transmission of less than 2,5%. A horizontal shading structure can never reach this level of protection, because scattered and reflected UV-radiation keeps penetrating from all open sides. Dense fabrics and pigmented textiles offer more protection than open meshes and permeable fibres. The formula for calculating UPF is given below:

$$UPF = 100 / \tau \tag{2}$$

The value of UV-transmission $[\tau]$ is introduced as a percentage. All transmission values below 2% are considered "best" [50+].

9 CONCLUSIONS AND RECOMMENDATIONS

Children and adolescents are generally not aware of the full dangers of over exposure to the sun. Courtyards, patios, playgrounds and public spaces in general need to be protected. Shade audits should be the basis of shade design projects that effectively combine natural with built shade. Designers use mainly tensile membrane structures for built shade because they combine lightweight with cost efficiency and ease of erection. The results of this research lead to the following eight-point summary of what should be good practice in climatic comfort and UV-protection under horizontal membrane structures:

- 1. The use of highly reflective, white fabric is the first and best option to minimize absorbed solar heat. White *Teflon* (PTFE) stands out as a particularly efficient fabric because it repels dirt and maintains its original solar optical properties.
- 2. Heat losses to the open sky should be as high as possible. Even under a burning sun a horizontal membrane can still lose up to 200W/m² of heat to the sky. White fabrics have excellent selective properties for shading purposes: they are white in the visible spectrum and appear black in the mid-infrared range, thus losing a maximum of heat to the cold blue sky.

- 3. Small shading structures exposed to high solar albedo radiation from below should combine a white side to the sun with a dark or even black side to the earth. The dark side avoids short wave solar reflection and leads to a lower total heat flux.
- 4. Low-e coatings on the inner side of membranes help to keep thermal radiation as low as possible. However, when applied to the upper side of the fabric, such coatings would prevent the intended heat loss to the sky and lead to higher temperatures.
- 5. The transmitted part of solar energy should be minimized. Small canopies should be completely opaque as there is enough light entering from the open sides.
- 6. Textile membranes are not in direct contact with the skin, so the highest possible ultraviolet protection factor should be chosen.
- 7. There is a direct relationship between the size of the visible part of the sky seen from underneath a shading structure to the diffuse and scattered UV radiation received. It is not enough to block only the direct solar radiation. Open sides have to be reduced and duly protected. For example, the classical *hypar*-shaped sail fails to offer the required protection around its high points.
- 8. Natural shade creates a higher degree of thermal comfort than built shade because plants regulate their temperature through evaporation and the use of solar energy in photosynthesis. It is highly recommended to combine natural and built shade in order to maximize the advantages of both systems.

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USING MOBILE DEVICES IN TEXTILE ARCHITECTURE DESIGN (iPad/iPhone)

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Key words: mobile devices, textile architecture, iPad, conceptual design.

Summary. The aim of this article is to explore the use of Mobile devices in the textile architecture field. Some Apps for iPad/iPhone/Android exist that cover general architecture aspects such as design, sketching, drawing, CAD viewers, calculators or project Management. A specific App for textile architecture conceptual design is proposed in this paper.

1 INTRODUCTION

This paper describes the use of mobile devices in different fields and shows how textile architecture can take benefit of it.

1.1 Mobile devices

Last generation of mobile devices, such as iPhone, iPad or Android based devices are changing the way the users interact with data and technology. Mobile devices bring the users a new way to deal with traditional computer tasks: browsing Internet, sending emails, accessing calendar or playing music and video.

Since the launch of the first laptop computers in the 80's, the term mobility has been a key factor in the design of computer devices. Laptop sales increased considerably compared with desktop computers sales in the 2000's.

Mobile devices have become more popular since the launch of the iPhone in 2007. Further devices as the iPad or the Samsung Galaxy Tab offer similar capabilities with a bigger screen (7 to 9.7 inches), using always a touch-based user interface.

The rise of tablets popularity shows that in a short time, they would become an alternative to laptops for a great of number of users, especially those dealing with Internet and multimedia.

Mobile devices are capable of running apps. There are a vast variety of apps, grouped in categories such as music, games, books, education, medicine, entertainment, business or social networking. Figure 1 shows a picture of the iPad apps being created for each category in the iTunes App Store.

Although games and entertainment seem to be the most popular categories, everybody wants to have presence in the mobile app market. Fields such as medicine or architecture have already started to migrate their products to this new platform.

The emergence of this new platform would be compared to the appearance of the Internet in the 90's, where most of the companies adapted their products to fit the new standard and have a dedicated space in the web.

Mobile devices will become the standard in the next few years, so we should be prepared for this change and create new products oriented to the textile architecture field.



Figure 1. Mobile devices. iPad Apps categories

1.2 Mobile Apps and Architecture

Mobile devices offer Apps that make life easier to field engineers and architects. Mobility is a key factor, which allows the users the possibility of working anywhere at any time.

Some of these apps are oriented to design, sketching or DXF viewers. As example of this, Autocad and Rhino [1, 2] have implemented mobile versions for iPhone/iPad in the last quarter of 2010.

Figure 2 shows some screenshots of iRhino 3D. The app allows users to view native 3DM files on the iPad, iPhone or iPod touch.



Figure 2. iRhino 3D for iPad/iPhone/iPod touch

It is a great app to show the design and ideas when the user requires mobility. It is just a viewer that allows the user to rotate, zoom or pan the model with a tap or drag of the finger. The user can't edit the models; they are represented as a collection of polygon and meshes. The models can be downloaded from websites, google docs, email attachments or iTunes. Views can be saved as images for markup or emailing.

Autocad has also presence in mobile devices. Figure 3 shows some snapshots of the iPad App, named Autocad WS and launched on Sept 2010. The app is also available for web browsers and allows the user to sync there models online using the file manager. The App includes the traditional drawing tools (line, polyline, circle, rectangle...) and editing tools (mode, scale, rotate, erase, copy...).



Figure 3. Autocad WS for iPad

Other popular apps between architects and engineers are those related to geolocation, unit conversion, measurement, material libraries, or technical calculators between others.

There are also some other specific tasks that would require using a tablet instead of a laptop, especially on field tasks such as surveying.

2 MOBILE APPS IN TEXTILE ARCHITECTURE

This paper introduces an iPad App for textile architecture conceptual design. Figure 4 shows some screenshots of the prototype. The purpose of this app is to create a 3D graphic scenario, in which the user can easily interact with the membrane structure, creating and editing its shape.

Dynamic movement as rotation, translation and zoom are performed by means of pinch or pan gestures, as it will be explained later in this section.

The basic geometric features allow the user the possibility of adding/removing/editing vertex to the model. Physical properties of the network and the boundary cable can be also easily defined.

The main purpose of this App is to sketch different design solutions. The designer uses basic gestures and works in a multi-touch environment. The model would be emailed in a standard neutral format (DXF), so the user can continue working in a desktop/laptop platform to add further details.



Figure 4. Screenshots of the proposed iPad App for textile architecture design

The app uses the Force Density Method [3,4], which allow obtaining the equilibrium shapes in a really fast, flexible and efficient way. The user can assign different force density values to the boundaries or to the network itself, obtaining new equilibrium shapes easily.

2.1 User interface design

In order to create a useful app, the design should be centered on the user. There are three basic rules that have been followed during the iPad app design, to obtain a good user experience.

- *Simplicity*: there is no need to overwhelm the user with a thousands choices, all at the same level. The iPad is a new platform, so we can't translate the desktop user experience. There is no menu system, no windows system, and no file system. The desktop tasks should be simplified on the iPad platform.
- *Prioritize*: Which features does the app really require? How is the user interacting with them? Which is the frequency of use of these features?.
- Innovate: We have created a new way of interacting with a 3D model.

Following these rules it's really easy, really efficient and really satisfying to manipulate the 3D model.

Although the core of creating and manipulating shapes is a structural problem based on the force density method, the success of the App will depend on how good the user interface is designed.

In other words, terms as real time, precision or accuracy are at a secondary level. So, the important point is how the user interacts with the model, which gestures, menu item or buttons are used to facilitate this interaction.

Next sections describe some of the user interface details which have been taken into account during the implementation.

2.2 Interaction with a 3D scenario using 2D gestures

We begin describing the virtual workspace that will be used by the designer while sketching the membrane structure. It is clear that a three-dimensional space will help the designer in the creation of the structure. Figure 5 shows a reference system and a point that represents the position of the user finger. As the user moves the finger, the point moves around the virtual workspace.

But, can you know exactly the location of the point represented on Figure 5a? The answer is no, and it is due to the lack of a reference related to the deep. It means that the same point represented in Figure 5a would have different representation, all of them with the same 2D screen xy location. Figure 5b & Figure 5c show two possible locations of the point given in Figure 5a. The projection of the point onto the three walls facilitates the user to locate the point.



Figure 5. Referencing a point in 3D space using 3 reference planes (a) unreferenced point (b) position A (c) position B

Should we conclude that the three walls and the projections are needed to locate a point in the 3D space? Not necessarily. As you can see in Figure 6b & Figure 6c, the point given in Figure 6a can be located in different ways in the virtual workspace by adding its projection on the horizontal plane.



Figure 6. Referencing a point in 3D space using 1 reference plane (a) unreferenced point (b) position A (c) position B

Although this way of representing a point in the 3D space seems to be something very common and trivial, the majority of 3D CAD systems use the traditional mouse as a 2D input device, using different strategies to give the user the feeling of being immersed in a 3D scenario. CAD applications usually combine the movements of the mouse with a keyboard input to restrain the movement of the pointer into a certain plane or in a specific direction.

In our case, we have to combine gesture movements with other controls that restraint the movement to a vertical direction or inside an specific plane.

2.3 Dynamic interaction using 2D gestures

It is important to define how the user interacts with the model in order to apply dynamic movements such as rotation, scale or pan. Figure 7 shows the user getures. Following are defined these actions:

- *Scale*: The iPad user experience associates the pinch gesture with zooming, so we are keeping it for our App. See Figure 7a.
- *Rotation*: a natural gesture to rotate the model is just moving one finger on the screen. The model will rotate around its center of gravity. See Figure 7b.
- *Pan*: Panning is achieved by touching the screen with two fingers and moving around. See Figure 7c.

The user can easily swap between the zoom and the pan action since both gestures use two fingers. Rotation and pan can also be easily combined. As example, if the user is rotating the model moving the finger, he can start panning just adding a second finger to the screen and continuing the gesture.



Figure 7. Main gestures. (a) Zoom/scaling (b) rotation (c) panning

2.4 Mesh control

Once the main scenario features have been described, it is time to talk about the representation of the equilibrium shape of the membrane structure. As stated previously, the formfinding method used for obtaining the equilibrium shape of the structure is the force-density method.

The model needs to be discretized in nodes and elements to calculate the equilibrium shape. This mesh is based on a control polygon, which contains the fixed vertices of the membrane. The example shown in Figure 8a shows the location of the seven fixed vertices of the membrane, which are joined to create a closed polygon.

The projection of this polygon on the bottom surface defines the boundaries of the mesh. This mesh consists of quadrangular elements as shown in Figure 8b. Figure 8c shows the final equilibrium shape.



Figure 8. Equilibrium shape (a) control polygon (b) mesh before equilibrium (c) equilibrium shape using formfinding

2.5 Restraining gesture movements

In order to facilitate the user interaction with the membrane vertices, it is possible to restraint the user gestures into a specific direction.

Figure 9a shows how the finger gets to the vertex and a control polygon appears in the scene. A button is added in the toolbar to allow the user to move the vertex only in the vertical direction. A vertical line is shown to tell the user visually that the vertical movement is allowed (see Figure 9b). While dragging the vertex vertically, the polygon control is updated according to the new vertex location. Once the desired position has been reached, the new equilibrium shape of the membrane is represented and the control polygon disappears of the scene. Figure 9c shows the new equilibrium position.



Figure 9. Moving a vertex vertically.

It is important to emphasize the importance of the shades projected on the three main planes. They can be used as reference while modifying the position of a vertex or when new geometry is added to the scene.

In some occasions, it can be interesting to have the possibility of moving a vertex inside the XY plane. The procedure is similar to the movement in the vertical direction. Again, the user should approach the finger to the location of the desired vertex (see Figure 10a). The control polygon appears again and the movement of the pointer is restrained to the horizontal plane.

A small 2D top view of the membrane structure is located on the top-left side of the main scene (see Figure 4). This is a very helpful view for the designer to see the appearance of the membrane while dragging the vertex horizontally.



Figure 10. Moving a vertex inside the XY plane.

2.6 Physical properties

As it was stated before, the force-density method has been used as formfinding method for the calculation of the equilibrium shape of the membrane. This method requires the introduction of the force-density values. Since the application is being use by designers at the conceptual stage of the design process, it is preferred to avoid the introduction of numerical values by the user. So, how can the user change the force-density values without introducing the numerical values?

Let's think about the meaning of the force-density value. This value is the ratio between the internal force and the length at each bar of the structure at the equilibrium shape. Although each bar of the mesh could have a different force value, it is not usually an efficient practice. The most common practice consists on assigning two force-density values to the internal mesh bars, and one force-density value to the bars of each edge of the membrane structure. The two internal force-density values are associated with the two main directions of the membrane and they use to have the same value, since the membrane material is isotropic.

As the force-density value just relates the relation between the internal bar force and its length, we can have the same equilibrium shape if all the force density values of the membrane are scaled.

So, to make easier the conceptual design process, the user will increase and decrease the value of the force density values by means of sliders. The force density value would be the same for all the edges, or have a different value for each edge of the membrane.

Figure 11 shows different equilibrium shapes of a membrane with different force density values at one of the edges. It can be seen the difference between Figure 11a with a 1:2 ratio and Figure 11c with a ratio of 1:50



Figure 11. Edge force-density ratio. (a)1:2 (b) 1:5 (c) 1:20 (d) 1:50

3 CONCLUSIONS

Mobile devices are changing the way the users interact with traditional computer tasks: browsing Internet, sending emails, accessing calendar or playing music and video. It seems that the future of technology tends to be the use of mobile devices. It is a great challenge to convert traditional desktop/laptop tools, using new multi-touch devices to make life easier to field engineers and architects.

Some CAD manufacturers as Autocad and Rhino have already point in this direction and have a specific product for this platform.

In this paper, we have explored the possibilities of using the iPad for textile architecture design purposes. Simplicity, prioritization and innovation are the key factors to the success of the app.

The app uses the force density method to calculate the equilibrium shapes. The main features of the app include the dynamic control of the 3D model, creation and edition of vertex and modification of physical properties of the membrane.

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EXPERIMENTAL MANUFACTURE OF A PNEUMATIC CUSHION MADE OF ETFE FOILS AND OPV CELLS

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Key words: EFTE, Organic Photovoltaics OPV, Optical properties, Carbon Nano-tubes, Tessellation, Building Integrated Photovoltaic (BIPV).

Summary: This paper presents the preliminary activity from which the fabrication of a new kind of ETFE foil fully integrated with a Smart, Organic, Flexible and Translucent Photovoltaic building component (named SOFT-PV) begins. A prototype of SOFT-PV cushion will be the object of optical, mechanical and thermal tests to preview all problematic aspects of the fabrication process and the environmental impact evaluation with LCA methodology.

1 INTRODUCTION

The paper deals with the first disciplinary goals obtained by the Building Technology researchers, during the first year of a biannual research activity, co-financed by Cariplo Foundation (Lombardy, Italy) and based on the collaboration between three different research groups (fig.1): the Department of Architecture and Building Technology of Polytechnic of Milan (POLIMI-BEST), the National Chemistry Research Institute (CNR-ISMAC and the Physics Department POLIMI-FISI).



Figure 1: The main "cross-fertilisation" loops between different fields of involved knowledge

2 TOWARDS THE INTEGRATION OF PHOTOVOLTAIC INTO ETFE CUSHIONS

Actually the current use of ETFE cushion as an inflatable substrate of Photovoltaic into buildings can be considered only as an added component, not fully integrated, which is represented now by PV manufacturers. SolarNext AG has now developed a form of photovoltaic technology that enables solar cells to be added directly onto membrane materials¹. This technology is based on extremely flexible, amorphous silicon thin-film solar cells (a-Si) laminated between two layers of ETFE foil. They can be substituted for the upper layer of pneumatically supported cushions. This process of lamination ensures the protection of the photovoltaic cells against loads and stresses, as well as against moisture and weathering. Depending on environmental aspects, they can be used in roofs or facades either in a single-layer form or as a part of a multilayer membrane cushion.

On one hand, the integration of 2nd generation PV cells into transparent ETFE foils could negatively impact the most important advantage of ETFE envelopes which is its transparency (using natural foil) or its great translucency (using printed foils). On the other hand, encapsulating PV cells into a double foil of ETFE reduces significantly the efficiency of the PV cells and the typical weight and thickness of 2nd generation PV cells are not perfectly suitable for the material properties of ETFE foils. When the photovoltaic elements are used as an intermediate layer of a cushion, they are, of course, optimally protected from the possibility of dirty. In such cases, however, the light-refracting effect of the upper film layer and the thermal gains that occur in the heat-absorbing middle layer would decrease the energy yield, but anyway the efficiency is higher than their set on the outsider layer, often easily dirty. Finally, the application of this kind of PV cell could cause problems in the management of flexibility and in the structural behaviour of pneumatic systems.

As the integration of Organic photovoltaic systems into building envelopes to replace conventional building materials has not been realized, combining two low cost technologies available on the market today - the ETFE cushion technology and the 3rd generation of organic flexible PV cells – should represent a more cost-effective solution for the creation of a new kind of smart façade employable both in renovation and new construction, overcoming the most of problems focused above on the integration between thin-film cells and ETFE foils. Moreover from an architectural perspective, photovoltaic elements don't only generate electricity, but they also provide shade, which may be often an essential design requirement.

This paper presents the preliminary activity from which the fabrication of a new kind of ETFE foil fully integrated with a Smart, Organic, Flexible and Translucent Photovoltaic building component (named SOFT-PV) begins. The origin of this idea came from the hypothesis of compatibility of the production processes of the ETFE and of the OPV: the extrusion of the film can be combined with the roll to roll OPV deposition technique. Based on the cross-fertilization of the three research groups (fig.1), the Building Technology researches tasks are involved in the design and experimental fabrication of a prototype of the first OPV integrated ETEE cushion and to test the optical mechanical and thermal

first OPV integrated ETFE cushion and to test the optical, mechanical and thermal performances of the new building component together with the LCA evaluation, while the CNR and FISI researches target is focused on maximizing the cell efficiency.

3 STATE OF THE ART

3.1 ETFE as A Shading System in Architecture

According to its numerous remarkable properties, ETFE foils offer a valid alternative to glazing in building envelopes. The reduced self weight is the most appreciated feature of ETFE structures. In comparison to equivalent glazing, pneumatic cushions achieve a comparable level of performance with less than 1% of the weight. This reduces the amount of secondary structure required to support the building envelope with consequent benefits on the primary structure and the foundations allowing unsupported spans up to 10m^{2,3}.

Furthermore, the high level of translucency over a wide spectrum represents the second main aspect which justifies the expectations placed in ETFE. The optical properties of ETFE foils are subjected to high variability from one producer to another due to the raw material used, the production process, the material colour and thickness⁴. In addition, the optical behaviour in the UV spectral range, with a light transmission around 70%, represents a significant benefit for structure such as solaria, swimming pools and greenhouses where the natural bactericidal and fungicidal properties of UV light reduce the demand of chemical treatments. The risk of an inadequate internal environment, due to the incorrect solar control, can be addressed by using different printing patterns which reduce the danger of glare and overheating.



Figure 2: Common formation of ETFE layers in a foil cushion¹³

Although the material ETFE does not offer exceptional thermal insulating properties, the use of multilayer solutions allows the achievement of considerable values of thermal insulation, comparable with those obtained by means of glazed envelopes, reducing overheating, internal condensation and the energy required for air conditioning, both during summer and winter. The environmental impacts of building systems based on ETFE foils represents one of the main interesting aspects in the comparison between different covering systems based on ETFE foils, PES/PVC and PVC Crystal foils, Polycarbonate panels and double glazing. Despite the absence of comprehensive studies in this field, recent researches^{5,3,6} showed the potential LCA performance of lightweight envelopes which is mainly related to the overall covering system rather than the embodied energy of the raw material, expressed in GJ per ton of material. However, side effects due to the resource consumption/savings in use, such as the pressuring system, the artificial light or the use of detergents and water, play a crucial role.

The acoustical behaviour is one of the weakest aspects of the envelopes based on this technology. The level of sound insulation provided by ETFE foil cushions is extremely low with a R_w of 8 dB⁵. However, the high transmission and the reduced reflection can have a positive effect reducing the amplification of sources of noise placed in the internal spaces. The level of noise due to the taut drum-like nature of tensioned ETFE and ETFE cushions² can be partially reduced by means of an addition external mesh layer.



Figure 3, 4: Membrane cushion movements, Atelier Brückner& Festo Technology, Cyclebowl, 2000, Hannover¹⁵

The load bearing capacity of ETFE foils can be achieved through two different approaches, tensioned double curved surfaces and pneumatic cushions. Although the increasing interest in single layer envelopes⁷, mainly due to the reduced maintaining costs, pneumatic cushions are the more common envelopes, especially for applications which require higher levels of thermal insulation or sun shading. The performances of the envelope can be improved increasing the number of layers and internal chambers and combining the reflectance properties of different frit patterns printed on the layers which can be moved by changing the air pressure of the chambers⁸. This solution becomes unavoidable for applications in the Mediterranean area where printed ETFE foils are necessary in order to reduce the sun irradiation and the air-conditioning loads. The high percentage of area covered with frit patterns, which increases with the amount of solar energy which have to be reflected, opens interesting prospects of applying flexible photovoltaic cells instead of reflecting patterns.

In 2000 Festo Technology^{9,10} developed an interesting solution to control the daylight conditions in an architectural space enclosed by an ETFE cushion system. As shown in fig. 3 and fig. 4, a variable skin has been created by printing overlapping gestalt graphics on multiple layers, integrating the cushions with sophisticated pneumatics, and finally by moving the different graphics together and apart from each other.

There are two main production techniques that obtain flat and blown ETFE foils: the blown film extrusion, in which the melted mass comes out of the extruder and is formed by a ring die into a tube which is expanded by blowing in air. And the flat extrusion is in which the film in a roll form coming flat from the extrusion¹¹. The extruded product, passing between rollers, is a 0,05-0,3mm thick and 150-220cm wide film. Then it is ready to be rolled up into cardboard tubes for storage and transportation to the cushion fabricators³. This part of the process, the lamination, is carried out by means of techniques with several aspects in common

with the technology used during the production of OPV. The potentials of industrial applications are numerous and represent the basis of this research.

4 RESEARCH WORK

4.1 Design and Investigation Of A New Active Shading System

4.1.1 The Optical Properties of Transparent and Printed ETFE Films

The first part of the research focuses on measuring the optical behavior of ETFE films. An optical test was performed on two samples of Clear and Printed ETFE (fig. 5) with 200 μ m thick and resulted in the transmission values indicated in fig. 6. Spectrophotometer was used to test the transmission and reflectance values of the samples. The optical data sheets were inserted in program Optics 5 in order to conclude values with the mean light transmission for each sample as follows: a. Mean Clear ETFE Transmission Value = 0,90; b. Mean Printed ETFE Transmission Value = 0,16.





Figure 6: Clear and Printed ETFE transmission graphs

4.1.2 Printing Techniques and Corona Treatment Currently Used for ETFE

ETFE may be produced as a clear film or with printed silver pattern on the surface, usually dots and squares. These printed patterns are designed to control light and heat transmission.

By combining one clear and two printed ETFE films into a three layer pillow and moving the middle layer up and down, it is possible to control the amount of light that is transmitted to the inside environment.

ETFE is chemically inert material with low surface tensions causing it to be non-receptive to bonding with printing inks. The molecular structure of the film's very smooth surface has to be opened up, for the chemically bond adhesion between the film and the ink. Corona treatment is a safe and economical technique that uses a chemical application, electrical discharge or subjecting the film to high intensity radiation to open the surface for printing¹². Patterns are printed with opaque or translucent fluoropolymer inks, leaving approximately 50mm not printed, along the edges of the film, for welds. The most common pigment is aluminum with the color of silver for the reflection of the solar light and heat. Also copper with its typical oxidized green colour has one time been used¹³.

4.1.3 Potentials of New Printing Patterns for Controlling Light Transmission

This part of the project focuses on trying to innovate concepts for patterns that could be printed on different cushion layers in order to optimize the visual aspects of ETFE printed cushions. These patterns ideas are still under progress trying to recognize and develop their integration into the cushion and optimize their environmental impact.

4.1.3.1 Tessellations

A tessellation is a pattern of plane figures that fills the plane with no overlaps and no gaps. As indicated in fig. 7, all patterns are composed by overlapping the same two geometries, the tessellation pattern on the left, but with different distances and angles. This idea can be applied onto the multi-layer ETFE cushions in order to optimize the aesthetic value of the envelope. The more the number of printed layers increase, the more the variability of patterns are; which will reflect actually on the percentage of transmitted light to the indoor environment.



Figure 7: An example of the patterns variety out of using the same tessellation geometry (left)

In order to design and apply practically these patterns, it was necessary to calculate the total printed area for each scenario and then conclude with the percentage of transmitted light through the overlapped layers. Using Grasshopper software¹⁹, it was easy to simulate the overlapped geometries while controlling different parameters such as the circles radiuses, the distances between circles, the displacement between the overlapped patterns in X, Y, Z coordinates. All these parameters values are controlled by a numeric slider tool. Figure 8

shows the definition used and examples of different shapes obtained by only changing the values of patterns displacement.



Figure 8: A screen shot for different obtained patterns using Grasshopper

4.1.3.2 Scanimation

A Scanimation is an animation technique invented by Rufus Butler that involves two layers: the bars (or "scanlines") and the background image. By splitting the frame of a potential animation up, according to the size of the bars, each image is fully revealed between the bars, in order, as the bar layer is moved horizontally over the background image. No computer chip is involved, only an acetate overlay, made up of stripes, flowed over a patented, scrambled image underneath, getting the fluidity of motion¹⁴. Figure 9 shows an example of this technique.

This idea is a part of the research interest but it's still in progress, trying to apply it on ETFE layers replacing the horizontal movement of the two layers (to get the fluidity of motion) by the movement of users themselves, walking under or in front of the cushion. In order to get the motion effect, specific distances should be kept between the two layers: it can be represented by the air chamber of the cushion. Specific distances, pattern area and number of frames should be recognized in a formula in order to get the clearest motion of the scene.



Figure 9: some shots for a scanimation pattern¹⁵

4.2 Sun-shading envelopes based on Organic Photovoltaic

4.2.1 Flexible Organic Photovoltaics

Due to the international orientation towards the use of renewable energies and the fact that harvesting solar energy using a photovoltaic panel costs more than burning fossil fuels pushed the researches to focus more on the organic solar cells. The OPV cost savings come from the possibility of using flexible substrates, printable organic inks for the active layers, low temperature and ambient pressure fabrication, and reduced materials costs¹⁸. However, the efficiency of the fabricated cell is not the target of the research. The research innovative idea is using fluoropolymers as substrate and encapsulating material and also replacing the typical ITO cathode with Carbon Nano-tubes (CNT).

As indicated in fig. 10, organic solar cells function as follows: the light absorption produces excitons, electron-hole pairs that are bound together and hence not free to move separately. To generate free charge carriers, the excitons must be dissociated. This can happen in the presence of high electric fields at the interface between two materials that have a sufficient mismatch in their energy levels. Thus, an organic solar cell can be made with the following layered structure: positive electrode/electron donor/electron acceptor/negative electrode. An exciton created in either the electron donor or electron acceptor layer can diffuse to the interface between the two, leading to electron transfer from the donor material to the acceptor, or hole transfer from the acceptor to the donor. The negatively charged electron and the positively charged hole is then transported to the appropriate electrode.



Figure 10: a graph indicates how the Organic Solar Cell is functioning and the SOFT-PV contribution

4.2.2 Fabrication of an OPV Cell based on CNT on ETFE Substrate.

The fabrication of the OPV cell is passing by three different phases. The first is the deposition of the Carbon Nano-Tubes and the PEDOT:PSS, the second is the deposition of the organic active layer PCBM/P3HT and the third is the deposition of the Aluminium electrode. These three phases are explained as follows:

a. First Phase - Deposition of the CNT & PEDOT: PSS:

Continuous CNT network in a film could offer a new class of transparent conducting materials which complements ITO for applications such as organic light-emitting diodes and organic photovoltaic (OPV) devices. CNT films are superior to ITO in terms of flexibility because the former can be bent to acute angles without fracture. In addition, although carbon is the most abundant element in nature, the worldwide production of indium is limited, which may soon find difficulty meeting the ever-increasing demand for large-area transparent conductive electrodes. The decreasing resources of crude indium induced a ten-fold price increase between 2003 and 2007¹⁶. Furthermore, CNT films may offer additional advantages such as tuneable electronic properties through chemical treatment and enhanced carrier injection; moreover CNT coatings can be produced at room temperature and do not require vacuum procedures. In this project we aim to create by electro spinning a 2D network of fibers deposited onto glass substrate. The same procedure will be tried using fluoropolymer substrates and CNT will result to be embedded into the electros pinned polymer. Polymeric lattices prepared by radical copolymerization of the monomer could offer the possibility to obtain the nanotubes directly embedded into the ETFE material. After this layer, a coating with Poly(3,4 ethylendioxythiophene- poly(stryrenesulfonate) (PEDOT:PSS) is applied.

b. Second Phase - Deposition of the active layer (PCBM/P3HT):

The realization of the flexible PV cell using P3HT and PCBM as donor and acceptor materials respectively, follow standard procedures, since the goal of the present project is not the specific performance and efficiency of the PV cell, but its fabrication using fluoropolymers as substrate and encapsulating material and its integration into the building component. Deposition of the active layer is based on the bulk heterojunction concept where the donor material (typically a polymer) is mixed with an acceptor (a soluble fullerene) in an organic solvent and then deposited in a spin coating device (fig.11).

c. Third Phase - Deposition of the Aluminum Electrode:

Finally, the Aluminum electrode is deposited using a vacuum evaporation device (fig.11) by putting the cells in a metal perforated template which is pre-designed according to the cells dimensions. The machine should be air-vacuumed before the evaporation starts then the process begins by increasing the temperature till the Aluminum evaporates and the molecules stick to the whole entire surface including the cells samples. After the deposition finishes, the final action is to stabilize the air pressure again.



Figure 11: Spin-Coating device (left) and Metal Evaporation Device (right)

4.2.3 Experimental data concerning the optical properties of the fabricated cell

The optical and optoelectronic characterization targets all the intermediate steps and the final product of the research. Transmission, absorption and reflectivity is measured on the new thin film electrode, on the organic active layer, on the final PV cell and will be measured on the final integrated structure. The goal is to assess the optical and consequently the optoelectronic performance of the device structure. In particular: The transmission, front and back reflections of the new flexible fluoropolymeric ETFE substrate deposited with the modified electrode, Carbon Nano-Tubes using a standard Spectrophotometer.

The light transmission is measured for three samples: each corresponds to a specific phase of the previous three deposition steps. As indicated in fig. 13, the first sample represented by the blue line corresponds to the deposition of the Carbon Nano-Tubes + PEDOT:PSS. Sequentially, the second sample represented by the red line corresponds to the deposition of the CNT + PEDOT:PSS + the active layer PCBM/P3HT. The third sample is not included in the transmission measurements as the aluminium layer has a transmission of zero value.



Figure 13: light transmission graph of the two samples; above: ETFE+CNT, below: ETFE+CNT+PCBM/P3HT

5 SOFT FUTURE WORK

5.1 The approach to design an innovative facade component

This part of SOFT research will focus on the creation of printable graphic motifs by integrating a CNT based organic cell into different ETFE layers of a building component, to preview all problematic aspects of the fabrication process. A demonstration prototype of this new kind of smart façade will be created by the integration of the new kind of SOFT-PV cells into an air-supported multi-layer fluoropolymeric building component trying to study the degree of flexibility of integrating OPV cells as units for various patterns.

5.2 Mechanical, environmental and LCA evaluation of the final prototype

First, this model of a new active building envelope (smart façade, smart transparent roofs) will be tested to investigate the mechanical behaviour of the new prototype. Second, the

thermal and the optical behaviour of the cushion will be simulated for different scenarios (orientation, time & season) of a building model. Finally, the life cycle assessment of the SOFT cushion with the integrated OPV has to be evaluated. On material scale, the life cycle analysis of all materials involved in this manufacturing process will be developed: the material inventory of the full process for a module production, the accountability of the energy embedded both in the input materials and in the manufacturing processes. Also, a computation of the energy payback time will be evaluated in order to compare this technology in progress with the current ones. At the scale of the cushion system the analysis of the environmental profile of the envelope technology will be carried out on the whole life cycle. The durability of the involved material and of the efficiency of the OPV cells has to be considered and compared.

6 CONCLUSION

Going through the research different phases, some problems and constraints were investigated by the researchers. Parts of them are related to the technical properties of used technologies such as The Spin-Coating device. Since the device technique is based on spinning the substrate while pouring the active layer, there are some restrictions regarding the maximum area to be covered as the more the substrate area the less homogeneous the deposited layer is.

During the service-life of a SOFT cushion some aspects would be deepen and previewed: the durability of the involved material and component and the efficiency of the OPV cells have to be considered and compared, in order to define the management and substitution cycles. In the design phase the compatibility of the durability between the involved materials and components has to be taken into account, in order to design the easy reversibility of each part, especially the OPV cells.

In the life cycle approach the end of life aspects has to be considered also in the design phase: actually the substrate ETFE is recyclable and a recycling chain is just operating. In Germany and in Italy commercial opportunities already exist to take the production waste or the "old" used ETFE and then to re-process them and close the cycle¹⁷. Actually one limit of the recycling practice of the ETFE is related to the printed foils, a technology to separate ETFE from ink has to be found. The same limit belongs to the SOFT cushion: how would be the end of life cycle scenario of the ETFE with the encapsulated OPV cells? Huge work has to be carried out, in order to improve the system during all the life cycle phases.

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LIGHTWEIGHT PHOTOVOLTAICS

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Key words: Lightweight Photovoltaics, Organic Photovoltaics (OPV)

1 INTRODUCTION

Lightweight structures and Photovoltaic are two elements in the building industry with hardly any relations to each other. Beside the rigid silicon based panels a new generation of thin and flexible panels will be ready for the market and will present useful qualities for tensile applications.

One of the main targets of tensile structures is the minimization of the materials used in their construction. It is my intention to combine this ecological factor with one of the biggest challenge of today: generating energy with consideration for our environmental responsibility. To build ambitious and mass reduced architecture with intelligent engineering and smart power generation.

2 PHOTOVOLTAIC TECHNOLOGIES TODAY

2.1 What are Photovoltaics?

Generally Photovoltaics (PV) means the technology of generating electrical power by converting solar radiation into direct current electricity by using solar cells to convert energy from the sun into a flow of electrons.

Today the majority of photovoltaic modules are used for grid connected power generation. An inverter is required to convert the DC to AC. Beside that off-grid applications are used to power small electronical equipment or to recharge batteries. Solar power is pollution-free during use.

Apart from thin-film modules, silicon cells and modules represent by far the largest

segment of solar cells and modules available on the market.

2.2 What different types of materials and systems are available?

Several materials with different qualities and efficiencies are used today for photovoltaics:

- Monocrystalline silicon
- Polycrystalline silicon
- Thin-film
- Amorphous silicon
- Cadmium telluride, and copper indium selenide/sulfide.
- Organic Photovoltaics (named as the "Third generation" in PV)
- Other systems with small value for the actual markets



Technology	Encapsulation	Weight
crystalline	glass	heavy
Thin-film	glass/PET foil	heavy/light
OPV	PET foils/ETFE foils	light

Crystalline wafers are mainly encapsulated in glass to stabilize their fragile structure. The silicon for thin-film modules is evaporated on glass or sometimes on PET. Both, crystalline and thin-film modules are not very useful in applications regarding lightweight and pliability.

3 CRYSTALLINE SILICON TECHNOLOGY



Very thin and shock-sensitive Wafer with mono-crystalline Silicon Technology by BOSCH®

These amorphic Silicon plates, called "Wafer" are very sensitive to any loads and need also to be covered with solid glass to avoid any contact with water or even air moisture.



A solar module is made of an array of solar cells (a panels with 60 cells is standard) encapsulated in glass and water- and weather-proof foils stabilized by a ridged aluminum frame.

These modules are heavy and need to be mounted to a riged steel structure on a certain angle towards the sun-position at noon, when the sun has the best light emission.

4 FLEXIBLE PHOTOVOLTAIC MATERIALS

Some of the various technologies are able to be used on flexible and less shock-sensitive solar panels. This allows them to be used on irregular curved surfaces of tensile membrane or cable structures.

Less information is available concerning the strength of the panels: how they will react under tension load with panel capacity while still generating power.



Global Solar® Copper Indium Gallium DiSelinide (CIGS) thin film solar material

Global Solar Energy has evolved into a major producer of thin-film photovoltaic Copper Indium Gallium DiSelenide (CIGS) solar cells. Global Solar is the leading manufacturer of CIGS thin-film solar on a flexible substrate.



These panels are developed for direct on-roof installation


Flexcell® Module with flexible amorphous Silicon (aSi) Thin Film Technology

Flexcell manufactures flexible PV modules using amorphous silicon ("a-Si"), which has various advantages over glass-based modules: lower material cost per Watt peak ("Wp"), wider application versatility. Beside direct-on-roof applications other uses are possible:



Ackermann + Partner, Munich, AMW Carport Munich, under construction.

In this roof application the solar panels are installed to the ETFE foil cushions fixed on the internal layer: generating shade and energy.

Scientists at Empa, the Swiss Federal Laboratories for Materials Science and Technology, have improved on their previous flexible solar cell and achieved an efficiency of 18.7 percent.



Flexible copper indium gallium selenide or C.I.G.S. solar cells are still an emerging field. But a team of scientists, led by Ayodhya N. Tiwari, have worked to improve the material's efficiency.



5 ORGANIC PHOTOVOLTAICS (OPV)

Semi-transparent and flexible POWER Plastic® Solar module from KONARKA®

Organic Photovoltaics (OPV) is a complete new technology in the photovoltaic industry and is beginning to be used commercially as the third generation in photovoltaic development. In contrast to silicon-based solar panels here the photo-reactive layer is based on (organic) carbon polymer. These thin and flexible solar panels will allow completely new uses and applications.

Together with the ability of generating power, the tensile OPV structures will be of evident environmental importance for our future.



OPV meets the basic targets of light weight structures: mass minimization:

- 1. Less than 1 gramm of the power-generating polymer material is necessary to produce one square meter of a solar panel.
- 2. The printing or vacuum-deposition process needs less energy and production temperature and makes OPV a sustainable product.
- 3. Less weight and transportation volume make OPV also easy to handle and transportation
- 4. The optical absorption coefficient of organic molecules is high, so a large amount of light can be absorbed with a small amount of materials.



Heliathek® Panel with flexible Organic PV based on vacuum-deposited small molecules

A photovoltaic cell is a specialized semiconductor diode that converts visible light into direct current. Organic photovoltaics (OPV) does so using organic molecules as light absorbers. These organic materials are engineered to have the correct band gaps to absorb most of the solar energy. The field of OPV has two main branches, the route of vacuum-deposited small molecules, and the route of usually wet-printed polymers.

Heliatek is a technological leader of vacuum-deposited small molecule OPV. First products ready for the market are expected end of 2012.

6 APPLICATIONS WITH OPV



Lightweight Photovoltaic tensile roof by Solartension® for SIEMENS outdoor working space



Tensile Structures using OPV Modules for a Solar Roof, designed by Solartension

Organic solar cells will achieve very low production costs when produced in high volumes, since they require only very small amounts of material. The production processes require less energy and thus lead to a significantly reduced energy payback time.



Tensile OPV façade for GREEN BUILDING VIS® in Hörbranz, Austria by Solartension



Vertical Solar sail at INTERSOLAR fair booth of LAPP group by Solartension



SOLARTENSION detail with structural and electrical cable connections

TEXTILE MEMBRANES FOR CASE OF EMERGENCY FLOOD PROTECTION

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Key words: environmental technology, environmental engineering, flood protection, technical application for membranes, water filled bodies, water filled container, envelope material, mobile flood barrier.

Summary: With this document you get an overview about textile membrane structures used for flood barriers in case of emergency. This project creates a connection between membrane construction and environmental technologies. It is a part of the German ZIM project of the government department. Insights of mobile flood protection using membrane structures are delivered. Two different kind of non rigid structures and their basically functions are described. A short overview about the selection procedure and requirements of the material is given. The simulation process is mentioned and serves insights about the form finding of water filled membrane structures and statically analyses. A short description of the system testing is given on the end.

1 INTRODUCTION

Nowadays more and more flooding appears which contain al low level of lead time and is not estimated on several locations. So it is an advantage to employ new recovery systems as mobile flood barriers. The needs of such a system are a short term implementation without heavy devices and a fast built up time. Such barriers are called none planned and in case of emergency barriers. This means a flood is more rising up then usual and the fixed barriers (for example dikes) are in the process of fail. The unplanned barriers assist for a short period the fixed system. With those Systems a maximum flooding level of 70 cm is allowed and there has to be no research of the consistence of the floor before.

Together with the University Leuphana of Lüneburg [1], the companies Optimal GmbH [2] and Daedler [3] and the University of Applied Science Munich [4] a research group was founded and were placed in a ZIM koop project of the German government [5]. The aim of the project is to realize an unplanned barrier out of membrane structures. The concept is to use a membrane container without any rigid structures. The stability is given by the connection of internal water pressure and the thin walled membrane structure.

The part of this project of the University of applied Science Munich/Karlsruhe Institute of Technologies is focused on the material behavior, the statically analysis and the structural properties under water pressure of such barriers.

2 SYSTEM DESCRIPTION

The system consists out of an envelope of membrane material, without any rigid structure. The barrier body is reached by filling this envelope with water. So you get a heavy structure and the end shape is given by an interaction of internal hydrostatic pressure and the cutting pattern. The principia of the system is the hydrostatic pressure, the water level of the barrier has to be higher than the flooding water level. The dam level shouldn't increase more than 70 % high of the head of water inside the barrier [6]. Otherwise the difference between the resulting loads is getting to low and the barrier receives too much buoyancy. Thus the friction force will be undervalued to hold the system.

Two different systems were observed. These thin-walled membrane constructions filled with water differ especially in the primary shape of the cross-section. The one system has a trapeze cross-section and the shape is enhanced by membrane stiffener walls. The other one has a circular cross-section and is attached to a second smaller cylinder to avoid rolling. (Figure 1)



Figure 1: Sketch of the systems with a trapeze and circular cross-section

The shape of the trapeze cross-section is marginal changing by the influence of the inner hydrostatic pressure. The circular cross-section changes into the shape of a water drop and looses about 20 % of the high given by the diameter in fact of normal filling. This means with an overpressure of 7000 Pa. The trapeze system was never used by an overpressure.

3 MATERIAL SPECIFICATIONS

A range of membrane material was observed to use it as the envelope. For this a profile of requirements was prepared. Flooding water has to be assumed as "contaminated". The membrane should be resistant against chemicals, UV radiation, oil and water. Since the barrier will be pulled over different surfaces during operation a certain abrasion resistance must be given. To ensure the utilizability of the system without being mounted to the ground, the friction coefficient with .respect to different surfaces, especially to wet surfaces, is important. The material should be easily in processing and cost efficient. The main criteria for the material are the structural requirements resulting out of different hydrostatic load cases.

Several testing procedures were applied to achieve the required results of the different materials. The resistances against environmental influences were dispute with the manufacturer. Also concerning to the proceeding and cost efficiency it results a PET fabric coated with PVC/PU.

Another decision criterion was the friction coefficient of the material on operation surfaces, for this the testing procedure DIN EN ISO 8295 [7] was used. Potential membranes where tested on:

- green dry/wet
- concrete dry/wet
- asphalt dry/wet

The table (Figure 2) shows the determined result of four different PET fabrics coated with PVC.

	friction coefficient			
	material 1 μ	material 2 μ	material 3 μ	material 4 μ
green	0,53	0,53	0,61	0,63
green wet	0,66	0,62	0,65	0,68
concrete	0,66	0,64	0,69	0,64
concrete wet	0,51	0,53	0,52	0,52
asphalt	0,77	0,75	0,79	0,81
asphalt wet	0,77	0,77	0,79	0,80

Figure 2: friction coefficients of PET fabrics PVC coated

The most significant criterion was the stiffness of the material. The membrane has to hold the stress out of the internal pressure and additionally the different load cases given by the BWK [6]. So the different materials were analyzed by uniaxial tensile testing under consideration of the DIN EN ISO 13934 [7]. Also different welding seams had been investigated, at this high frequency and hot air seams had been used. As a result of this test it derived that the seams don't decrease the stability of the material. The tearing was tested on the same way.

For determining the material properties for the simulations the most valuable material was loaded in a biaxial tensile testing application.

Out of the operational demands and requirements it appears that a safety factor of 2.5 is an adequate value.

4 SIMULATION

A part of the simulation was the form finding. This means how the end shape appears under the internal hydrostatic pressure. The structure of the barrier has no rigid body and is an interaction between membrane and the hydrostatic pressure. This complex behavior demands a geometrical non linear calculation of the force equilibrium. An adequate tool is the program "Easy" by the Technet GmbH [8]. It is intended for textile roof form finding and stress analysis. The theory is based on a cable grid and calculates with force density.



Figure 3: iteratively increase of the internal load

For the adoption on the water filled flood barriers that is a non common problem, some extra tools had to be integrated. A contact algorithm was implemented. One point was the "floor" contact, to transfer this physical behavior in the program the internal pressure was iteratively applied and step wise increased (Figure 3). During these steps the algorithm controlled the element nodes which move below the z-axis and force them back onto the floor plane (figure 4 Pic. 1). So the geometric is changing and the equilibrium has to be recalculated that is achieved by the geometrical non linearity of the program. With this effect it is also possible to simulate rolling of the water filled cylinder.



Figure 4: floor contact and rolling

Because this kind of simulation was a new operation field for the program verification experiments had to be done to improve the virtual conditions. For this purpose a water filled membrane cylinder was measured by the laser grid distance method. The original cylinder diameter was 0,5 m and was filled with an overpressure of 7000 Pa on the top. The blue triangle in figure 5 describes the head of water that was applied. The simulation (black line) was done with the same parameters and reflects accurate the measured shape (red line). Under consideration of error in measurement matches the simulation excellent.



Figure 5: comparison of laser distance measurements and "Easy" simulation

After the form finding the algorithm was used to improve and optimize the stability and failure under different load cases and combinations. The required loads for the following analysis are given by the German BWK [6]. The considered loads for the analysis are:

- inside hydrostatic pressure (Figure 6 Pic. 1)
- internal over pressure
- external hydrostatic load (Figure 6 Pic. 2)
- external hydrodynamic load
- wave pressure (Figure 6 Pic. 3)
- impact (Figure 6 Pic. 4)



The structural analysis with respect to the different load cases was done with volume consistency. The enclosed water was considered as incompressible. As a result of a load case the structure has different force equilibrium, so the cross-section shape is changing geometrically while the case (Figure 7). The results of the simulation also reflect the tensile stresses in the membrane.



Figure 7: deformed shape and statically analysis results

Local stress peaks are decisive for the tensile stress calculation and were compared to the material tests regarding the safety factor. These peaks vary in their location and dimension regarding the different load cases. This is due to the fact, the geometry of system is totally changing, and therefore the load imitation into the membrane is completely different. So it is possible that smaller but similar load cases create other stress peaks. With this the iterative increase of a load case could be an advantage.

During the time of the project both systems were modified and emerge a large amount of versions. An initial selection had been chosen by simulating and much system testing time was saved by this.

Nevertheless there were still an amount of versions which had been tested. For this the Technicians in heavy-duty fabric needed several cutting pattern of the parts. This was partly made with the program "Easy" as well. At the circular cross-section system it is important for corner elements to avoid bigger angles than 22,5 °. Otherwise the cross-section of the corner increases to such an extent that the load distribution is so different by changing the geometry and the barrier high level decreases on that point.

5 SYSTEM TESTING

Several versions of Prototypes had been built and tested in field experiments. During the project the different versions had been tested in an area at the banks of the Elbe River near by Hamburg, where the river has a good defined tidal range. So the water tightness and the stability could be investigated on a basic level. This field experiments served a good wealth of experience by optimizing the construction and handling. More specific test on the aim of the different load cases and defined water tightness was made at a water channel of the University of Hamburg Harburg [9].

6 CONCLUSION

The construction of mobile flood protection with textile membranes affords new and advantageous options. The investigations show the structural behavior of water filled flood barriers and serve the possibility of optimization regarding cutting pattern, stability, stiffness and economical aspects. Geometric non linear calculation methods generate accurate results of water filled membrane structures without rigid structures. This was verified on a first way. The products already ready for the marked and the next step will be the certification. For further research the dynamic behavior has to be mentioned, like slushing. Rules for mobile flood protection in case of emergency have to be more prepared. At the point of statically the structural concept has to be more discussed on the base of safety factors.

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NUMERICAL INVESTIGATIONS FOR AN ALTERNATIVE TEXTILE INVERTER BUILDING IN THE AREA OF SOLAR POWER GENERATION

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Key words: Textile composites, environmental technologies, photovoltaics, cooling, converter, latent heat, radiation, convection, phase change material, CFD, inverter building, numerical methods, power generation

Abstract. Within electrical power generation by solar applications such as photovoltaics, the direct electric current is transformed into alternating current by converters. By the conversion of the current, heat is released from the electrical equipment into the building where the converters are placed. The building itself is made from concrete. Hot air (mediterranean area like Spain) sucked from the ambient by the converters into the building is the only heat sink. Water is not available. By the heat release from the inverters the indoor temperature increases up to 50° C which decreases the convective cooling effect for the inverters. Furthermore during a day cycle the concrete is heated too and the heat can not sufficiently be released from the concrete during the night.

To avoid this critical thermal situation an alternative inverter building based on textile structures (membrane) has been designed. The indoor climate and energy balance of textile buildings depend highly on the optical properties of the roof's membrane material. The heat capacity and heat resistance can be neglected (no storage effects). So the radiation becomes dominant for the thermal situation in the building during day and night. By covering the textile material, to ensure specific optical properties, the thermal impact from outside can be minimized and heat release during night to the sky becomes possible. To validate the concept, a complex three-dimensional numerical model has been set-up including heat sources inside the building. To prohibit temperature increase during the day a passive cooling system based on phase change materials [2] has been integrated into the model. Based on the numerical model a time dependent computation of the thermal behaviour of the textile building has been performed including irradiation of the sun (solar, diffuse). The influence of the wind on the building has been modeled, too. The results of the simulation show that the complex thermal transport mechanism for textile buildings can be well predicted by the used computing technology (CFD). Furthermore it can be demonstrated that the concept of the textile building can be a good alternative to a building made of concrete.

1 Introduction

Photovoltaic systems need beneath the solar panels an additional equipment like converters which transform the direct current into alternating current. Thereby heat is released by the electrical components of the converter. Therefore they must be cooled. This is mostly realized by air which is conducted through the converters. The converters are sucking air from the interior of the building. The buildings have openings to ambient. Therefore air from ambient is indirectly conducted into the building. The ambient temperature in hot areas like Spain may be above 40°C, so the temperature gradient is not sufficient to cool the inverters effectively. The concrete is heated by the inverters inside and by thermal solar load from outside. During the night the heat stored in the concrete is partially released into the building. The building itself stores most of the heat until the morning. Therefore the interior air temperature in the building is not decreasing much. The new current production cycle starts in the morning with a room temperature of nearly 30°C, whereas the ambient temperature is below 20°C.

To improve the thermal situation inside the building a new innovative concept was developed and thermally analysed by numerical computations. The alternative concept for the building is based on textile materials instead of concrete. The approach of textile materials for walls and roofs leads to an improved heat transfer to the environment during the night. The reason for this is that textile materials do not store much energy and have a good heat transmission by radiation. The benefit of this effect helps to reduce the temperature of structures and air inside the building during night. Furthermore textile materials can be made with appropriate surface properties to achieve good radiation reflectivity during day, so that the heat impact on inner structure components is reduced. The numerical results show that the thermal problems in inverter-buildings can be solved using a design of a coated fibre cloth shell and an integrated latent heat cooling system.

2 Thermal behaviour of an inverter-building made of concrete

Mostly two inverters are placed in a special building. The walls of the building consist of concrete. Also, there is a cellar embedded in earth for better installation of wire leads. Interior room and cellar are seperated by a medium density fibreboard. In figure 1 (a) the building is schematically pictured in a sectional drawing. The walls of the building (concrete) are green shaded. Additionally in the figure the heat transfer processes are represented. Following heat transport processes occur:

- thermal conduction
- convection
- thermal radiation

In figure 1 (b) the measured time-depended temperature development of room and ambient temperature is shown. All measurement data in this paper is provided by Phoenix Solar AG. Furthermore the according solar irradiation is plotted (San Clemente, Spain). It can be seen that the room temperatur is about 7 K higher than the ambient temperature (within a time shift). The air in the building is heated up by the power loss of the inverters, the released heat of the walls and the sucked air from ambient.



(a) Thermal load and heat transfer processes

(b) Measured temperatures in a inverter-building placed in Spain, San Clemente (August 2009, 16^{th})

Figure 1: Thermal behaviour of an inverter-building made of concrete

The inverter-building made of concrete has pros and cons. A big con is the material concrete. Concrete has a bad thermal conductivity. The properties are tabulated in table 1. In comparison to concrete copper has a thermal conductivity of 400 W/m/K. As a consequence the thermal conduction through a wall made of concrete is very bad. Figure 2 exemplarily shows the steady thermal conduction through a flat wall. Because of the temperature gradient between the inner and outer face a heat flow through the wall arises

density	ρ	2400	kg/m^3
thermal conductivity	λ	2	W/m/K
specific heat	c_p	1000	J/kg/K

 Table 1: Properties of concrete [3]

(Eq. 1). The heat flow depends on the thermal resistance R, the temperature gradient $\Delta T = T_1 - T_2$ and the surface A. The thermal resistance is a function of wall thickness s and thermal conductivity λ of the material (see Eq. 2). For example an inverter building has a wall thickness of $s = 0.1 \ m$ with a thermal conductivity of $\lambda = 2 \ W/m/K$. The outcome of this is a thermal resistance of $R_{concrete} = 0.05 \ m^2 \ K/W$ (Eq. 3). Due to the bad thermal resistance the walls made of concrete store a part of the conducted heat by day. At night the walls partially conduct the stored heat to ambient and interior. To clarify the bad thermal conduction through the concrete a numerical example is used at this point.

A wall may have a surface area of $A = 1 m^2$. The temperature of the outer wall is $T_1 = 25^{\circ}C$ and the inner wall is $T_2 = 30^{\circ}C$. From this a temperature gradient of $\Delta T = 5 K$ results.

- If the wall is made of concrete $(\lambda = 2 W/m/K)$ with a thickness of s = 0.1 m, the heat flow amounts to $\dot{Q} = 100 W$.
- If the wall is made of copper ($\lambda = 400 W/m/K$) with a thickness of s = 0.1 m, the heat flow amounts to $\dot{Q} = 20000 W$.
- If the wall is made of PVC (membrane, $\lambda = 0.17 \ W/m/K$) with a thickness of $s = 0.001 \ m$, the heat flow amounts to $\dot{Q} = 850 \ W$.

The comparison shows that the heat flow through a wall made of concrete is very bad. This demonstrates that concrete stores heat during day because of the bad thermal conduction. Also the heat is conducted very slow to ambient at night. For this reason the air inside the inverter-building is not cooled effectively during the night. In figure 1 (b) it can be seen that the temperature inside the building is 7 K higher than the ambient temperature at night. A solution is to replace the walls made of concrete with a textile reinforcement (membrane). Membranes are very thin whereby marginal heat can be stored. Because of their coating membrane has special optical properties (transmittance, emissivity, reflectivity) which influence the thermal radiation. Based on radiation exchange membrane conduct more energy from inside to ambient day and night. The aim of the concept study is to use this special membrane properties to cool the interior (inverters) indirectly at night.



Figure 2: Thermal conduction through a flat wall (steady)

heat flow through a plate

$$\dot{Q} \equiv \frac{dQ}{dt} = \frac{1}{R} \cdot A \cdot (T_1 - T_2) \quad (1)$$

with the thermal resistance

$$R = \frac{s}{\lambda} \tag{2}$$

thermal resistance of a wall made of concrete (thickness 0.1 m):

$$R_{concrete} = \frac{0.1 \ m}{2 \ \frac{W}{m \ K}} = 0.05 \ \frac{m^2 \ K}{W} \quad (3)$$

3 Concept of a textile building

3.1 Membrane

Today there are many configurations of fibre cloth on the market like fibre cloth consisting of PET¹-fibre coated with PVC² or glass-fibre reinforcement coated with PTFE³. The main difference are the optical properties. In table 2 the properties of different membrane configurations are displayed. Thermal conductivity, specific heat or density of the textile structures are insignificant in this case because the membranes are very thin. The surface coating is affected by the optical properties (transmittance, emissivity and reflectivity) in the solar and infrared radiation band. For example ETFE-foil has a high transmission factor and Glass/PTFE has a low transmittance in the solar radiation band (see table 2). Because of the optical distinctions a study has been made to find the optimal properties of the membran coating for the alternative, textile inverter-building.

A variation of transmittance, reflectivity and emissivity of the membranes were accomplished in the solar and infrared radiation band separately. For the computation a simulation model for two-or-more-layered membrane roofs [6] based on the thermal net radiation method [4] was used. The variation of transmittance, reflectivity and emissivity has shown that the heat exchange between interior and exterior is influenced by the membrane coating. The membran of the alternative, textile building should reflect all of the solar irradiation. Therefore the reflectivity of the coating must be high and the transmittance and emissivity must be low in the solar radiation band. The transmittance in the infrared radiation band has to be large so that the thermal radiation is submitted

 $^{^{1}\}mathrm{PET} = \mathrm{Polyethylentherephtalat}$

 $^{^{2}}$ PVC = Polyvenylchlorid

 $^{^{3}}$ PTFE = Polytetrafluorethylen

to ambient. In addition to a large transmittance a low reflectivity in the infrared range occurs. Because of these criteria a glass-fibre reinforcement coated with PTFE is selected for the concept study.

spectral range		glass/PTFE	PET/PVC	PET/silicone	ETFE-foil
	ε_{sol}	0.22	0.36	0.28	0.05
solar	$ au_{sol}$	0.13	0.04	0.20	0.90
	$ ho_{sol}$	0.65	0.60	0.52	0.05
	ε_{ir}	0.82	0.91	0.90	0.60
infrared	$ au_{ir}$	0.02	0.01	0.02	0.35
	ρ_{ir}	0.16	0.08	0.08	0.05

 Table 2: Properties of different membrane configurations [9]

3.2 Membrane-building

The idea is to design a round membrane-building to minimize the surface area. Also, a round floor is innovative and forms a contrast to the straight solar panels of a solar power plant. Figure 3 shows different designs of a round membrane-building. The three different designs vary in the roof construction. The design in figure 3 (c) is chosen because water drains off and does not retain on the roof. Due to the design the inverters are placed at the center of the building.



Figure 3: Designs of the alternative, textile buildings

3.3 Latent Heat Storage (LHS)

Phase Change Materials (PCM) like paraffin have the advantage that the melting temperature can be fixed to a specific temperature by additives. If PCM's are melting

the latent heat stores energy applied by any heat source without significant temperature change. The stored heat may be released at a later time by any heat sink. This time shift can be used for inverters if heat is applied to the PCM during the day and the stored heat is removed during the night. As long as material is available for melting, the temperature is nearly fixed to the melting temperature. To develop a cooling system based on PCM, the main problem of low thermal conductivity of PCM's was solved by using metal foam. Metal foam exists as foam with open porosities and foam with closed porosities. The foam with open porosites can be flown through. This effect is helpful to fill PCM into the open porosities.

A prototype of a latent cooling system has been built in La Solana (Spain) in cooperation with Phoenix Solar AG. The cooling system was installed in the cellar of the inverterbuilding and consists of fifteen single storage modules. Each of these storage modules contains the same mixture of paraffin and metal foam and then have a optimized ordering scheme. Therefore a channel was designed to conduct the flow from ambient to the latent cooling system. The melting temperature of around $29^{\circ}C$ has been chosen. Figure 4 (a) shows the cooling system during the installation process. From the figure the single storage modules on the left side can be seen how they are embedded into the channel for air circulation. The temperature development in the building with and without cooling system is shown in figure 4 (b). The two buildings are placed at the same solar power plant.

Due to the good results of the LHS in Spain the same configuration of single storage modules is chosen for the concept of the alternative, textile building. Only the flow channel has to be adapted to the design of the membrane-building.



(a) LHS during installation in Spain



(b) Comparison of measured temperatures in buildings with and without cooling system

Figure 4: LHS in La Solana, Spain

4 Numerical analysis

4.1 Model

After having found a practicable design, a three-dimensional model for CFD (Computational Fluid Dynamics) calculations has been set up. The numerical model consists of two inverters (heat source), a latent cooling system, a membrane-shell and other elements. In figure 5 the whole numerical model with all components is represented. In the simulation a closed air circuit is concepted, because only the influence of the membrane properties are demonstrated. The surface area of the inverters is the heat source in the model. The surface temperature is fixed at 50°C for the first 12 hours (inverters are running). That is a worst case scenario. After 8 p.m. the inverters are shut down and have no fixed surface temperature any more.



Figure 5: Numerical Model of the textile building

In figure 5 the latent heat cooling system and the constructed flow channel is pictured, too. The configuration is the same as the prototype. The location of the LHS is under the floor assembly of the building. There is no extra cellar needed. The melting temperature is 29°C. The flow channel has an inlet and an outlet at the floor. Both are closed until the average temperature of 41°C is reached inside the building. It becomes clear that no connection to ambient exists. So the effects of the membrane properties can be analyzed. After all the paraffin in the cylinders is melted the LHS is shut down. That means that there is no heat exchange between the LHS and the air inside the inverter-building, because the openings are closed. The re-cooling of the LHS and the solidification of the paraffin is not considered in our simulation, because this must be realized by ambient air. The simulation model consists of two regions, a flow region (ambient atmosphere, air inside the building, air in the channel) and a solid region (paraffin-metal foam composit cylinders). By defined interfaces between the fluid of the interior and the fluid of the ambience (membran-interface) and between the fluid and the solid the heat exchange can be calculated. The used method to simulate the phase change is called method of Volume of Fluid. This additional equation (to the conservation equations) solves for the volume fraction of the liquid phase per finite volume. The flow around the single storage modules and in the other fluid regions have been solve for mass and momentum. As turbulence model a $k - \varepsilon$ model has been selected.

4.2 Results

The computation has been performed with the CFD-Solver StarCCM+ [8]. To evaluate the developed configuration a full day-night-cycle using the measured ambient temperature and the solar irradiation (San Clemente, Spain) as boundary conditions has been computed. Also, the different solar altitudes and the sun movement in San Clemente, Spain, has been computed for a full day cycle in August. In figure 6 the solar irradiation at the building surface is pictured for three different time stamps. According to the sun movement the solar irradiation is moving, too. In the afternoon the sun is standing low. Therefore the shadow of the building is quite long.



Figure 7: Temperature inside the building during the day

The temperature inside the building increases (see figure 7) until the afternoon up to about $45^{\circ}C$. The reason for this is the hot surface area of the inverters and the solar impact. During the night the heated structures are releasing heat by radiation through the membrane to the sky. Figure 8 shows the temperature drop inside the building during the night with the help of three time stamps. The room temperature decreases below $13^{\circ}C$ until the next morning because there is no heat source left inside (inverters shut down) and due to the radiation heat exchange with the environment.



Figure 8: Temperature inside the building at night

Figure 9 shows the temperature profile of the membrane roof and shell. Further the ambient temperature is plotted, too. The averaged temperature of the roof and the shell increases very fast. After three hours and 48 minutes (at 11:48 a.m.) the LHS is activated. The fan blows the cool air from the LHS channel into the room. So the surface temperature of the membrane decreases a short time. Because the radiation intensity and the elevation angle of the sun increases until the afternoon the temperature of the membrane surface increases, too. Also the hot inverter surface area influences the temperature of the membrane. The developing of the membrane roof and shell temperature depends on the ambient temperature (see figure 9). The maximum temperature of membrane roof and shell were achieved in the afternoon at 3:00 p.m.. After 12 hours (8:00 p.m.) the temperature of the membrane surface area decreases very fast because the inverters were shut down and the temperature of the environment drops down. Due to the membrane coating (optical properties) the temperature of the membrane surface is falling below ambient temperature at night. That can be traced back to the radiation exchange with the sky because the temperature of the sky falls below $0^{\circ}C$ during a clear night. Figure 10 summarizes the achieved results. The graphic shows the temperature gradient

of

- the room temperature of the membrane building with LHS cooling (case 1)
- the room temperature of the membrane building without LHS cooling (case 2)
- the room temperature of an inverter building made of concrete (case 3)
- the ambient temperature.



Figure 9: Area averaged temperature of membrane roof and shell

The temperature development inside the building is the same at the first two hours for all three cases. This can be traced back to the heated inverter surface area. After that time interval the room temperature of the membrane buildings continue increasing. In contrast the developing of the room temperature of case 3 is gently declined in comparison with case 1 and 2 because the concrete stores a part of the released heat. The LHS cooling system (case 1) is activated if the temperature inside the building exceeds $41^{\circ}C$. This mark is achieved after three hours and 48 minutes. In the graphic it becomes apparent on the sudden drop of temperature because cold air stored in the channel of the LHS gets into the building. Thereby the room temperature temporary falls below $35^{\circ}C$. The room temperature of case 2 and 3 increases above $45^{\circ}C$ until the afternoon. Due to LHS the air temperature of the cooled inverter-building does not exceed the temperature of $45^{\circ}C$ during the day. The temperature inside the textile inverter-building can be further decreased by a modification of the LHS. The cooling effect of the LHS can be optimised by a larger LHS or a different melting temperature.

After 12 hours (8:00 p.m.) the inverters were shut down and the ambient and sky temperature decreases. The paraffin of the LHS is completely melted at this time and the LHS is deactivated. These factors become apparent in figure 10. The temperature inside the building of case 1 and 2 quickly decrease within two hours. The developing of the room temperature of case 1 and 2 is similar again after 15 hours (11:00 p.m.). The decrease of the air temperature inside the building (case 3) is very slow in comparison with case 1 and 2. This can be traced back to the walls (thickness of 0.1 m) made of concrete because of the bad thermal conduction. At night the room temperature of the membrane building is 4 K less than ambient temperature. As conclusion it can be shown that the fibre coated reinforcement conduct more energy to the environment at night than applied to them. Equally textile structures do not store much heat because of their thinness. After a day-cycle of 24 hours in the morning the temperature inside a membrane building is $12.5^{\circ}C$ and inside a building made of concrete $25^{\circ}C$. Due to the special properties of textile structures the interior temperature of an inverter-building can be reduced about 12.5 K.



Figure 10: Volume averaged temperature inside different inverter-buildings

5 Conclusion

The concept study shows that a textile inverter-building of coated fibre cloth with integrated LHS solve the thermal problems of the current inverter-building. The numerical computation has shown that membrane-buildings have considerable advantages compared to buildings of concrete at night. Due to their optical and material properties and their low density they conduct energy to ambient at night. Thereby the temperature inside the building decreases after an operating day under ambient temperature at night.

From the numerical computation it follows that

- the room temperature is under ambient temperature at night
- the room temperature (membrane building) is lower than the room temperature of a concrete-building during the day and at night
- the coated fibre cloth conduct more energy to the environment during the day and at night than applied to them
- the integration of a LHS shows, that the temperature inside the building can further be decreased
- the membran structures do not store much heat

Because of these scientific findings a realisation may be possible. A prototype has to be build to test the functionality of the building for one year in a field test, because the simulation needs much time and there is no possibility to compute the whole circle of a year.

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Membrane structures with improved thermal properties

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Key words: Multilayered membranes, thermal active membranes, thermal behaviour .

Summary. Typical for single layer membrane structures is the less isolation against temperature and sound. A de monstration building is de veloped to demonstrate the possibilities of membrane structures being more environmental and sustainable for conditioned indoor use. The solar and therm al principles of the basis of the skin of the ice bear are converted in the technical solution of a membrane structure. A membrane structure is developed based on a translucent multi layer, double curved and pretensioned system with high therm al isolation properties. The demonstration building shows different solutions of membranes with improved isolating properties.

The structural task is the development of a multi layer, double curved membrane with layers of different stiffness, load carrying and environ mmental properties. The different solutions of coupled layers are designed and an alyzed. This includes the design of the membrane in relation to the different mechanical and thermal properties of the single layers as well as the mechanical connection of the layers. The main aspects are the position of the load carrying layer and the load carrying connection between the layers and the influence of the different tem - peratures of the single layers to the load carrying behaviour of the total system. Furthermore it needed to be examined the influence of enclos ed air if one solution has an inner and outer load carrying layer similar to an inflated structure.

The design steps towards a dem onstration building are presented which are divided into the examination of the membrane materials available on the market for the functional layers, analysing of different layouts of the functional membranes, design proposals for the demonstration building and detailing of the multi layer system including the thermal active membrane and thermal isolation. Advantages and disadvant ages of the different solutions are discussed considering air flow, load carry ing behaviour, detailing, long term behaviour and m ounting on site.

1 INTRODUCTION

The target of the presented design project is tied together the theoretical knowledge of the fields textile of materials, structural design of m embranes, mechanics of enclosed gas volumes and thermodynamics of heated gas flow, with the technological experience concerning energy technology, supply technology, connecting methods, cutting p attern and practical know-how with regard to m anufacturing new and efficient use of solar energy in mem brane structures.

The actuality of the design lies in developing ne w ways to the use of solar energy, m inimizing the use of fossil fuels for heating and cooling of buildings. The presented design is also suitable to rearm existing buildings with a lightweight second skin increasing the thermal isolation and having the potential of harvesting solar energy.

In the presentation a textile building is developed with all necessary structural components including a gas layer which can heat up with high solar irradiation on 100° C to 150° C is incorporated actively in the energy balance. This encloses the production of energy from the hot gas layer, preventing the solar radiation in the interior and cooling out in cold winter nights. The design is a cooperation of several companies and ITV Denkendorf.

ITV Denkendorf has developed on the solar-therm al functions of the polar bear's fur spacing textiles which connects the flexibility of a fibre-reinforced material with a translucent solar insulation. The knitted spacing f abric is able to let the sunlight though to an tex tile absorber which transfers visible and UV-light into hot gas. The gas is guided in canals to energy storage system, figure 1.



2 MATERIAL REQUIREMENTS

The design of a multi-layer membrane is examined which is used f or harvesting of solar energy and requires a higher therm al resistance than usual membranes in the textile a rchitecture such as coated fabrics or foils made of polymers. The multilayer system for harvesting of solar energy is called subsequently functional membrane.

For demonstrating the capability of the textil es a free span of app. 10 m is given of the south side of the building. The surface is double curved in relation to the necessary pretension stress of the membrane avoiding fluttering or snow and water sags. This includes that all layers are made of flexible membranes, should have the same curvature and being pretensioned. Among the layers are several which are strong enough to carry the dead load of the thermal isolation, wind and snow. Depending on the layout the load carrying m embranes needed to withstand temperatures up to 150 °C. For the load carrying layer fabrics of polyester and glass as well as ETFE foils are tested under higher temperature. The unidirectional tensile strengths of the polyester and glass fabrics lies between 60 kN/m and 100 kN/m and allows the span of 10 m. Differences between polyester and gla ss fabrics are the tem perature expansion and temperature resistance. Usual polyester fabrics have an im proved load-carrying capacity and suitability up to a temperature of 70°C. If polyester fabrics are used as load bearing elements, it should be made sure that the m aximum temperature on the m embrane surface lies under 70°C. Glass fabrics has a higher temperature resistance, the stiffness and unidirectional tensile strength shows are only little reduced up to temperatures of 150°C as result of tests carried out by the ITV Denkendorf. The absorber m embranes is chosen as a silicon coated glass fibre fabric

ETFE foils perm it a maxim um span of 1.0 m in mechanically pretensioned membrane structures and the location of the dem onstration building is in the surrounding of Stuttgart. The values of the load-carrying capacity and suitability is given up to a temperature of 70 °C on the foil s urface. If temperatures increases to more than 70°C in the functional membrane and the foils have, in addition, a load carrying f unction, investigations are to be carried out to get values for the unidirectional tensile strength and seam strength under higher temperatures.

The task needed to be solved is how high is the loss of pretension stress if the m embranes is heated up to a surface tem perature of 120° C. To grasp the influen ce of the temperature expansion on the load bearing behaviour, the following coefficients of tem perature expansions and Young's Moduli are assumed, table 1. Even for the silicon coated glass fabric the pretension stress is lost if the temperature increase of more than 40°C and the m embrane can only be used to carry the dead load of he system.

Membrane material	Temperature	Coefficient of	Youngs-Modulus kN/m
	range	expansion	warp/weft
ETFE-Foil	20 – 150 °C	$\alpha_{\rm T} = 13 \cdot 10^{-5} \ 1/{\rm K}$	280/300
Polyester fabric	20 – 70°C	$\alpha_{\rm T} = 7 \cdot 10^{-5} \ 1/{\rm K}$	800/1000
Glass fabric	20 – 150°C	$\alpha_{\rm T} = 5 \cdot 10^{-5} \ 1/{\rm K}$	900/1200

Table 1: Data of the assumed materials

3 DESIGN OF THE MULTI LAYER SYSTEM

The lost of pretension under higher temperatures for any type of membrane material results in a system of multi layer membranes of an outer and inner load carrying membrane and the functional membrane in between. For the layout of the multi layer system several solutions are developed and evaluated. The functional laye r for harvesting energy is designed as an ETFE foil on top, a black spacing polyester fabric and the black silicon coated glass fibre fabric. The layers above are to increase the therm al isolation with the constrain of having a high translucency. The layers below are to prevent heating up the inner space, the silicon/ glass fibre is hugh radiator and the heat should be ke pt in the spacing fabric. The differe nt systems had been tested in a test equipment to increase the air temperature in the spacing fabric under radiation of 400 W/m² to 800 W/m². The air flow is given between 0 m /s and 0,6 m/s. The layout which turned out be feasible for energy harvesting are shown in figure 2, the most favoured one is layout 2 concerning the layers above the functional layer. The thermal isolation in layout 1 reduces the light com ing to the abso rber and in layout 3 loss of te mperature is higher.



The test equipment has a span of 0.5 m and a length of 4 m and the targ et is to bring th is layouts to a free span mem brane structure of a pp. $10 \text{ m} \times 10 \text{ m}$. Possible layers to carry the load over the assumed span are coloured blue in figure 2. To earn very high profits from the solar irradiation, the ETFE foils which are permeable for 99% of the visible light and ultraviolet rays are suited. The disadvantage of ETFE foil is the less strength and the top layer is rein-

forced by additional polymer coated steel cables. The size and the distance of the steel cable is defined by carrying the external loads such as snow and wind. The distance to the next ETFE foil is given to avoid convection of the air in the gap on the one side and to prevent contact under snow on the other side.

Designing the absorber m embrane as load carrying element for a span of 10 m means to take into account the change of the sag under di fferent temperature ranges. It is assumed temperature drops to 10 °C in a cold winter n ight and increases to 150 °C in hot summer days. These temperature range causes a change of the length of 0,7 % for a span of 10 m which seems to be relative less. The change of the sag is related to the curvature of m embrane and a elongation of 0,7 % may increases the sag of the membrane, see table 2.

		Temperature	sag incl. temp.	difference
Span	sag	Elongation	elongation	in the sag
10,00 m	0,00 m	0,7 %	0,36 m	0,36 m
10,00 m	0,20 m	0,7 %	0,55 m	0,35 m
10,00 m	0,50 m	0,7 %	0,72 m	0,22 m
10,00 m	1,00 m	0,7 %	1,12 m	0,12 m

Table 2: Influence of the temperature elongation to the sag of a membrane

In case the layer above are connected to the functional layer, these layers have to be able to fulfill the same deformations. The two other so lutions are shown in layout 1 space between the absorber membrane to avoid contact or the ab sorber membrane lays on the thermal isolation and has no load carrying function, layout 3. In layout 1 the air gets hot between the absorber membrane and the isolation leads to an internal pressure because of the closed gas volume necessary for the required tightness. In layout 3 a solution needed to be found how to fix the functional membrane onto the inner membrane and tension the whole package.

The different polymers for the functional m embrane make it hardly possible to laminate the ETFE foil to the polyester spacing fabric and the spacing fabric to the silicon coating of the glass fabric. The connection of the m embranes to the spacing fabric is only possible m echanically either by using snap clips, clamps or air pressure. Developed are two solutions in relation to the air flow through the spacing fabr ic. For energy harvesting the functional lay er is divided into canals of a width of app. 0,5 m. Using a second layer of air above for thermal isolation this layer is also divided into can als. The ETFE foils and the absorber mem brane have to be clamped air tight every 0,5 m. To prevent lifting of the foil from the spacing fabric the air pressure in the second has to be little hi gher than in the spacing fabric. This causes a controlled air flow not only in the spacing fabric but also in the air chamber above. Depending on the stiffness of the different layers and assu ming an incompressibility of the enclosed air all layers are participating on the load carrying. The other possibility is suck ing the air through the spacing fabric and the negative pressure holds the foil and fabric to the spacing fabric. This requires an air pressure in the second chamber in case of a hold up in the air flow between the spacing fabric. Numerical simulations are carried out to know the change of the pressure and temperature to the deflections and stresses in the different layers, see figure 3.



Figure 3: Three chamber system with four layer membranes

4 DESIGN OF THE STRUCTURE

The design of the demonstration building is defined by following parameter:

- The functional membranes is given by an area of app. 10 m x 10 m
- The span of the 100 m² is without supports, pretensioned and double curved
- The functional membrane is oriented to the south
- The inclination of the surface is between 30° and 40° related to the sun radiation of the location
- The lower and upper boundary is rigid to connect the membranes to the air supply system
- The functional membrane is integrated in m embrane structure which encloses a inner space and can be used for exhibition and small conferences.
- Most of the roof and wall structures are designed as textile elements
- The parts of conventional elements such as glass facades are to be minimized
- The bending elements are as less as possible

Several proposal are designed and improved. Any type of saddl e shaped membrane has the disadvantage of integrating vertical facades or textile walls. Arches need either m ore space than required or the membranes are also orientated to east and west sides. Radial systems are less useful because the functional membrane has to rectangular shap ed with can als of the nearly the same width. Finally a solution com e up which fulfill most of the assumed requirements. The structure is based on two inclined arches and only the surfaces between the arches are made of rigid elements, the whole surface is made of textiles and foils. The outer membranes are pulled of the inner layers and are responsible for carrying the external loads. On the south side the inner part is transparent and made of ETFE-foils with cables. On the north, east and west side the outer membrane is a high translucent PCV coated Polyester fabric, the inner fabric is on all sides a open mesh PVC coated Polyester fabric which carries the thermal isolation. The functional membrane is of course tota lly opaque which requires a certain translucency of the other parts of the structure to ensure working conditions for the day see figure 4.

The membranes and the steel arches are desi gned for 0.85 kN/m^2 snow load and 0.5 kN/m^2 wind load. Numerical calculations are carried out for the entire structures including the vertical and textile facades. One of the m ain topics of calculations is on the south side the influence of the therm al behavior of the membranes itself, the change in deflection in relation to the temperature and the interaction of enclosed gas volume. The air value in the single canals in very small and the air is assumed a incompressible which leads to an load transfer trough all layers under external loads, which is depending on the stiffness of the layers changing in relation to the actual temperature in the different layers and membranes.



Figure 4: Design of a membrane structure for energy harvesting

5 CONCLUSION

- The design of membrane structure is present for harvesting solar energy
- The material chosen for the structure are discussed as well as the layout of the functional membranes is described
- The requirements for the structure are listed and the final design is shown.

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Development and Testing of Water-Filled Tube Systems for Flood Protection Measures

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Key words: Flood Control; Emergency System; Water-Filled Structure; Buoyancy, Field Tests, Laboratory Tests

Summary: This paper provides information on mobile flood protection systems in general and on water-filled tube systems for flood control in particular. Results of field tests and laboratory tests of prototypes developed in the research project HWS-MOBILE are described and recommendations for the use of water-filled tube systems are given.

1 INTRODUCTION

Floods are one of the most frequent natural hazards worldwide. According to the NatCatSERVICE database of the reinsurance group Munich Re approximately 38% of the total number of natural catastrophes since 1980 are a consequence of hydrological events like river floods, flash floods, storm surges as well as resulting landslides¹. The flooding caused by the Hurricane Katrina can be assumed as the flooding with the highest economical losses for more than 100 years amounting to US\$ 81 billion at the US coast^{2, 3}. The second most expensive flooding took place in China in 1998 with more than US\$ 30 billion at the rivers Yangtse and Songhua. In Europe, the flooding at the rivers Elbe and Danube in 2002 caused damages of US\$ 21 billion⁴.

Such major events cause public attention; however, a fact frequently overlooked is that small local events cause approximately 50 % of total flood damage⁵. To avoid fatalities and damage it is necessary to select the appropriate measure of flood control for a specific site. Even with technical measures it is not possible to eliminate the risk but it is obvious that taking precautions pays off.

Beneath dykes and floodwalls, mobile constructions are a solution for flood protection especially in densely populated areas where no space for permanent structures is available. In addition, permanent structures may obstruct heavily the view onto the water body. In these cases, mobile flood protection measures may be a solution to fit both requirements: protection in case of flooding and open access to the floodplain over the remaining time. Furthermore, mobile protective systems can be used as emergency tool against flooding in unprotected lowlying areas and for heightening of permanent flood protection structures in extreme events.

Mobile flood protection systems differ in material, construction, permanent facilities, and

available protection height. In the following, planning criteria of mobile flood protection and a systematization of different mobile protection systems are given. Project results of the research project HWS-MOBILE on the development and testing of water-filled tube constructions for the use in flood protection are shown in detail resulting in recommendations for the use of such construction in emergency flood control. The project HWS-MOBILE was conducted in 2009 to 2011 by the Leuphana University Lüneburg together with the Hochschule München and the business companies Optimal Planen GmbH Menden and Karsten Daedler e.K. Trittau.

2 PLANNING CRITERIA FOR MOBILE FLOOD PROTECTION

Considering the use of mobile flood protection systems, in particular safety-related aspects have to be accounted for. The mode of operation, construction and the applicable materials are dependent on available early warning time, static and dynamic loads from water level, waves, ice pressure and flotsam impact as well as physical stresses due to weathering effects and required protection height.

Beside the general stability with regard to static and geotechnical aspects, the risk of failure of mobile protection systems is mainly dependent on the possibility of a safe assembly of the system. Important parameters are available early warning time, number of skilled helpers mobilized in a short time as well as manageability of protective components even under bad weather conditions.

A strict assembly schedule is mandatory based on locally defined threshold values of forecasted water levels defining action steps. The assembly schedule of mobile flood protection must not leave to the discretionary power of the decision maker. All in all, a low failure risk of mobile flood protection can only be guaranteed, if technical components as well as administrative conditions are suitable designed.

Generally, the structural failure of mobile flood protection systems can be distinguished into five types:

- Sliding (also rolling)
- Tilting
- Failure of stability (due to poor layout, capacity overload, or vandalism)
- Leakage without overall failure
- Geotechnical failure

If the static friction between system and underground is not sufficient due to minor friction coefficient or small normal force in interaction with buoyancy, the system may slide in case of acting lateral loads from water levels, waves, currents, and wind. A special case of sliding is the lateral rolling of cylindrical constructions.

A system is in a stable position as far as its centre of gravity is lying normal above the contact patch. If the centre of gravity is normal above the tilting line, the position is unstable and the system may topple over due to smallest interferences if no additional fastening is existing. The steady position of a body is impacted by the geometry of the body itself as well as lateral forces due to wind (static / dynamic), hydrostatic water loads, and hydrodynamic loads from wind, waves, and currents.

The inner stability can fail in case of capacity overload and/or incorrect installation. Especially high puncture loads, e.g. due to flotsam impact, can lead to failure. Furthermore, mobile systems can fail due to vandalism, which can be encountered only by safeguarding of the system.

Leakages can occur especially at the underground contact area and lateral connection surfaces resulting from design aspects or incorrect installation. Minor leakages are normally acceptable whereas larger leakages with higher current velocities may soak the underground leading to wash out of soil particles at the contact patch and consequently to stability problems.

Geotechnical failure occurs if the system possesses no stable foundation, unstable slopes exist in the protection line or the safety against hydraulic base failure or erosion is not guaranteed.

3 SYSTEMATISATION OF MOBILE FLOOD PROTECTION SYSTEMS

Available mobile flood protection systems differ in material, construction, permanent facilities, and available protection height. In the following, a systematization of different mobile protection systems is given.

Mobile flood protection systems can be divided in stationary and non-stationary mobile systems, see Figure 1. Stationary mobile systems may be partly or completely preinstalled whereas non-stationary systems are fully mobile and can be installed on different locations. The systems may be sub-divided in wall, container, mass, and flap systems. Further information on the different mobile flood protection systems is given in ⁶.



Figure 1: Classification of mobile flood protection systems⁶

4 WATER-FILLED TUBE CONSTRUCTIONS

Like other mobile flood protection devices water-filled tube constructions can be distinguished in stationary and non-stationary systems.

4.1 Stationary water-filled tube constructions

Stationary, i.e. permanently installed tube systems, consist of a foundation made of concrete, a hull made of synthetic or rubber as well as the filling material water. For filling of elements pumps are in use, whereas a redundancy of filling technique is obligatory. In case of flood protective use of the system must be protected in idle time by a cover.

Permanently installed water-filled tube systems are mainly used as weirs in rivers (Figure 2). Up to now, no longer system lengths are realized in flood protection. Permanent installed tubes offer the opportunity of easy installation but investment costs for foundation, construction and coverage are high.



Figure 2: Construction principle of a permanent installed tube system (left) and an example of a water-filled weir in Marklendorf at the river Aller, Germany⁷ (right)

4.1 Non-stationary water-filled tube constructions

Non-stationary water-filled tubes are made of synthetics like a reinforced plastic liner and are filled with water. In most cases, an initial filling with air is required for alignment. For filling, special equipment is needed like compressors and pumps. Normally, no additional anchorage like mounting bars and end constructions are necessary and the fixation of the construction is done only by its mass effect. The use of ground spikes is often not possible as the ground might be asphalted or the soil might be water-saturated and is consequently too weak for the use of spikes.

Also a non-destructive installation at walls or buildings is advantageous and can often be realized by these flexible constructions, which are able to follow existing structures.
Nevertheless, it is important to avoid any gaps between flood protection tube and structure as these will result in water leakage and, depending on the erosion stability of the ground, wash out of soils and therefore instability of the overall system. The emergency use of protection measures always requires some scope for improvisation and it is advisable to keep at hand materials like rubber mats and filled sandbags to construct waterproofed wall connections.

Because of their flexibility, tube systems are able to follow an uneven ground, but problems occur if smaller gaps and joins exist as e.g. in case of paved surfaces. These potential water passages cannot be sealed by the structure itself. Laboratory tests executed within the research project HWS-MOBILE show that the use of foam rubber mats 6 cm thick (only charged by dead load) underneath the structure shows good sealing results.

Further substructures are normally not required, but the ground must:

- be stable to bear structural and hydraulic loads,
- offer a sufficient static friction between the system ground / tube or the system ground / foam rubber mat / tube to enable a stable position of the flood protection device and
- not show any sharp edges (in case a foam rubber mat is used underneath eventually existing sharp edges will not damage the tube material)

Difficulties may arise in the fixation of water filled tubes. Especially in case of dynamical loads, e.g. wave loads, cylindrical elements may roll aside and change their position uncontrollably. This can be avoided by e.g. the use of two cylindrical inner tubes and one cylindrical outer tube (Figure 3). Due to the friction between the two inner tubes as well as between the inner and the outer tubes the movement of the construction can be minimized.



Figure 3: Exemplified non-stationary tube system with two inner and one outer tube

Position control can also be achieved by a special shaping with internal reinforcements applied to the tube construction or the coupling of two tubes. The prototypes shown in Figure 4 and Figure 5 have been developed within the project HWS-MOBILE and have been produced by the involved companies Karsten Daedler e.K. Trittau and Optimal Planen GmbH Menden.

The length of the single element of water-filled tube systems varies from 5 to 60 m. By coupling the elements any system length can be achieved. The element joints must be largely water proofed, which can be achieved easily if the construction is designed that way that the single elements are pressed together by their own weight during water-filling.



Figure 4: Tube system prototype with inner reinforcements made by the company Karsten Daedler in Trittau, Germany (photo: M.W. Jürgens)



Figure 5: Tube system prototype made of two jointed cylindrical tubes made by the company Optimal in Menden, Germany (photo: M.W. Jürgens)

Large radius curves are easy to realize with water-filled tube systems. For smaller curve radiuses special angle or corner elements are required, see Figure 6.

The use of water filled containers saves material and personnel and enables a quick installation. Drawbacks are that the density of the filling material is identical with the density of the source of loading and buoyancy. Also horizontal shifting is a problem in case of high water levels bearing the potential of sudden failure. Laboratory tests on an even concrete ground with different water-filled tube prototypes (Figure 7 and Figure 8) show that a horizontal shifting of the structure occurs system-dependent already at a flood water level of 70% of the filling water level. In the laboratory tests the highest achieved flood water / filling water ratio was 97% applying a foam rubber mat underneath the tube structure to increase the



water tightness and the static friction.

Figure 6: Corner element of the prototype Optimal in a laboratory test of the research project HWS-MOBILE (photo: B. Koppe)



Figure 7: Laboratory test of the prototype Daedler within the research project HWS-MOBILE (photo: B. Koppe)

Several producers of mobile flood control systems (e.g. ^{8, 9, 10}) recommend the installation of plastic sheets on the waterside of the construction to decrease the hydraulic pressure underneath the system. This shall lead to a decrease of leakage and to an increase of the overall system stability. From theory, this assumption can only be valid if the plastic sheet offers 100% water tightness or if the underground is highly permeable to dissipate any available hydraulic gradient below the plastic sheet. In praxis, these conditions are not existent and a waterside placed plastic sheet will not increase the functionality of a flood protection system. This could be validated by executed laboratory tests within the project



HWS-MOBILE as leakage and stability measurements show no improvements by placement of a plastic sheet at the waterside of the structure (Figure 9).

Figure 8: Laboratory test of the prototype Optimal within the research project HWS-MOBILE (photo: B. Koppe)



Figure 9: Laboratory test of the prototype Daedler with a waterside placed plastic sheet on an impermeable ground within the research project HWS-MOBILE (photo: B. Koppe)

Beneath static loads also dynamic loads from waves, currents and flotsam impact have to be considered in flood control. Because of low dead load and related buoyancy problem, water-filled constructions are not applicable in regions with higher quasi-regular dynamic loads like waves. In contrast, flexible water-filled tube construction are able to sustain flotsam impact very well. The constructions have been able to stand flotsam weights of 1 ton and impact velocities of up to 2 m/s within laboratory tests conducted in the project HWS-MOBILE (Figure 10).



Figure 10: Flotsam impact test at water-filled tube prototype Optimal within the research project HWS-MOBILE (photo: B. Koppe)

Also the use of water-filled tubes during frost periods shows no difficulties. Freezing tests in the severe winters 2009/2010 and 2010/2011 within the research project HWS-MOBILE show that even after freezing periods of weeks to months no structural or material damages occurred at water-filled tube systems (Figure 11). The only problem is that water filled constructions cannot be drained during frost but must be remained installed up to the beginning of the thaw period.



Figure 11: Water-filled, sandbag loaded prototype Optimal after a several months lasting freezing period in January 2011, research project HWS-Mobile (photo: B. Koppe)

5 CONCLUSIONS

A variety of mobile flood protection systems are on the market fulfilling different security and manageability levels. Therefore, it is necessary to analyze properly the requirements and site conditions in every specific application.

Within the research project HWS-MOBILE water-filled tube systems have been developed and tested, both in field and in laboratory tests. It can be stated that the developed prototypes are able to withstand a hydraulic load of 70 to 97% of the filling water level, depending on the construction. As testing underground an even concrete surface was chosen with low static friction coefficient.

Furthermore, a loading by flotsam impact with a flotsam weight of 1 ton and an impact velocity of 2 m/s showed no structural or functional damages. In contrast, in field tests quasi-regular wave loads in conjunction with higher static water loads lead to sudden failure. These tests have been conducted at the shore of the river Elbe where mainly ship-induced wave loads have been applied.

Additionally, the theoretical assumption that water-filled tubes equipped with plastic sheets at the waterside of the construction show no better performances than plain water-filled tubes has been validated in laboratory tests.

It can be concluded from the experiences made in the research project HWS-MOBILE that properly designed water-filled tube systems can serve as an appropriate tool for emergency flood control up to a flooding height of 0.60 m^1 as the constructions offer the following advantages:

- Low consumption of resources
- Short installation time
- Small number of personnel required
- Deployable at different undergrounds without any destructive installations

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A SIMULATION MODEL FOR THE YEARLY ENERGY DEMAND OF BUILDINGS WITH TWO-OR-MORE-LAYERED TEXTILE ROOFS

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Key words: Optical Properties, Thermodynamics, Radiation Heat Transfer.

Summary. This document provides information and instructions for preparing a Full Paper to be included in the Proceedings of *MEMBRANES 2011* Conference.

1 INTRODUCTION

The indoor climate and the energy balance of textile buildings depend highly on the optical properties of the roof's membrane material. The main factors for heat transfer from the environment into the building and v. v. are radiation and convection – coming from solar irradiation and outdoor weather conditions - whereas thermal conduction in the membrane itself can be neglected. The simulation of energy balances for a textile building by computational fluid mechanics (CFD) is well understood, but results normally in large models and time expensive calculations. The determination of the yearly energy demand for heating and cooling of a membrane building by CFD is therefore nearly impossible.

Based upon an extended thermal net radiation method [1] and some simplifications we present here a simulation model for two or more layered membrane roofs, that allows to estimate quickly the yearly energy demand of a membrane building under realistic weather conditions. By this method it is furthermore possible to compare and optimize the optical configuration of membrane materials in an early stage of the design of a building.

2 OPTICAL PROPERTIES

The optical properties of a membrane material are characterized by its spectral absorptivity, reflectivity and transmissivity that vary over the wavelength. The emissivity of a material equals the absorptivity due to Kirchhoff's law. It is common in engineering to assume

that absorptivity and emissivity do not depend on the wavelength (gray body assumption), but examination of the measured spectral values of the optical properties of a typical membrane material (see figure 1) show the differences in two ranges of the spectrum:



Figure 1: Optical properties of membrane material over the wavelength

In the range from 2500 nm to 18000 nm the optical properties are nearly constant. This range corresponds with the mid-wavelength, long-wavelength and far infrared thermal radiation spectrum. The range below 2500 nm represents the range of the main part of solar radiation including UV radiation, the range of visible light and short-wavelength infrared radiation. For the analysis of the thermal equilibrium of membrane buildings as shown below these two ranges are distinguished and the optical properties are assumed constant in each range. The mean values of the optical properties are found by averaging the measured spectral data over the two ranges.

3 NET RADIATION METHOD FOR ENCLOSURES

To analyze the radiation exchange between the surfaces of a membrane building (membrane or cushion, walls and floor) the interior of the building is regarded as enclosure. If the roof is composed by a two-or-more-layered membrane or by membrane cushions, each interior volume forms an additional enclosure. The radiation exchange between the surfaces of these enclosures is quite complex, because radiation that leaves a surface area will be partially reflected and absorbed by other surfaces. The reflected part itself will be many times re-reflected by other surfaces. In the case of translucent materials there are additional effects when radiation is transmitted from one to the neighbor enclosure.

Fortunately it is not necessary to follow all this multiple reflections and transmittances when we use a net radiation method for enclosures [1]. For each surface area A_k we consider first the total incoming radiant energy $q_{i,k}$ per unit area of A_k . A part of this will be reflected or transmitted and does not change the energy balance of the surface element. The internal

energy flux which is supplied to the regarded surface by the volume the surface belongs to (e.g. by conduction) will be q_k . Other external effects like heat transfer by convection will be written as external heat flux $q_{ext,k}$. Finally the surface element at temperature T_k emits temperature radiation $\varepsilon_k \theta_k = \varepsilon_k \sigma T_k^4$. This leads to an equilibrium equation on each surface element A_k :

$$q_{i,k} - \rho_k q_{i,k} - \tau_k q_{i,k} + q_{ext,k} - q_k - \varepsilon_k \theta_k = 0 \tag{1}$$

On the other side the total incoming flux is the sum of all fluxes $q_{o,j}$ leaving the other surfaces A_j , weighted by the view factors $F_{j,k}$:

$$q_{i,k}A_k = \sum_j F_{j-k}A_j q_{o,j} \tag{2}$$

The view factor F_{j-k} shows which part of the flux leaving surface A_j is received by the surface A_k . From view factor algebra we get:

$$F_{j-k}A_j = F_{k-j}A_k \tag{3}$$

and hence:

$$q_{i,k} = \sum_{j} F_{k-j} q_{o,j} \tag{4}$$

Finally the total outgoing radiation flux $q_{o,k}$ leaving surface A_k results from the reflected part of the incoming flux, the temperature radiation of the surface and the part which is transmitted from the backside of the surface element if the surface is transparent:

$$q_{o,k} = \rho_k q_{i,k} + \varepsilon_k \theta_k + \tau_n q_{i,k^*}$$
⁽⁵⁾

The index k^{*} marks the backside of the surface with index k (if any). Putting all this together we get an equation in θ_k and q_k for each surface element of the enclosure:

$$\sum_{j} \left(\delta_{kj} - F_{k-j} \left(\rho_{j} + \varepsilon_{j} \right) \right) \theta_{j} - \sum_{j} F_{k-j} \tau_{j^{*}} \theta_{j^{*}} + \sum_{j} \left(\frac{\delta_{kj}}{\varepsilon_{j}} - F_{k-j} \frac{\rho_{j}}{\varepsilon_{j}} \right) q_{j} - \sum_{j} F_{k-j} \frac{\tau_{j^{*}}}{\varepsilon_{j^{*}}} q_{j^{*}} = \sum_{j} \left(\frac{\delta_{kj}}{\varepsilon_{j}} - F_{k-j} \frac{\rho_{j}}{\varepsilon_{j}} \right) q_{ext,j} - \sum_{j} F_{k-j} \frac{\tau_{j^{*}}}{\varepsilon_{j^{*}}} q_{ext,j^{*}}$$

$$(6)$$

Splitting the wavelength spectrum into two ranges, "solar" and "infrared" as shown above, two equations (6) for each surface area are necessary. The internal variable q_j has to be split into a solar and an infrared component ($q_{j,SOL}$ resp. $q_{j,IR}$). In the solar wavelength range the emissivity is neglected, so that the terms in θ_j will vanish. In the case of outside surface areas oriented to the sky we find the absorbed direct or diffuse solar radiation on the right hand side of (6) in the solar range.

4 BOUNDARY CONDITIONS AND EQUATION SYSTEM

Generally, equation (6) is valid for all surface areas of the enclosure, but the different types of surfaces of a membrane building (floor, wall, membrane / cushion) need different boundary conditions as shown below. So we have three unknowns θ_k , $q_{k,SOL}$ and $q_{k,IR}$ for each surface

area k of the membrane building to be analyzed on one side and two equations (6) on the other side. Hence there is necessary one boundary condition equation per surface area k to solve the resulting equation system.

Floor: Boundary conditions for floor areas could be constant temperature or constant heat flux in the infrared wavelength range. It is also possible to define a constant "earth" temperature and calculate the heat flux by conduction through the thickness and the heat capacity of the floor. Another possibility is to set the interior temperature to a constant value and to calculate the corresponding floor temperature by a additional heat balance of the air in the interior room of the building.

Wall: A boundary condition for wall surface areas can be defined by heat conduction through the wall regarding convection effects to the interior of the building and to the environment. Also an adiabatic insulation can be formulated setting $q_{k,SOL} + q_{k,IR} = 0$.

Membrane: The boundary conditions for membrane surfaces have to be formulated for a pair of membrane surfaces, because each membrane surface consist of a front and a backside. A simple condition is that the temperature of both sides is the same. This is realistic if the membrane layer is thin. For a heat insulation layer the condition equation is formulated as relation of internal (infrared) heat flux and front and backside temperatures. In both cases the internal heat flux composed of the infrared and solar parts of the front and backside must also be in equilibrium.

The structure of the equation system for each surface element follows:

$$\begin{bmatrix} solar \ equilibrium \\ infrared \ equilibrium \\ boundary \ conditions \end{bmatrix} \begin{bmatrix} \theta_k \\ q_{k,SOL} \\ q_{k,R} \end{bmatrix} = \begin{bmatrix} solar \ irradiation \\ convection \\ conduction \ or \ 0 \end{bmatrix}$$
(7)

The resulting equation system is linear in $\theta_k = \sigma T_k^4$, but the temperature dependent terms of convection and conduction on the right hand side of (7) have to be iteratively updated to get the final equilibrium. Once inverted the left hand side of (7) the iteration only consists of matrix vector multiplications.

4 WEATHER DATA

For the analysis of the yearly energy demand of a membrane building, so-called "Test Reference Years" available from German Weather Service (DWD) are used. Test Reference Years (TRYs) are datasets of selected meteorological elements for each hour of the year. They provide climatological boundary conditions for simulating calculations on an hourly basis and are part of several German engineering standards. For each of 15 German regions a representative station has been determined and a Test Reference Year has been prepared.

A Test Reference Year contains characteristic weather data of a representative year. The weather sequences have been chosen on the basis of an analysis of general weather situations in such a way that the seasonal mean values of the individual meteorological elements (especially of temperature and humidity) of the representative stations mainly correspond with their 30-years mean values. As heating and air conditioning equipment also have to be designed for extreme situations, additional datasets with the same data structure for a very cold winter and an extremely hot summer have been compiled [2]. An example for the irradiation data is shown in figure 2.



Figure 2: Solar and athmospheric irradiation, July, TRY region 13

5 EXPERIMENTAL STUDIES

During a German research project on an alternative design of a textile building, which is capable to generate its energy demand for air conditioning of the interior and water heating from renewable (solar) energy, some test stands ("thermoboxes") were constructed and equipped by measuring devices [3]. Generally the test stands consist of a insulated box with two membrane frames in the upper area and a heatable blackbody area element on the ground. The air flow through the space between the ground and the lower membrane as well as the space between the two membrane layers is controlled by a adjustable fan. If desired a heat absorber element can be positioned between the two membrane layers.



Figure 3: Components of the thermoboxes. 1: heat store, 2: blackbody, 3, 4: flow channels, 5: lower membrane frame, 6: upper membrane frame, 7: absorber, 8: insulation, 9: construction frame, 10: flow outlet, 11: flow inlet, 12: adjustable flat

The thermoboxes are equipped by thermocouples for temperature measurement, hot-wire anemometer and Pitot-tubes for volume flow rate measurement as well as by pyranometers and pyrgeometers for radiation measurement.

The comparison of analysis results of the thermoboxes was started with a simple CFD model. A hot summer day with high solar irradiation as well as a cold and clear winter night were used as boundary conditions. Free convection is assumed. The temperature of the floor is set to constant 20°C. Table 1 and 2 show the good accordance of the temperature and heat flow results of the CFD and the simple two-layer analysis.

	Hot	Hot summer day			Cold winter night		
Temperatures	CFD	Simple	$\Delta T/T$	CFD	Simple	$\Delta T/T$	
		Model			Model		
Component	[K]	[K]	[%]	[K]	[K]	[%]	
Upper Membrane	309.85	309.59	-0.08	277.27	277.55	0.10	
Cushion	308.66	308.19	-0.15	282.38	282.56	0.06	
Lower Membrane	307.45	306.78	-0.22	287.59	287.57	-0.01	
Interior	300.18	299.96	-0.07	290.35	290.29	-0.02	
Floor	273.00	273.00	0.0	293.15	293.15	0.0	

Table	1	:	Tem	peratures	in	the	thermobox
1 uore		٠	1 0111	perutures		une	unennooox

	Но	Hot summer day		Colo	1 winter n	night
Heat fluxes	CFD	Simple	Δq	CFD	Simple	Δq
		Model	-		Model	-
Boundary irradiation (downwards)	$[W/m^2]$	$[W/m^2]$	$[W/m^2]$	$[W/m^2]$	$[W/m^2]$	$[W/m^2]$
Upper Membrane (solar)	768.95	788.00	19.05	307.38	315	7.62
Upper Membrane (infrared)	385.47	366.42	-19.05	222.34	214.72	-7.62
Upper Membrane (convection)	-62.37	-50.27	12.1	-31.49	-23.20	8.29
Lower Membrane (solar)	250.55	248.37	-2.18	100.15	99.28	-0.87
Lower Membrane (infrared)	518.38	517.79	-0.59	349.40	345.71	-3.69
Lower Membrane (convection)	1.54	-0.42	1.12	-18.44	-11.78	6.66
Floor (solar)	55.97	56.45	0.48	22.38	22.56	0.18
Floor (infrared)	485.42	488.30	2.88	395.26	393.58	-1.68
Floor (convection)	8.94	10.28	1.34	-9.33	-5.16	4.17
Boundary irradiation (upwards)	$[W/m^2]$	$[W/m^2]$	$[W/m^2]$	$[W/m^2]$	$[W/m^2]$	$[W/m^2]$
Upper Membrane (solar)	140.80	151.28	10.48	56.28	60.47	4.19
Upper Membrane (infrared)	510.71	505.33	-5.38	373.59	379.98	6.39
Upper Membrane (convection)	-1.54	-0.44	1.1	18.21	11.78	-6.43
Lower Membrane (solar)	13.31	11.29	-2.02	5.33	4.51	-0.82
Lower Membrane (infrared)	439.81	432.65	-7.16	411.34	413.71	2.37
Lower Membrane (convection)	-9.23	-10.28	-1.05	9.35	5.16	-4.19
Total heat flux	$[W/m^2]$	$[W/m^2]$	$[W/m^2]$	$[W/m^2]$	$[W/m^2]$	$[W/m^2]$
Upper Membrane	18.25	16.16	-2.09	60.23	58.14	-2.09
Lower Membrane	-59.98	-63.67	-3.69	29.19	26.19	-3
Floor (heating / cooling)	107.07	111.09	4.02	-10.22	-7.23	2.99

Table 2 : Heat fluxes in the thermobox

Small differences in the heat fluxes occur for two reasons: First, in the simple model the incoming boundary irradiation at the upper membrane is strictly separated into solar and

infrared components, whereas the CFD model takes into account that there are small parts of solar radiation in the infrared wavelength range and v. v. Second, convection at the outer side of the upper membrane is slightly underestimated by the simple model, because the heat transfer coefficients for free convection are calculated for a theoretically infinite plate, where the real model has small dimensions. However, the interesting results - membrane and interior temperatures as well as the energy demand for heating and cooling - are well estimated by the simple model.



Figure 4: Temperatures and velocities in the simple CFD model (hot summer day)

A second more complex CFD model was examined to analyze the behavior of the thermobox with constant air flow through the volume between the membranes as well as the volume between lower membrane and the blackbody at the ground. Also here a good accordance between this CFD model and the simple model could be reached.



Figure 5: Temperature distribution in CFD model (extreme summer day)

	Extreme summer day					
Temperatures	CFD	Simple Model	ΔT			
Component	[K]	[K]	[K]			
Upper Membrane	315.65	315.67	0.02			
Temperature increase	1.10	1.04	-0.06			
Lower Membrane	305.65	305.55	-0.10			
Floor	302.89	302.89	0.00			

Table 3: Temperatures in the thermobox

Finally, the analysis results have been compared to the long-time measurements which are taken on four thermoboxes with different material configurations. First simulations of single states gave also good accordance in temperature results. However, long-time measuring with the boxes goes still on; so integral results will be presented at the conference.

6 APPLICATIONS

For a storage building located in Mannheim, Germany, the yearly energy demand had to be determined. The building's roof consists of 6 PVC-PES cushions with optical properties as shown in figure 1. Two extreme weather situations have been chosen to compare CFD results with the simplified analysis presented above: A clear summer day with high environment temperature and an intense solar irradiation and a cold, clear winter night with extreme low environment and sky temperatures.



Figure 6: Temperature distribution [K] in the storage building. (a) hot summer day, (b) cold winter night

	Hot summer day			Cold winter night		
Temperatures	CFD	Simple	$\Delta T/T$	CFD	Simple	$\Delta T/T$
		Model			Model	
Component	[K]	[K]	[%]	[K]	[K]	[%]
Upper Membrane	334.36	330.59	-1.13	258.88	260.21	0.51
Cushion	329.81	325.79	-1.22	266.3	268.09	0.67
Lower Membrane	321.48	316.97	-1.40	274.09	276.49	0.88
Interior	308.36	305.15	-1.04	283.14	285.17	0.88
Floor	273.00	273.00	0.0	293.15	293.15	0.72

Table 4: Temperatures in the building



Figure 7: Boundary heat flux [W/m²]. (a) hot summer day, (b) cold winter night

	Hot summer day			Cold winter night		
Heat fluxes	CFD	Simple	Δq	CFD	Simple	Δq
		Model	-		Model	•
Component	$[W/m^2]$	$[W/m^2]$	$[W/m^2]$	$[W/m^2]$	$[W/m^2]$	$[W/m^2]$
Upper Membrane	80.96	85.95	4.99	79.51	87.83	8.32
Cushion (convection)	11.38	11.99	0.61	27.80	29.53	1.73
Lower Membrane	108.21	113.63	5.42	79.68	88.34	8.66
Interior (convection)	9.19	8.27	-0.92	29.53	28.33	-1.2
Floor (cooling/heating)	117.25	125.22	7.97	-94.64	-95.06	-0.42

Table 5 : Heat fluxes in the building







Figure 8 : Calculated temperatures and heat fluxes for a summer day

Finally, the energy demand for heating and cooling of the building under the weather conditions of a test reference year has been calculated by integration of the hourly heat fluxes through the floor of the building. For a ground area of 225 m² the estimation leads to a realistic value of about 45.000 kWh/year.

6 CONCLUSIONS

The simple model approach as presented above is able to estimate the energy behavior of membrane buildings within a small error range which can be estimated by few CFD simulations for extreme boundary conditions. So the simulation of the behavior of a building for each hour in a year can be accomplished in a very short time. By this approach it is also possible to study and compare different configurations of the membrane roof of a building under realistic weather conditions.

It should be noted that the output of the simple model simulation is only averaged, that means that peaks in temperatures or heat fluxes could not be determined.

To approve this approach for energy demand calculations and to be able to simulate more complex building geometries, further work will be done on modeling a building by a higher number of surfaces. However, it must be taken into account that the simulation time increases considerably the more radiation surfaces are used in the model.

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INVESTIGATION OF AN ELASTOFLEXIBLE MORPHING WING CONFIGURATION

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Key words: Membrane wing, adaptive wing, experimental methods

Abstract. This paper considers the experimental investigation of a morphing wing using an elastic membrane for the lifting surface to allow large variations of the planform. Measurements of the membrane deflection of two different wing configurations at various flow conditions (i.e. dynamic pressure and angle of attack) are presented to provide insight into the complex flow-structure interaction mechanisms governing the behavior of the wing and help optimizing its aerodynamic performances. The results allow identifying the relative influence of the aerodynamic, geometric and structural parameters of the wing on the membrane deflection. In particular, the non-linearity of the interaction between the aerodynamic load and the membrane deflection is pointed out.

1 INTRODUCTION

1.1 Membranes in airplane design

In the history of airplane design, the use of thin, compliant membranes for the wing surfaces originates back to the beginnings of human flight. The wings of the "Wright Flyer" airplane, the world's first successful powered airplane developed by the Wright Brothers and flown in 1903, were made out of a thin membrane spanned over a rigid, load bearing structure [1]. Later, as aircraft had to fly faster and carry more payload, much stronger structures were required to withstand the aerodynamic loads acting on the airframe, discarding the use of membrane wings.

However, the low weight, low cost and structural simplicity of membrane wings still continued to make of them an attractive technical solution for low speed applications (typically hang glider, para glider). In the mid of the 20th century, membrane wings received attention with the development of the so called Princeton sailwing (Fig. 1a). It basically consists of a rigid leading edge spar and a trailing edge wire spanned between a tip rib and a root rib with a flexible membrane wrapped around the leading and trailing edges, forming the upper and lower wing surfaces. Initially conceived as an advanced sail for a boat, it was later converted to an airplane wing. In Refs. [2] and [3], experimental tests of different sailwing configurations show that their aerodynamic characteristics compare favorably with conventional rigid wings in terms of maximum lift and maximum lift-to-drag ratio (i.e. aerodynamic efficiency). In particular, a notable feature of sailwings is their ability to naturally adapt their shape to changing flow condition, resulting superior stall characteristics.

More recently, membrane wings gained increased attention for their potential application in micro-sized aircraft design. The flow physics associated with the reduced size (similar to natural flyers) and low speed flight of these vehicles (i.e. low Reynolds number) differs passably from the behavior of conventional full scale aircrafts, and the use of flexible wing surfaces is found to be advantageous for this application. Investigations presented in Refs. [4] and [5] show that using a flexible wing surface facilitate shape adaptation, resulting in overall better performances.



Figure 1: a) Schematic structure of a Sailwing [6]. b) Sketch of the elastoflexible morphing wing: extension of the Princeton sailwing concept to a biologically inspired, form-variable wing.

1.2 The elastoflexible morphing wing

In recent years, an increased amount of resources is spent for the development of aircrafts able to considerably alter their shape with the goal to improve efficiency and expand flight envelope compared to conventional rigid configurations. In fact, the flight performances (efficiency and maneuverability) of an aircraft are directly related to its geometry and thus, in flight reconfiguration would allow a single aircraft to accomplish different mission roles efficiently and effectively ([7], [8]). A big challenge in the design of such morphing aircrafts lies in the contradictory requirements for the structure of high compliance to allow big deformations on the one hand, and structural integrity to withstand aerodynamic loads on the other hand. In this context, this paper considers the investigation of a wing concept that uses the high compliance, low weight and adaptability advantages of membrane wings as a technical solution for such an extreme form-variable aircraft. The construction of this biologically inspired concept,



Figure 2: a) Lift coefficient C_L as a function of the angle of attack α measured at different freestream dynamic pressures, illustrating the pronounced dependence of the aerodynamic characteristics on the flow conditions, [9]. b) Diagram showing the parameters involved in the aeromechanics of the elastoflexible morphing wing.

shown in Fig. 1b, basically consists of an articulating frame structure (leading and trailing edge spars) over which an elastic cover is spanned to form the aerodynamic surface. While the configuration of the leading edge spar, actively controlled, sets the overall planform of the wing, the highly extensible membrane used for the wing surface naturally adapts to the changing shape. Like this, a seamless aerodynamic surface can be obtained and the actuation can be done with reasonable actuation energy. With this particular design, the wing planform can be continuously varied between a high aspect ratio, straight wing and a highly swept back, low aspect ratio configuration (the so called "loiter" and "maneuver" configurations, respectively).

The investigation of this concept is carried out mainly experimentally by means of comprehensive tests of an appropriate wind tunnel model. So far, measurements of the lift, pitching moment and drag characteristics of several wing configurations allowed assessing the general behavior of the wing. The results (reported in Ref. [9]) show that overall, the active morphing of the wing planform effectively alters the lift and drag characteristics in such a way that a relatively high efficiency can be maintained over a wide range of flight conditions. However, due to its high compliance, the wing surface passively deforms under aerodynamic loading leading to a pronounced dependence of the aerodynamic characteristics on the flow conditions (see Fig. 2a). Thereby, the design of the membrane plays a determinant role.

In the present paper, the focus is set on quantitative measurements of the membrane deflection under various flow conditions, with the intention to better understand the behavior of the wing. The general trends are also compared with a simple model describing the aeromechanics of elastic sailwings.

1.3 Aeromechanics of sailwings

Due to its intrinsic construction, the shape of a membrane wing is not fixed in advance like for most common rigid wings. Rather, its shape results from the interaction between an aerodynamic flow field and a deformable surface. In Ref. [10], it is suggested that the maximum deflection of an elastic single-membrane wing is completely characterized by two parameters, the prestrain, ε_0 , and a Weber number, We, expressing in this context the ratio of the aerodynamic load to the structural parameters of the membrane.

$$z_{max}/c = f(\varepsilon_0, We) \tag{1}$$

$$We = \frac{C_L qc}{Et} \tag{2}$$

where $q = \frac{1}{2}\rho U_{\infty}^2$ is the freestream dynamic pressure, *c* and *t* are the chord length and membrane thickness, respectively, and *E* the modulus of elasticity of the membrane¹. C_L is the lift coefficient, expressing the fraction of the freestream dynamic pressure effectively converted into lift force, i.e., the actual aerodynamic load on the membrane. In Ref. [10], this model is found to match very well with experimental data. As far as a sailwing (i.e. double membrane) is considered in this paper and not a single membrane wing, additional considerations are necessary to extend the validity of Eq. (2). The main difference comes from the fact that, for a sailwing, the relative contribution of the upper and of the lower surfaces in the production of lift is different and therefore, the load acting on each surface is only a fraction of the whole C_L . As long as the upper and lower surfaces can be treated separately, Eq. (2) can be modified by a factor k_{up} and k_{low} , respectively, with $k_{low} = 1 - k_{up}$. Taking this into account, Eq. (2) results in

$$We_{up} = \frac{k_{up}C_Lqc}{Et}$$
(3)

$$We_{low} = \frac{k_{low}C_Lqc}{Et} = \frac{(1-k_{up})C_Lqc}{Et}$$
(4)

Typically, the pressure load is less on the lower surface than on the upper one and a ratio of $k_{up} \approx 0.7$ and $k_{low} \approx 0.3$ can be assumed to be representative for positive angles of attack. As shown in Fig 2b, in the case of the elastoflexible morphing wing considered here, the large planform variability increases the complexity of the interaction since a variation of the configuration simultaneously affects the membrane characteristics (via the prestrain) and the aerodynamic load. Beside the chord length *c* in the numerator of Eq. (2), the wing planform influences the aerodynamic loading via C_L , too. The influence of the planform on C_L can be taken into account using results of rigid wing theory ([1]) as follows. For angles of attacks in the attached flow regime, the lift coefficient C_L for a rigid wing is a linear function of the angle of attack α ,

$$C_L = C_{L0} + C_{L,\alpha}\alpha \tag{5}$$

where C_{L0} is the lift coefficient at $\alpha = 0$. C_{L0} depends on the camber of the wing which, in the case of a membrane wing, is largely a function of the prestrain ε_0 ([6]). $C_{L,\alpha}$ is the slope of the lift curve, which directly depends on the planform of the wing via its aspect ratio (*AR*). Wings

¹A linearly elastic material is assumed in this analysis.

with high aspect ratio (like the loiter configuration) provide a higher $C_{L,\alpha}$, thus produce more lift at equivalent flow conditions, than a low aspect ratio wing would do (like the maneuver configuration). Including Eq. (5) into Eq. (2) gives

$$z_{max,i}/c \propto \frac{k_i(C_{L0}+C_{L,\alpha}\alpha)qc}{Et} = \underbrace{\frac{k_iC_{L0}qc}{Et}}_{\propto 1/\varepsilon_0} + \underbrace{\frac{(k_iC_{L,\alpha}\alpha)qc}{Et}}_{\propto AR}$$
(6)

where *i* stands for either the upper or the lower surface. Eq. (6) models the aeromechanics of a sailwing taking into account the relevant aerodynamic, structural and geometric dependencies. The first term on the right hand side models the "initial" deflection at $\alpha = 0$ deg whereas the second term models how the deflection is expected to change with the angle of attack. The influence of the planform on the prestrain will be addressed in section 2.1.

2 EXPERIMENTAL TECHNIQUE

2.1 Wind tunnel model

The wind tunnel model used for the experimental investigations is shown in Fig. 3. An asymmetric cross section is used for the leading edge spar in order to provide better aerodynamic performances compared to a more simple rounded one. Table 1 gives a summary of the morphing capability of this model in terms of the relevant geometric parameters. The theoretical slope of the lift curve, $C_{L,\alpha}$, calculated for rigid wings of equivalent planforms and to be used in Eq. (6) are also given in this table 1².

The membrane currently used consists of an elastic fabric coated on one side with rubber layer³ to ensure air impermeability. Table 2 gives an overview of the mechanical properties of this fabric as provided by the manufacturer (Eschler Textile GmbH, [12]). This material features anisotropic stiffness, and for its application in the wind tunnel model, the stiffer direction is aligned in chordwise direction. A suitable cut has been designed in order to avoid crinkles and provide the membrane with a certain pre-tension when mounted on the model even when the wing is in maneuver configuration. Obviously, due to the large planform variation, the pretension in the membrane depends on the configuration. Fig. 4 shows the prestrain of the membrane when mounted on the model for the loiter and maneuver configuration, measured on the lower side of the wing using a 2D photogrammetry technique. Here, the prestrain $\varepsilon_{0, chord}$ and $\varepsilon_{0, span}$ represent the elongation of the grid lines in the chordwise and spanwise directions, respectively, compared to the initial membrane cut. While the loiter configuration exhibits a much higher prestrain in spanwise direction due to its larger span (up to 40%) and almost no prestrain in the chordwise direction, the maneuver configuration exhibits just the opposite pattern with prestrain in chordwise direction up to 15%. Thereby, the prestrain in chordwise direction is supposed to have a greater influence on the deflection of the aerodynamically loaded membrane.

²Computed with a vortex lattice method ([11])

³Material composition as given by the manufacturer: 58% polyamide, 28% polyurethan, 14% spandex (Elastan).



Figure 3: Morphing wing model. a) articulating structure, b) Loiter configuration, c) Maneuver configuration

	Loiter conf.	Maneuver conf.	$\Delta_{Maneuver ightarrow Loiter}$
Aspect Ratio AR [-]	8.6	4.6	+87%
Sweep angle $\Lambda_{1/2}$ [^{<i>o</i>}]	6	36	-30^{o}
Area $S[m^2]$	0.23	0.2	+15%
Half span $b/2 [m]$	1	0.6	+67%
Mean chord <i>c</i> [<i>m</i>]	0.232	0.263	-12%
$C_{L,\alpha}$ (theoretical) [1/rad]	4.84	3.66	+32%

 Table 1: Geometric characteristics of the morphing wing model.

2.2 Test setup

The elastoflexible morphing wing model presented in Sec. 2.1 was tested in the low-speed wind tunnel facility "Windkanal A" of the Institute of Aerodynamics and Fluid Mechanics of the Technische Universität München. This wind tunnel has an open rectangular test section of 1.8 m height by 2.4 m width and 4.8 m length. It generates wind speeds up to 65 m/s with freestream turbulence below 0.4%.

The deflections of the upper and lower wing surfaces of the loiter and maneuver configurations have been measured at two wind speeds, namely $U_{\infty} = 15 m/s$ and 30 m/s (q = 135 Pa and

	Warp	Weft
Breaking load	450 N/5cm	290 N/5cm
Max. elongation	240 %	460 %
Modulus of elasticity (approx.)	0.307 MPa	0.616 MPa
Specific weight	$250 g/m^2$	
Thickness	0.5 <i>mm</i>	

 Table 2: Overview of the mechanical properties of the membrane.



Figure 4: Prestrain for the membrane mounted on the model.

 $q = 535 \ Pa$) and for angles of attack of $\alpha = 0, 5, 10, 15$ and 20 deg⁴. The corresponding Reynolds numbers (based on the mean chord length) are $Re_{mac} = 0.232 \cdot 10^6$ and $0.464 \cdot 10^6$, resp., for the loiter configuration and $Re_{mac} = 0.263 \cdot 10^6$ and $0.526 \cdot 10^6$, resp., for the maneuver one.

2.3 Wing deflection measurements

The general setup for the deflection measurements is shown in Fig. 5. The stereo photogrametry system used is composed of two FlowSense 2M cameras placed outside of the wind tunnel test section with 40 deg angle of separation between the respective optical axes. The cameras have a resolution of 1600×1186 pixels which, in conjunction with the imaging optics (Nikon Nikkor, focal length 135 mm, aperture 1:28) and the distance to the model, provides an average spatial resolution of 0.15 mm per pixel. A custom software using Direct Linear Transformation (DLT, [13]) technique is used to recover the 3D coordinates of markers put on each on the wing surface. A total of 230 markers (23 in spanwise and 10 in chordwise direction) consisting of white stickers of 5 mm diameter are used on each wing surfaces. A calibration target consisting of a 2D grid of markers defining the x-y plane and moved to several z-positions is used to obtain the transformation parameters necessary to reconstruct the coordinates in the object space with DLT. The reconstruction of the calibration points in the object space from the calibration images, using the transformation parameters obtained, indicates an average measurement uncertainty of 0.085 mm within the control volume. Finally, due to the small size of the control volume imposed by the imaging optics, the measurement of a whole wing consists of 11 single measurements patched together. For this, a traversing unit is used to move the measurement volume along the wing into the desired position. However, the traveling length of the traversing system used to measure the upper side is not sufficient to measure the complete wing in the loiter configuration. For this reason, results at the the wing tip for this side are not

⁴Results for the maneuver configuration at $\alpha = 20$ deg and $U_{\infty} = 30 \text{ m/s}$ could not be analyzed because of the strong vibration of the membrane.



Figure 5: a) Schematic of the experimental setup (measurement of the wing upper side, similar setup used to measure the lower side). b) Real setup.

available.

3 RESULTS AND DISCUSSION

3.1 Membrane deflection

To show the overall deformation pattern, Fig. 6 presents the normal membrane deflection Δz measured for the case q = 535 Pa and $\alpha = 5$ deg. Due to the airflow accelerating over the wing upper side, a lower pressure occurs there that creates a suction force, thus the upward deflection of the membrane (positive z-direction). On the lower side, the pressure increases and the membrane deflects in the same direction than for the upper side but with lower amplitude since the pressure load is less on this side. In the case of the loiter configuration, the influence of the inner wing structure (cf. Fig. 3a) is visible on the lower left part. The membrane comes in contact with the linkages, limiting the deflection in this region. Regarding the deflection pattern, both configurations exhibit a maximum deflection in the inner part of the wing with a "zero" deflection on the boundary since the membrane is there constrained by the frame structure. Similar deformation patterns were observed in the other cases.



Figure 6: Normal deflection of the membrane (Δz) at q = 535 Pa and $\alpha = 5$ deg.



Figure 7: Spanwise distribution of the maximum deflection of the upper side.

Fig. 7 shows the spanwise distribution of the deflection $\Delta z/c$ (i.e. maximum deflection in each spanwise section) of the upper side of the loiter and maneuver configurations as a function of the angle of attack for both dynamic pressures investigated. In all cases, similar shapes are found with a "zero" deflection at the root as well as at the tip because the membrane is fixed on the structure there. Generally, the camber (i.e. deflection) grows roughly monotonically with the angle of attack. In all cases, except the one in Fig. 7b, the deflection grows more rapidly between $\alpha = 0$ deg and $\alpha = 5$ deg than for higher angles of attack. This so called hysteresis effect



Figure 8: Evolution of the maximum deflection $\Delta z_{max}/c$ with the angle of attack α .

occurs because the situation around $\alpha = 0$ deg is unstable since the direction of the aerodynamic load is not clearly defined. At higher angles of attack, however, the direction is uniquely defined and the membrane "snaps" to a stable shape. This effect is known to be strongly dependent on the pretension of the membrane, which is also verified here since it is more pronounced in the case of the loiter configuration where the chordwise prestrain is much lower than for the maneuver configuration (see Fig. 4). Fig. 7c exhibits some additional features. The deflection increases only between $\alpha = 0$ deg and $\alpha = 10$ deg but not further. This may be explained twofold. On the one hand, the onset of stall as a result of the high camber and relative high angle of attack limits the aerodynamic load, discarding a further increase of the deflection. On the other hand, due to the high deflection, the membrane has reached an elongation from which its stiffness increases, making a further increase of the deflection impossible. Finally, comparing the respective shape of the deflection along the span, the loiter and maneuver configuration show some clearly different characteristics. In the maneuver configuration, the decrease of the deflection after the maximum towards wing tip is steeper but the camber overall higher at the tip than for the loiter configuration. This is supposedly due to the difference in spanwise pretension between both configurations.

3.2 Comparison of the maximum deflection with the theoretical model

Fig. 8 shows the evolution of the maximum deflection with the angle of attack of the upper and lower wing surfaces. The characteristics observed in the previous section concerning the upper side can be generalized to the lower surface. In particular, the rapid increase in $\Delta z_{max}/c$ between $\alpha = 0 deg$ and $\alpha = 5 deg$, characteristic of the hysteresis effect, can be well observed. In the following, the principal trends observed in Fig. 8 are compared to the predictions of the theoretical model presented in Sec. 1.3. First of all, Eq. (6) predicts that the maximum deflection scales with the dynamic pressure. This can be well observed here since the amplitude of $\Delta z_{max}/c$ at q = 535 Pa is much higher than the deflections measured at q = 135 Pa. However, the dependency is not linear since the lift coefficient C_L , which also plays a role in Eq. 6, also depends on q in the case of a sailwing (see Fig. 2). Here, the non-linearity of the aeromechanics of a sailwing becomes clear: a higher dynamic pressure induces a higher deflection, which in turn increases the aerodynamic load until an equilbrium is reached. The simple linear relation for C_L considered in section 1.3, Eq. 5, is not sufficient to completely describe the behavior of a sailwing and a more complex relation in the form of $C_L = f(\alpha, q, \varepsilon_0)$ should be used.

The deflection in the case of the maneuver configuration is overall lower than for the loiter one (at least for the upper side), which confirms the dependency between the prestrain and the deflection amplitude assumed in Sec. 1.3, namely that a higher prestrain limits the deflection. Surprisingly however, the deflections of the lower surfaces of both configurations are very similar, which may be explained by the presence of the linkages of the inner wing structure, as mentioned in Sec. 3.1.

From Eq. 6, the planform is supposed to influence the rate of growth of the deflection with the angle of attack, via $C_{L,\alpha}$ and c. Looking at the respective values of these two parameters for both configurations in Tab. 1, the lower $C_{L,\alpha}$ of the maneuver configuration is somehow compensated by its higher chord but overall, a slightly lower rate would be expected. In Fig. 8, the curves of both configurations feature very similar slopes and the hysteresis effect makes it difficult to discern a clear trend.

Finally, as a result of the different pressure load on the upper and lower wing surfaces, the model of Sec. 1.3 predicts lower deflections of the lower than of the upper surface. This trend can be fairly observed in Fig. 8.

4 CONCLUSION AND OUTLOOK

This paper considers the experimental investigation of a morphing wing using an elastic membrane for the lifting surface to allow large variations of the planform. Measurements of the membrane deflection of two different wing configurations at various flow conditions (dynamic pressure and angle of attack) were performed to investigate in more details the complex flow-structure interaction mechanisms governing the behavior of the wing. For this, a stereo photogrammetry system using the Direct Linear Transformation method is used. Also, a simple theoretical model describing the aeromechanics of elastic sailwings taking into account the principal geometric, structural and aerodynamic dependencies is also presented.

Measurement results highlights the strong dependency of the membrane shape on the flow conditions, which is responsible for the unconventional aerodynamic characteristics of the wing. The influence of the planform variability on the deflection of the membrane is pointed out and thereby, the major influence comes from the varying prestrain associated with the different configurations. The pure influence of the planform on the aerodynamic load can not be clearly identified in this case. Finally, the results highlights the strong non-linear dependency between the aerodynamic load and the membrane deflection, indicating a limited validity of the basic aeromechanic model.

Taking advantages of these results, further work will consider new membrane design with the main goal to optimize the aerodynamic performances of the wing. For this, not only wind tunnel tests but also numerical simulations are planned.

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PRESSURE-VOLUME COUPLING IN INFLATABLE STRUCTURES

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Key words: Deformation-dependent forces, Inflatable structures, Pressure-volume coupling, Multi-chambers, Membrane finite element

Abstract. Inflatable structures or pneumatic structures are air-supported structures and find application in a variety of new engineering projects. By these structures the gas gives shape and strength to the structure in such a way that with larger volumes and higher pressures larger spans can be abridged. Their architectonic flexibility and the search for better structural efficiency are strong favorable arguments to the use of this kind of structure. Furthermore, inflatable structures can be erected or dismantled quickly, are light, portable and materially inexpensive. Also, some characteristics such as the utilization of natural lighting and ventilation and its possibility of reuse contribute to the pursuit of sustainable development. In this work the influence of the pressure-volume coupling on structure behavior is studied. A membrane formulation with deformationdependent forces was implemented in a finite element model to take into account the influence of gas volume variation, with corresponding change in pressure for enclosed gas on the stiffness of membrane structures. An analytical solution of inflatable membranes was developed for the validation of the numerical results. Comparisons are carried out on a clamped circular membrane inflated by a uniform pressure.

1 INTRODUCTION

Inflatable structures or pneumatic structures are according to Dent [6] structures acted on by air or gases and relate particularly to architecture and construction. Membrane foils are used in these structures and it is defined according to Pauletti[5] as a constructive element in tension-structures that balance the external loads with tensile and shear stresses tangent to the structure surface. Otto[14, 13] defines a membrane as a flexible skin stretched in such a way to be subjected to tension. The membrane finite element formulation used is described in the works of Wüchner and Bletzinger [18], Vázquez [17] and Linhard [12].

The properties and the different shapes of pneumatic structures is described in the work of Herzog [10]. A large number of parameters determine the final shape of these structures: type of internal pressure, magnitude of internal pressure, boundary conditions, number of membranes, type of utilization, membrane material, surface curvature, etc.

The pressure-volume coupling will be considered in the analyses of the pneumatic structure. The numerical implementations of the finite element coupling model was carried out on the program used in the Static Chair at TUM (Technische Universität München). This program is called CARAT++ (Computer Aided Research Analysis Tool) and it has been started by Kai-Uwe Bletzinger, Hans Stegmüller and Stefan Kimmich at the Institut für Baustatik of the University of Stuttgart in 1987. In accordance with its title, CARAT served first of all as a research code.

An analytical solution of inflatable membranes for the validation of the numerical results was developed. The results for the analytical and numerical analysis clarify the influence of the pressure-volume coupling and highlight the importance to consider this coupling in these type of structure.

2 DEFORMATION-DEPENDENT FORCES

The objective to implement the pressure-volume coupling is to take into account influences of gas volume variation, which leads to the change in pressure for an enclosed gas on the stiffness of membrane structures. According to Jarasjarungkiat ([11]) numerical examples demonstrate not only the efficiency of the model but also the need to consider the volume (pressure) variation in addition to the change of surface normal vector. This work reveals the observable feature that the pressure of an enclosed fluid provides additional stiffness to the inflatable structure, analogous to the behavior of a membrane on elastic springs.

For this implementation the concept of deformation-dependent forces was studied. The formulation used within this work refers to the works of Hassler and Schweizerhof [9], Rumpel and Schweizerhof [16], Rumpel [15], Bonet et. ali. [2] and Berry and Yang [1].

Hassler and Schweizerhof [9] presented a formulation for the static interaction of fluid and gas for large deformation in finite element analysis that can be applied to pneumatic structures. Moreover it provides a realistic and general description of the interaction of arbitrarily combined fluid and/or gas loaded or filled multi-chamber systems undergoing large deformations static or quasi-static .

The use of a deformation-dependent force formulation brings along the drawback of a fully-populated stiffness matrix whose triangular factorization requires large numerical effort. To circumvent this problem Woodbury's formula was used to solve the fullypopulated stiffness matrix as discussed in the work of Hager [8]. The Woodbury's formula updates the inverse of a matrix with the update tensors without performing a new factorization of the stiffness matrix.

2.1 Numerical analysis model

The formulation presented in the work of Hassler and Schweizerhof [9] concern an enclosed volume filled with combined fluid and gas. Rumpel and Schweizerhof [16] treat the case of structures filled with gas, which is the common case in civil engineering. Therefore, the formulation considered in this work concern an enclosed volume filled with gas. This formulation will be briefly present:

Taking the principle of virtual work as basis for the problem formulation, the external virtual work of a pressure load is given by:

$$\delta \Pi_{press} = \int_{a} p \,\mathbf{n} \cdot \delta \mathbf{u} \, da \tag{1}$$



Figure 1: Surface under pressure loading.

where $\mathbf{n} = \mathbf{x}_{\xi} \times \mathbf{x}_{\eta} / |\mathbf{x}_{\xi} \times \mathbf{x}_{\eta}|$ is the normal, $da = |\mathbf{x}_{\xi} \times \mathbf{x}_{\eta}| d\xi d\eta$ is the surface element and $p = p(v(\mathbf{x}))$ is the internal pressure. The surface vector $\mathbf{x}(\xi, \eta)$ depends on local co-ordinates ξ and η represented in Figure 1. Substitution of the definition 1 gives:

$$\delta\Pi_{press} = \int_{\eta} \int_{\xi} p \frac{\mathbf{x}_{,\xi} \times \mathbf{x}_{,\eta}}{|\mathbf{x}_{,\xi} \times \mathbf{x}_{,\eta}|} \cdot \delta\mathbf{u} |\mathbf{x}_{,\xi} \times \mathbf{x}_{,\eta}| d\xi d\eta = \int_{\eta} \int_{\xi} p \left(\mathbf{x}_{,\xi} \times \mathbf{x}_{,\eta}\right) d\xi d\eta \qquad (2)$$
$$= \int_{\eta} \int_{\xi} p \mathbf{n}^* \cdot \delta\mathbf{u} d\xi d\eta$$

According to Poisson's law, the constitutive behavior of the gas is describe by the following equation:

$$pv^{\kappa} = PV^{\kappa} = const \tag{3}$$

where κ is the isentropy constant, P and V are the initial pressure and volume for a closed chamber. This equation demonstrates that the final pressure is inversely proportional to the final volume, in other words, when the volume decreases the internal pressure inside the enclosed volume will increase.

When $\kappa = 1$ the adiabatic change simplifies to Boyle's law.

The volume for the enclosed chamber v is computed through the equation:

$$v = \frac{1}{3} \int_{\eta} \int_{\xi} \mathbf{x} \cdot \mathbf{n}^* \, d\xi \, d\eta \tag{4}$$

The external virtual work is linearized for the solution with a Newton scheme. Equation 2 and the constraint 3 are expanded into a Taylor series up to the first order term:

$$\delta\Pi_{press}^{lin} = \delta\Pi_{press} + \delta\Pi_{press}^{\Delta \mathbf{p}} + \delta\Pi_{press}^{\Delta \mathbf{n}}$$
(5)
$$\delta\Pi_{press}^{lin} = \int_{\eta} \int_{\xi} (p\mathbf{n}^* \cdot \delta\mathbf{u} + \Delta p\mathbf{n}^* \cdot \delta\mathbf{n}^* + p\Delta\mathbf{n}^* \cdot \delta\mathbf{u}) \, d\xi \, d\eta$$

with

$$\Delta \mathbf{n}^* = \Delta \mathbf{u}_{,\xi} \times \mathbf{x}_{,\eta} - \Delta \mathbf{u}_{,\eta} \times \mathbf{x}_{,\xi}$$
(6)

$$\Delta(pv^{\kappa}) = 0 \quad , \ pv^{\kappa} = constant$$

$$\Delta p \cdot v^{\kappa} + \Delta v^{\kappa} \cdot p = 0$$

$$(7)$$

where

$$\Delta v^{\kappa} = \kappa \frac{v^{\kappa}}{v} \Delta v \tag{8}$$

$$\Delta v = \frac{1}{3} \int_{\eta} \int_{\xi} \left[\Delta \mathbf{u} \cdot \mathbf{n}^* + \mathbf{x} \cdot \Delta \mathbf{n}^* \right] d\xi d\eta = \Delta v^{\Delta \mathbf{u}} + \Delta v^{\Delta \mathbf{n}}$$
(9)

Equation 7 results in:

$$\Delta p + \frac{\kappa p}{v} \Delta v = 0 \tag{10}$$

In this work the final results for the partial integrations of equation 5 will be presented. The solution for each part of the partial integration of the external virtual work is shown in References [9], [16] and [15]. The partial integration of the external virtual work due to the change in the normal vector is given by:

$$\delta\Pi_{press}^{\Delta\mathbf{n}} = \frac{p}{2} \int_{\eta} \int_{\xi} \begin{pmatrix} \delta\mathbf{u} \\ \delta\mathbf{u}_{,\xi} \\ \delta\mathbf{u}_{,\eta} \end{pmatrix} \cdot \begin{bmatrix} 0 & \mathbf{\underline{W}}^{\xi} & \mathbf{\underline{W}}^{\eta} \\ \mathbf{\underline{W}}^{\xi T} & 0 & 0 \\ \mathbf{\underline{W}}^{\eta T} & 0 & 0 \end{bmatrix} \begin{pmatrix} \Delta\mathbf{u} \\ \Delta\mathbf{u}_{,\xi} \\ \Delta\mathbf{u}_{,\eta} \end{pmatrix} d\xi d\eta \tag{11}$$

The partial integration of the external virtual work due to the change in the pressure is:

$$\delta\Pi_{press}^{\Delta p} = -\frac{\kappa p}{v} \int_{\eta} \int_{\xi} \mathbf{n}^* \cdot \Delta \mathbf{u} \, d\xi d\eta \, \int_{\eta} \int_{\xi} \mathbf{n}^* \cdot \delta \mathbf{u} \, d\xi d\eta \tag{12}$$

Replacing equations 11 and 12 in equation 5 gives:

$$\delta\Pi_{press}^{lin} = 0 \qquad \delta\Pi_{press}^{\Delta p} + \delta\Pi_{press}^{\Delta \mathbf{n}} = -\delta\Pi_{press} \qquad (13)$$
$$-\frac{\kappa p}{v} \int_{\eta} \int_{\xi} \mathbf{n}^* \cdot \Delta \mathbf{u} \, d\xi \, d\eta \qquad \int_{\eta} \int_{\xi} \mathbf{n}^* \cdot \delta \mathbf{u} \, d\xi \, d\eta$$
$$+\frac{p}{2} \int_{\eta} \int_{\xi} \begin{pmatrix} \delta \mathbf{u} \\ \delta \mathbf{u}_{,\xi} \\ \delta \mathbf{u}_{,\eta} \end{pmatrix} \cdot \begin{bmatrix} 0 & \mathbf{W}^{\xi} & \mathbf{W}^{\eta} \\ \mathbf{W}^{\xi T} & \mathbf{0} & \mathbf{0} \\ \mathbf{W}^{\eta T} & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{pmatrix} \Delta \mathbf{u} \\ \Delta \mathbf{u}_{,\xi} \\ \Delta \mathbf{u}_{,\eta} \end{pmatrix} d\xi \, d\eta$$
$$= -p \int_{\eta} \int_{\xi} \mathbf{n}^* \cdot \delta \mathbf{u} \, d\xi \, d\eta$$

The discretization for the finite element is applied taking the equations 13 and the isoparametric representation:

$$\mathbf{x} = \mathbf{N}\mathbf{x}, \qquad \Delta \mathbf{u} = \mathbf{N}\mathbf{d} \quad and \quad \delta \mathbf{u} = \mathbf{N}\delta\mathbf{d} \tag{14}$$

The global stiffness matrix and the global load vector are given:

$$\left[\mathbf{K}_T - (\mathbf{K}_{press} - b\mathbf{a} \otimes \mathbf{a})\right] \mathbf{d} = \mathbf{f}_{ext} + \mathbf{f}_{press} - \mathbf{f}_{int}$$
(15)

$$\mathbf{K}_{press} = \frac{p}{2} \int_{\eta} \int_{\xi} \begin{pmatrix} \delta \mathbf{N} \\ \delta \mathbf{N}_{,\xi} \\ \delta \mathbf{N}_{,\eta} \end{pmatrix}^{T} \cdot \begin{bmatrix} 0 & \mathbf{\underline{W}}^{\xi} & \mathbf{\underline{W}}^{\eta} \\ \mathbf{\underline{W}}^{\xi T} & 0 & 0 \\ \mathbf{\underline{W}}^{\eta T} & 0 & 0 \end{bmatrix} \begin{pmatrix} \Delta \mathbf{N} \\ \Delta \mathbf{N}_{,\xi} \\ \Delta \mathbf{N}_{,\eta} \end{pmatrix} d\xi d\eta$$
(16)

$$\mathbf{a} = \int_{\eta} \int_{\xi} \mathbf{N}^T \mathbf{n}^* \, d\xi d\eta \tag{17}$$

$$\mathbf{f}_{press} = p \int_{\eta} \int_{\xi} \mathbf{N}^T \mathbf{n}^* \, d\xi \, d\eta \tag{18}$$

$$b = \kappa \frac{p}{v} \tag{19}$$

where \mathbf{K}_T is the stiffness matrix containing linear and non-linear terms, \mathbf{K}_{press} is the load stiffness matrix for each structural element in contact with gas, **a** is the coupling vector, \mathbf{f}_{press} is the load vector, \mathbf{f}_{int} is the residuum vector and \mathbf{f}_{ext} is the vector of the external forces. According Rumpel [15] the load stiffness matrix \mathbf{K}_{press} reflect the direction dependent of the internal pressure and the fully-populated coupling matrix $b\mathbf{a} \otimes \mathbf{a}$ is the volume dependent of the internal pressure.

Equation 15 can be rewritten as:

$$\left[\mathbf{K}^* + b\mathbf{a} \otimes \mathbf{a}\right] \mathbf{d} = \mathbf{F} \tag{20}$$

where $\mathbf{K}^* = \mathbf{K}_T - \mathbf{K}_{press}$ and $\mathbf{F} = \mathbf{f}_{ext} + \mathbf{f}_{press} - \mathbf{f}_{int}$.

The stiffness matrix is fully-populated, and therefore triangular factorization requires for its great computational effort. To circumvent this problem Woodbury's formula was used to solve the fully-populated stiffness matrix, as discussed in the work of Hager [8]. The Woodbury's formula updates the inverse of a matrix through the update tensors dismissing factorization of the stiffness matrix.

3 ANALYTICAL ANALYSIS

To perform an analytical analysis, circular inflated membrane clamped at its rim is inflated by a uniform pressure. The membrane is supposed to have large displacements, we refer to works from Hencky (see [7]), Fichter [7], Campbell [4] and Bouzidi et. ali. [3]. Fichter [7] considered that the lateral stress for this problem has a radial component. This radial component is neglected in Hencky's problem. Note that, Fichter and Hencky consider the membrane without initial tension. Campbell [4] generalized Hencky's problem to include the influence of an arbitrary initial tension.

In this work an analytical solution was developed for inflated circular membranes considering the radial component in the lateral stress and an arbitrary initial tension in the membrane.

3.1 Hencky's solution

Hencky's solution consider an uniform lateral loading, in other words, the radial component of pressure on the deformed membrane is neglected. The equation for radial equilibrium is:

$$N_{\theta} = \frac{d}{dr}(r \cdot N_r) \tag{21}$$

and for lateral equilibrium:

$$N_r \frac{d}{dr}(w) = -\frac{pr}{2} \tag{22}$$

where N_r and N_{θ} are the radial and circumferential stress resultants, respectively, r is the radial coordinate, w is the lateral deflection, and p is the uniform lateral loading. It is assumed that the material is linear and it has elastic behavior, then the stress-strain relations are:

$$N_{\theta} - \nu \cdot N_r = E \cdot h \cdot \epsilon_{\theta} \tag{23}$$

$$N_r - \nu \cdot N_\theta = E \cdot h \cdot \epsilon_r \tag{24}$$

The strain-displacement relation is given by:

$$\epsilon_r = \frac{d}{dr}(u) + \frac{1}{2} \cdot \left(\frac{dw}{dr}\right)^2 \tag{25}$$

$$\epsilon_{\theta} = \frac{u}{r} \tag{26}$$

where u is the radial displacement and μ is Poisson's ratio.

3.2 Fichter's solution

The equation of the radial equilibrium for Fichter's solution has, in comparison with the Hencky's solution (see equation 21), an addition term:

$$N_{\theta} = \frac{d}{dr}(r \cdot N_r) - p \cdot r \frac{w}{dr}(w)$$
(27)

This additional term comes from the normal pressure which is neglected in Hencky's solution. The equations of Fichter's solution for the lateral equilibrium and the stress-strain relation remain the same as those of Hencky's solution (see equations 22 through 26).

4 RESULTS

A circular membrane clamped at its rim is inflated by a uniform pressure. The membrane is assumed to have large deflection and small strain for the analytical and numerical solutions. The large strain is assumed just in the numerical solution because it is not available in the literature an analytical solution with large strain. The data used for the



Figure 2: Mesh for a circular inflated membrane.
numerical and analytical analysis is from the work of Bouzidi et. ali. [3]. The membrane characteristics are: E = 311488 Pa.m (Young's modulus), $\nu = 0.34$ (Poisson ratio) and the radius is 0.1425 m. The static analysis was carried out in two steps. Initially the configuration for an internal pressure of 400kPa is obtained. After the inflation external pressures were applied. The mesh for the numerical solution is composed of 641 nodes and 640 membrane elements (see Figure 2). The membrane element has 4 nodes and 4 gauss integration points. It is also considered the plane stress state for the membrane element.



Figure 3: Comparison between the Hencky's and Fichter's solution for the applied external pressure of 150kPa and 300kPa.

In Figure 3 the results of Hencky's and Fichter's solution for the applied external pressure of 150kPa and 300kPa are presented. The difference between both solutions is due to the additional term present only in Fichter's solution associated to the normal pressure. The analytical solution from Fichter is closer to the solutions from numerical and experimental analysis. This can be observed in the Figure 4, which presents the results for the analytical and numerical solution due to the applied external pressure of 150kPa and 300kPa. The difference result obtained with Fichter's solution and the numerical solution is accredited to the presence of finite strains, which are included in the finite element formulation and are precluded in the analytical solution. In equation 28 the strain components with the finite strain contribution are shown.

$$\epsilon_r = \frac{du}{dr} + \frac{1}{2} \cdot \left(\frac{dw}{dr}\right)^2 + \frac{1}{2} \cdot \left(\frac{du}{dr}\right)^2$$

$$\epsilon_\theta = \frac{u}{r} + \frac{1}{2} \cdot \left(\frac{u}{r}\right)^2$$
(28)

The terms $\frac{du}{dr}$ and $\frac{u}{r}$ are the small strain, the term $\frac{1}{2} \cdot \left(\frac{dw}{dr}\right)^2$ arises in the presence of large



Figure 4: Result for the analytical and numerical solution without pretension and $\kappa = 0$ due to the applied external pressure of 150kPa and 300kPa.

displacements and the terms $\frac{1}{2} \cdot \left(\frac{du}{dr}\right)^2$ and $\frac{1}{2} \cdot \left(\frac{u}{r}\right)^2$ account for the finite strains.



Figure 5: Comparison between the numerical solution with a pretension of 1kPa for $\kappa = 0$ and 1 due to the applied external pressure of 150kPa and 300kPa.

Figure 5 presents the result of a numerical solution for the circular membrane with pressure-volume coupling ($\kappa = 1$) and the case without the pressure-volume coupling ($\kappa = 0$). It is observed that the pressure-volume coupling is more noticeable for higher external pressure, in agreement with Poisson's law (see equation 3). It is important to observe according to the amount of coupling that different final configurations are obtained.

Next, the influence of pretension is investigated. In Figure 6 the results for the analyt-



Figure 6: Result for the analytical and numerical solution with pretension of 1kPa and $\kappa = 1$ due to the applied external pressure of 150kPa and 300kPa.

ical and numerical solution with pretension of 1kPa and with pressure-volume coupling subjected to external pressures of 150kPa and 300kPa are presented. The analytical solution considers both the term from the normal pressure, which is neglected in Hencky's solution, and an initial tension, which is not considered in the Hencky's and Fichter's solution. It is observed that the results obtained with the analytical solution are in accordance with the numerical results. The relation between the internal pressure versus volume are illustrated in Figure 6, stressing that when the volume decreases due to the external pressure the internal pressure increase.



Figure 7: Result for the analytical and numerical solution with pretension of 4kPa and $\kappa = 1$ due to the applied external pressure of 150kPa and 300kPa.

In Figure 7 results for analytical and numerical solution with pretension of 4kPa and with pressure-volume coupling subjected to external pressures of 150kPa and 300kPa are presented. It is observed, comparing the results from Figures 7 and 6, that the deformed configuration and consequently the volume of the circular membrane decrease for the case with the pretension of 4kPa. Although the increase in the pretension is 4 times more, the decrease in the volume is not large. It can be observed in the graphics internal pressure versus volume in the Figures 7 and 6.

5 CONCLUSIONS

The present work analyzed pressure-volume coupling for a circular inflated membrane clamped at its rim. The results for the circular inflated membrane obtained with the analytical solution are in good accordance with the numerical solution. The pressurevolume coupling for the pneumatic structures was compared with the case when this coupling is not considered. It was shown that the final configuration with the coupling for the static analysis is very different in comparison with the analysis without this coupling, as defined by Poisson's law. It is also highlighted that higher external forces lead to higher internal pressure, due to the change in the configuration of the structure resulting in a decrease in the volume.

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NUMERICAL TOOLS FOR THE ANALYSIS OF PARACHUTES R. FLORES*, E. ORTEGA AND E. OÑATE

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Summary. The design and evaluation of parachute-payload systems is a technology field in which numerical analysis tools can make very important contributions. This work describes a new development from CIMNE in this area, a coupled fluid-structural solver for unsteady simulations of ram-air type parachutes. For an efficient solution of the aerodynamic problem, an unsteady panel method has been chosen exploiting the fact that large areas of separated flow are not expected under nominal flight conditions of ram-air parachutes. A dynamic explicit finite element solver is used for the structure. This approach yields a robust solution even when highly non-linear effects due to large displacements and material response are present. The numerical results show considerable accuracy and robustness. An added benefit of the proposed aerodynamic and structural techniques is that they can be easily vectored and thus suitable for use in parallel architectures. The main features of the computational tools are described and several numerical examples are provided to illustrate the performance and capabilities of the technique.

1 INTRODUCTION

The numerical simulation of parachutes is a challenging problem as the geometry is complex in design and behaviour and, in addition, is continuously changing in time. Factors contributing to the complexity and unsteadiness of the aerodynamic field are massive flow separations, complex aerodynamic interactions between the structural components and the presence of large unsteady wakes. The structural analysis requirements are also challenging. Braced membranes, such as parachute canopies, cannot equilibrate an arbitrary set of loads unless drastic geometrical changes take place. The structural response is thus extremely nonlinear. In the case of follower loads (e.g. pressure loading) the matter is further complicated by the fact that the equilibrium solution may not exist at all (i.e. the structure is in fact a mechanism). This extreme geometrical nonlinearity can give rise to severe numerical convergence problems. Due to the lack of bending stiffness of the structural components, the materials are able to resist tensile stresses but buckle (wrinkle) under compressive loads^{1,2}. This asymmetric behaviour should also be accounted for. Finally, the nature of the applied forces which depend heavily on the structural response of the parachute adds an extra layer of complexity to the analysis. As the magnitude and direction of the aerodynamic forces are not known in advance but are a function of the deformed parachute shape, they must be computed as part of the solution in an iterative process.

An effective numerical model of parachutes must deal with all the issues listed above in a robust way while keeping computational cost at an acceptable level. The magnitude of the challenges faced explains why the current design of parachute systems relies mostly on empirical methods. As an example, 15 worldwide parachute manufacturers were recently surveyed about the use of computational tools in the design and evaluation of parachute systems³. None of those 10 who provided feedback declared using computer tools beyond CAD packages for geometry modelling. This is a clear indication that computational mechanics does not yet enjoy wide acceptance among the parachute design industry. The numerical simulation tool described in this work is intended to address the needs of this sector.

In the following, the main features of the parachute simulation tool currently being developed at CIMNE are described and several numerical applications are presented with the aim of illustrating the performance and potential of the proposed techniques. The rest of the document is organized into 4 main sections. The core features of the coupled fluid-structural solver are given in section 2. Next, validation cases for the structural and flow solvers are presented in section 3 and numerical applications are shown in section 4. The main conclusions are summarized in section 6.

2 COUPLED SOLUTION STRATEGY

In view of the important challenges involved in modeling parachute systems, the choice of the structural and aerodynamic solvers as well as the coupling methodology were thoroughly examined from two different points of view. First, considering the capabilities of the techniques to deal with the typical situations encountered during the flight of parachutes; second, evaluating its robustness and the chances of achieving low computational costs through efficient numerical implementations.

Regarding structural modeling, it was decided to use a FE dynamic structural solver. An unsteady analysis is not affected by problems caused by the lack of a definite static equilibrium configuration. In fact, since for dynamic problems the structure is constantly in equilibrium with the inertial forces the solution is always unique. Even when only the long-term static response is sought, the dynamic approach offers some advantages. Furthermore, the extension to transient dynamic problems becomes trivial.

There are two basic kinds of dynamic solvers, implicit and explicit⁴. Implicit solvers can be made unconditionally stable, which allows for large time steps although the computational cost is high because a non-linear problem must be solved at each time step. When the structural response does not show a high deviation from linearity the implicit treatment is usually preferred, as it allows for large time steps. Also, the static equilibrium (when it exists) can be reached after a small number of iterations. However, it should be stressed that the radius of convergence of the iterative algorithms employed for solving the non-linear system is limited. Thus, the time step cannot be made arbitrarily large. In addition, when the structural behavior is heavily non-linear, the time increment must be cut back to ensure convergence of the iterative algorithm and the computational cost is rapidly increased. Implicit solvers also exhibit a lack of robustness due to the possibility of the scheme failing to

converge. On the other hand, although the explicit solvers are conditionally stable (the stability limit is determined by the material properties and the geometry of the FE mesh) the cost per time step is low. The explicit method is extremely insensitive to highly nonlinear structural behavior and requires a number of time steps that does not change substantially as the system response becomes more complex. Material nonlinearities and large displacements, which are extremely detrimental for the convergence behavior of the implicit scheme, do not affect adversely the explicit algorithm. In view of the difficulties expected, the choice was made to use an explicit FE structural solver. A further benefit is the ability of the algorithm to be easily vectored and thus take advantage of modern parallel processing architectures. Linear cable and membrane elements were selected due to their ease of implementation. The fabric is modeled using three-node membrane elements due to their geometric simplicity. The three nodes of the element will always lie in the same plane so the definition of the local coordinate systems is straightforward. A local co-rotational reference frame is used for each cable and membrane element in order to remove the rigid-body displacements and isolate the material strains. Inside each element a simple small-strain formulation is used due to the properties of the fabric. Tensile deformations are always small; on the other hand compressive strains can become extremely large due to the inclusion of a wrinkling model (zero compression stiffness). There is, however, no stress associated with the compressive strains and, correspondingly, no strain energy. Therefore, the small-strain formulation is adequate as only tensile deformations must be taken into account to calculate the stress state.

In spite of the fact that the structural solution approach is general and can be applied to any kind of parachute system, the computational cost of a general flow solution was not feasible from a practical point of view (at least during the first stages of the work) and a decision had to be taken regarding the scope of the aerodynamic solution. Consequently, following previous in-house developments, the focus was initially placed on ram-air type parachutes for which a potential flow approach is valid as no extensive separation regions are present during nominal operation. The main advantage of the potential model is that boundary methods can be employed. Hence, most problems related to grid generation can be avoided. Due to the large geometric changes expected, methods based on discretization of the surrounding air volume would need multiple remeshing steps. For the boundary mesh, however, only changes to the nodal position are required as the topology remains unchanged. As an added benefit, the computational cost is significantly reduced with respect to volume techniques (e.g. Finite Differences, Finite Volumes and Finite Elements). Even in cases when extensive flow separation occurs, alternative potential approaches such as vortex methods could be used. In other cases, for problems going beyond the scope of potential methods, the modular approach adopted for the code allows changing the flow solver with minimal modifications.

3 VALIDATION EXAMPLES

The accuracy of the structural and aerodynamic solvers has been tested with widely accepted benchmark cases. Two validation examples are shown next.

3.1 Structural solver validation

The most challenging aspects of the structural response are the large displacements

involved and the asymmetric material behavior due to wrinkling. The airbag inflation test is commonly used to assess these capabilities. This benchmark computes the vertical displacement at the centre of an initially flat square airbag of side length 840mm. An internal pressure of 5kPa is applied. The deformed configuration is strongly affected by the no-compression condition on the fabric, so this is a very popular benchmark for wrinkling models. The textile properties are:

$$E=588MPa \ t=0,6mm \ v=0,4b$$
 (1)

A mesh composed of 16x16 squares is used for each side of the airbag. Each square has then been divided into 4 equal triangles in order to eliminate mesh orientation effects (**¡Error! No se encuentra el origen de la referencia.**). The total number of triangular elements is therefore 2048.



Figure 1: Square airbag inflation test. Final deformed geometry

Table 1 shows the comparison of the result from the parachute simulation tool (named PARA STR in the table) with several sources^{5,6,7}. The differences are negligible.

3.2 Aerodynamic solver validation

A two-bladed rotor for which experimental results are available⁸ is solved. This is a case which needs an accurate representation of the wake shape. The rotor geometry is discretized with a structured mesh of 4000 quadrilateral panels (40 along the span and 25 in the chordwise direction). Figure 2 shows the Cp distribution computed for a collective pitch angle $\theta c = 8^{\circ}$.



Figure 2: Wake shape and pressure distribution for spinning rotor

Comparisons of the experimental measurements and the computed pressure distribution at several spanwise stations are given in Fig. 3. Numerical predictions closely match the measured values.



Figure 3: Numerical and experimental Cp distributions at different sections along a blade

4 EXAMPLES OF APPLICATION TO PARACHUTES

In this section some application examples are presented to demonstrate the capabilities of the simulation software.

4.1 Stationary analysis of a large ram-air parachute

Steady aerodynamic characteristics of a large ram-air parachute are investigated in this example. The model is a high glide-performance parachute aimed at delivering very heavy payloads designed and manufactured by CIMSA in the framework of the FASTWing Project⁹. The model canopy discretization consists of an unstructured distribution of 11760 triangular elements and 11912 cable elements model the suspension and control lines as well as the reinforcement tapes integrated into the canopy. The movement of the suspension line's confluence points is restricted to follow the experimental setup. To obtain a faster convergence to the equilibrium position of the parachute, some degree of under-relaxation is employed when transferring the aerodynamic loads to the structure. Initial and equilibrium parachute configurations are shown in Fig. 4.



Figure 4: Initial (top) and equilibrium (bottom) configurations

The time history of force and moment coefficients is displayed in Fig. 5, where the moment coefficients are computed about a point located between the suspension line's confluence points. Note that the moments plotted include only the contribution due to the canopy; if the contribution of the drag from the suspension lines were included the total value would be zero. It must also be stressed that the transient behaviour lacks real physical meaning as under-relaxation has been employed to accelerate convergence to the steady state.



Figure 5: Computed evolution of force and moment coefficients

4.2 Parachute manoeuvre analysis

A left-turn manoeuvre of a small CIMSA parachute-payload system in free-flight is studied in this example.



Figure 6: Payload centre of mass computed trajectory

The canopy discretization consists of an unstructured distribution of 9548 triangular elements and 3077 cable elements model the suspension lines and the canopy reinforcements. The payload is 100 kg and the parachute is released with North heading at a velocity of 12 m/s. The simulation starts with a partially inflated parachute configuration and, once steady descent flight is achieved, the manoeuvre is initiated by applying a 0.5 m downward deflection of the left brake line. After 5 seconds, the brake line is released and the parachute recovers a straight down descent flight. The trajectory described by the payload center of gravity during the manoeuvre is displayed in figure 6. Snapshots of the parachute-payload system taken at different times during the simulation are shown in figure 7. The dynamic evolution of the model shows a good agreement with the observed behavior of a real ram-air parachute performing the same manoeuvre. This demonstrates the capability of the present methodology to provide not only aerodynamic and structural data for design but also for performance and trajectory analyses



Figure 7: Snapshots of the parachute-payload configuration at different stages of the manoeuvre simulation: (a) 3.564s, (b) 15.58s, (c) 19.05s, (d) 19.88s., (e) 23.17s

4.3 Inflation process of a conventional parachute

This is a simple inflation test aimed at exploring the capabilities of the computational code to simulate parachute deployment and inflation. The model proposed for the aerodynamic loads is quite simple. The parachute is initially deployed by applying a force at the canopy apex in the direction of the incident wind. The inflation stage, which begins after line and canopy stretching, occurs due to a variable pressure force applied on the canopy accounting for relative wind direction and velocity. In this way, the maximum pressure force corresponds to the fluid stagnation condition and the value is reduced according to the orientation of the elements in relation to the incident wind. The parachute is discretized with an unstructured distribution of 3390 triangular elements and 2040 cable elements modeling the suspension lines and the fabric reinforcements. Some snapshots of the parachute at different times during the inflation process are presented in figure 8.



Figure 8: Parachute views at different stages of the inflation process

5 ACKNOWLEDGEMENTS

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6 CONCLUSIONS

- There is currently a severe lack of computational tools for the analysis of parachute systems.
- A new development from CIMNE in this field has been presented. The simulation

package contains a coupled fluid-structural solver tailored for the unsteady simulation of ram-air type parachutes.

- The capabilities of both the dynamic explicit structural code and the unsteady potential flow solver have been successfully validated through relevant benchmark cases.
- The coupled solution approach has been successfully applied to a variety of problems encountered during parachute design activities.
- The solution strategy is robust and the code shows a notable efficiency, being able to treat complex systems with only limited computational resources (all the examples presented have been run on mid-range desktop computers).

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ON THE HYDRAULIC AND STRUCTURAL DESIGN OF FLUID AND GAS FILLED INFLATABLE DAMS TO CONTROL WATER FLOW IN RIVERS

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Key words: Inflatable dams, composites, design.

1 INTRODUCTION

The German Federal Waterways and Shipping Administration operates about 280 weirs, half of which are more than 50 years old. Many of these weirs will therefore need to be refurbished in the near future, even though budget resources are shrinking. An inflatable dam is a relatively new gate type, which enables savings to be made on the capital spending and maintenance costs. It consists of a multi-ply rubber membrane (Figure 1), is filled with air or water and clamped to the weir body with one or two fixing bars (Figure 2). Inflatable dams have a number of advantages when compared with steel gates [2]:

- The design is simple and does not include any moving parts (hinges, bearings); there are no problems due to corrosion or sealing and no lubricants used, which might be harmful to the environment. Inflatable dams are not affected by settlements or earthquakes.
- Drive mechanisms, such as hydraulic cylinders, electrical actuators or chains, which require a great amount of maintenance are not needed. Inflatable dams are controlled by inflating or deflating by injecting and discharging air or water.
- The cost of recesses and reinforcement is low and the transfer of forces into the weir sill is evenly distributed. Major refurbishments are thus facilitated considerably, especially if the existing concrete structure has to be included.
- Inflatable dams can be operated safely and can always be deflated to prevent blocking. The membranes can be installed or replaced within a few weeks so that the construction times and periods for inspection and refurbishment are considerably reduced.

In spite of their advantages, there is still much scepticism regarding the use of inflatable dams. This is partly due to the damage that has occurred in the past and partly due to the lack of design principles.



Figure 1: Micrograph of a three-layer, 12 mm thick rubber membrane: cross section (left) and plan view (right) [3].



Figure 2: Water-filled inflatable dam – Isometric drawing

2 HYDRAULIC DESIGN ASPECTS

2.1 **Overflow characteristics**

Physical models were used to investigate the hydraulic engineering problems (Figure 3). The effect of various hydraulic boundary conditions, internal pressures and types of clamping

systems on the discharge capacity, the geometry of the inflatable dam, the sensitivity of the rubber body to vibrations and the function of countermeasures, such as the installation of deflectors and breakers, were investigated in numerous series of tests.



Figure 3: Physical Model in the laboratory

One of the characteristics of air-filled inflatable dams is that the water flow over the dam ceases to be evenly distributed when the internal pressure drops. The inflatable dam will then collapse at one point, usually near one of the abutments. This is due to the fact that the pressure differential on the headwater side is not constant, as it is in the case of the water-filled type, but varies with the overflow depth. As membranes are very thin two-dimensional load-bearing structures with relatively low bending stiffness, the system will become unstable and the membrane will be folded or dented. The resulting V-shaped "dent" will cause the inflatable dam to be loaded on one side only and the downstream riverbed to be subjected to locally higher loads. Stationary vortexes can develop in the tailwater which may result in sloped banks being subjected to higher loads. Practical experience has shown that this does not adversely affect the regulation of the headwater level. Air-filled types used to control water levels will collapse in this way even if the overflow depths are low [1].

2.2 Causes of vibrations and the effects of countermeasures

Due to their elasticity, inflatable dams change their geometry in dependence of the pressure distribution along the surface, so that the occurrence of vibrations can be very different in their characteristics (mode shape, amplitudes and frequencies). Generally four types can be distinguished: vibrations of the nappe, vibrations due to pressure fluctuations, vibrations due to uplift forces and vibrations of the deflated membrane [5].

In order to reach a dynamic similarity of the vibration behavior in experimental models, the bending stiffness EI must be taken into account. Tensile tests were carried out on samples, to estimate the Young's modulus in model and nature. In order to examine the range of vibrations, the vertical amplitudes at the crest of the rubber body were measured, using a laser distance measuring device which works according to the triangulation principle. A further evaluation took place with the help of a Fast Fourier Transform (FFT) [1].



Figure 4: (a) Dam height vs. time during the in-situ investigations with range of vibrations, (b) amplitudes of the vibrations depending on the distance of the breakers [4]

As a result of the above experiments, it can be shown that vibrations of the water-filled type can be observed only in a small range of dam heights when no countermeasures are provided and a constant upstream water level is considered. Then vibrations appear suddenly and disappear during further deflation of the dam. It is induced by an unstable separation point of the nappe and the resulting pressure fluctuations on the downstream side of the dam. Through the adaptation of a row of breakers vibrations can be avoided or can be at least significantly reduced. In order to improve the shape, the location and the separation distance of the breakers, several test series have been carried out at an existing water-filled dam with two spans of a width of 15.60 m each and a dam height of 0.84 m [5].

3 STRUCTURAL DESIGN ASPECTS

3.1 Geometry and membrane force

In the case of inflatable dams, the internal pressure should be taken as the control variable and the relationship between the internal pressure and the design dam height h_d is defined as the internal pressure coefficient α . In the design case for the components, i.e. with-out overflow, there is a good correlation between the results of the calculations performed with analytical and numerical methods (finite-element model with ABAQUS) and the geometries measured in the model test (Figure 5). Thus, for many applications, the geometry and membrane force can be computed as a function of the internal pressure coefficient α , the density of water ρ_w , the gravitational constant g and the dam height h_d , which corresponds to the upstream water depth when there is no overtopping. Analytical solutions allow developing design charts for all relevant parameters of a cross section [1,4].

Water-filled-type
$$T = \frac{1}{4} (2 \alpha - 1) \rho_w g h_{d^2}$$
(1)

Air-filled-type
$$T = \frac{1}{2} \alpha \rho_w g h_{d^2}$$
 (2)

By contrast, when designing inflatable dams with overflow, the differences between the results of numerical and analytical models and the results of the model tests become greater as the overflow depth increases. This is due to the fact that the deviation from the hydrostatic pressure distribution increases with the overflow depth owing to the conversion of static to kinetic energy [2].

3.2 3D Finite-Element model of inflatable dams using quasi-static fluid-structure interaction

To optimize inflatable dams in terms of stresses, stability and impact of swimming structures a 3D finite element model was created. Provided that the filling of the inflatable dam proceeds quasi-statically and both top water and bottom water can be considered quasi-static and the water flow is relatively small, the fluid can be described by an energetically equivalent load vector [6] which can directly be correlated to certain water depths and water or gas volumes. The fluid parameters pressure and density describe completely the fluid filling and are input parameters of the finite element simulation.

The structural model is created using the geometric input parameters like the length of the inflatable dam or the angle of the flanges. In total the structural geometry and thus the model depend on eight independent geometric parameters. The concrete part of the structure is modeled by rigid walls.

Simulations have been done with various geometric parameter sets, with fluid and/or gas filling of the inflatable dam and different heights of top and bottom water using the currently implemented routines in LS-Dyna [7], which permits the use of different material models and contact formulations in combination with multi-chamber quasi-static fluid-structure interaction.

An empty and filled inflatable dam is shown in Figure 6. The internal volume of the inflatable dam is described by the red, yellow and brown part of the model, while the volume of the head water is calculated with the green and yellow part and the bottom water volume is described by the blue and red part. In total the inflatable dam and the tub are discretized with about 75000 shell elements. A fairly fine mesh is chosen for the locations where folding of the membrane is expected.



Figure 5: Comparison of the geometries of the rubber body determined with the FE model, the physical model and the analytical calculation [2]



Figure 6: Model: a) initial state of uninflated tube and b) final state of inflation with gas

3.3 Impact of swimming trees and the effect on stresses

The standard loading of inflatable dams is by top and bottom water thus pressure. The top water however, can carry trees or sediment. The impact of a tree increases the stresses in the inflatable dam near the impact zone and for this reason affects the stress concentration factor.

In the finite element simulations the tree is modeled by a cylinder of length 5m and weight 4t which swims with a constant velocity and hits the inflated dam under an angle between 0° and 90°. To show the effects of the filling the dams have been filled with different water heights and gas pressures.

Considerations show, that – not unexpected - a higher velocity of the tree or a completely gas filled dam with a low gas pressure would cause the longest deformations and highest stresses.

In Figure 7 the finite-element setting in an inflated state with the tree at initial position and the impact of the tree is shown. To show a case with a relatively large deformation of the dam the initial pressure has been set to a low pressure of 0,2bar and the velocity of the tree at time of impact 4 m/s.



Figure 7: Inflated dam with idealized tree before and during impact

To compare stresses in this extreme example the stresses of the area of impact have been compared with the stresses of the complete dam. The stresses in the area of impact increase during the impact, but reduce shortly after the tree leaves the dam, see Figure 8. After a longer time period which is not included in the figure, the stresses and the geometry return to the state before the impact.



Figure 8: Von Mises stresses in area of impact

Comparing these maximum stresses in the area of impact with the maximum stresses near the flanges shows that these impact stresses do not change the stress concentration factor which is important for design. The maximum von Mises stresses in this particular inflatable dam are about 6 MPa, while the maximum stresses in the impact area do not exceed 2,9 MPa.

For lower velocities of the tree or dams filled with water the effect of the impact on the stresses is even smaller. So in general we can conclude that the impacts of object such as trees do not affect the maximum stresses of a dam. Further investigations will include the effect of sharp edges and the overrun of trees with sharper parts.

3.4 Comparing cross-section deformation of fluid and/or gas filled dams

At built gas filled inflatable dams it can be observed that these dams are prone to buckling, see Figure 9, while fluid filled dams do not develop such a V-notch.



Figure 9: V-notch in gas filled dam

To simulate the effects of top and bottom loading on the cross section of the inflatable dam, the dam was filled up to a water level of 3m. Three different states have been investigated: a head water level of 2m, a head water level of 1.5m and a head water level of 2m while the bottom water level was set to 1m. Figure 10 shows that the deformation of the cross section under head and bottom water is relatively small.



Figure 11: Cross-section deformation of gas filled dam

In comparison the dam is now filled with gas (gas pressure p=0.0125 N/mm²), see Figure 11. As a consequence the cross-sections show much larger deformations. In addition, gas filled inflatable dams are more prone to buckling.

4 CONCLUSIONS

The contribution covers some results of the investigations conducted during an interdisciplinary Research & Development project and gives an outline of first experiences varying design, construction and operation.

The results of extensive investigations with physical models form the basis for discussing the causes of vibrations and the effects of countermeasures. As a result of the experiments, it can be shown that vibrations can be observed in a small range of dam heights when no countermeasures are provided. Vibrations appear suddenly and disappear during further deflation of the dam. By the adaptation of a row of breakers vibrations can be avoided or at least significantly reduced. Information on the location and form of the breakers will be discussed.

The practical application of the results has been carried out on two inflatable dams within the area of responsibility of the WSV: Figure 12 shows the row of breakers in Bahnitz: the breakers were spaced by 1.0 m and located at 85 and 95 % of the deflated membrane length, alternately.



Figure 12: Weir Bahnitz east of Berlin: row of breakers (left), weir in operation (right)

In addition the important issues of cross-section deformation and impact of trees are investigated by an explicit finite element analysis. The filling is described by energetically equivalent loading which takes into account the considerable deformations and volume change during the filling phase and in the loading phase. In LS-DYNA such a feature was already included for gas and has within the project been enhanced for fluid filling and combinations of gas and fluid loading. With this new feature a numerical study on the crosssection deformation showed large deformations of gas filled dams while fluid filled dams are far less prone to buckling. Investigating the effect of trees hitting the dam showed that stresses in the area of impact are increased compared to the standard loading but remain still smaller than the maximum stresses in the complete dam which are usually in the region of folds at the flanges.

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MULTISCALE SEQUENTIALLY-COUPLED FSI COMPUTATION IN PARACHUTE MODELING

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Abstract. We describe how the spatially multiscale Sequentially-Coupled Fluid–Structure Interaction (SCFSI) techniques we have developed, specifically the "SCFSI M2C", which is spatially multiscale for the structural mechanics part, can be used for increasing the accuracy of the membrane and cable structural mechanics solution in parachute FSI computations. The SCFSI M2C technique is used here in conjunction with the Stabilized Space–Time FSI (SSTFSI) technique, which was developed and improved over the years by the Team for Advanced Flow Simulation and Modeling (T \star AFSM) and serves as the core numerical technology, and a number of special parachute FSI techniques developed by the T \star AFSM in conjunction with the SSTFSI technique.

1 INTRODUCTION

The spatially multiscale Sequentially-Coupled Fluid–Structure Interaction (SCFSI) techniques were introduced in [1] as spatially multiscale for the fluid mechanics part, which is called "SCFSI M1C", and then in [2] as spatially multiscale for the structural mechanics part, which is called "SCFSI M2C". In SCFSI M2C, the time-dependent flow field is first computed with the (fully) coupled FSI (CFSI) technique and a relatively coarser structural mechanics mesh, followed by a structural mechanics computation with a more refined mesh, with the time-dependent interface stresses coming from the previously carried out CFSI computation. With this technique, the FSI computational effort is reduced where it is not needed, and the accuracy of the structural mechanics computation is increased where we need accurate, detailed structural mechanics computations, such as computing the fabric stresses. We can do this because the coarse mesh is sufficient for

the purpose of FSI computations, and using more refined meshes does not change the FSI results that much. However, mesh refinement does make a difference in detailed structural mechanics computation.

The SCFSI M1C and SCFSI M2C techniques have been used in conjunction with the the Stabilized Space–Time FSI (SSTFSI) technique, which was developed and improved over the years by the Team for Advanced Flow Simulation and Modeling (T \star AFSM) and serves as the core numerical technology, and special techniques developed in conjunction with the SSTFSI technique. In the case of the SCFSI M2C technique, the applications have been in parachute FSI modeling, and therefore the special techniques developed in conjunction with the SSTFSI technique have targeted parachute computations.

The SSTFSI technique was introduced in [3]. It is based on the new-generation Deforming-Spatial-Domain/Stabilized Space–Time (DSD/SST) formulations, which were also introduced in [3], increasing the scope and performance of the DSD/SST formulations developed earlier [4, 5, 6, 7] for computation of flows with moving boundaries and interfaces, including FSI. This core technology was used in a large number of parachute FSI computations (see, for example, [3, 8, 9, 2, 10, 11, 12, 13]). The FSI coupling is handled with the direct and quasi-direct FSI coupling techniques, which were introduced in [14] and are generalizations of the monolithic solution techniques to cases with incompatible fluid and structure meshes at the interface. They remain robust in computations where the structure is light compared to the fluid masses involved in the dynamics of the FSI problem, which is the case in parachute modeling. They were used in a large number of parachute FSI computations (see, for example, [3, 8, 9, 2, 10, 11, 12, 13]).

Computer modeling of large ringsail parachutes by the T \star AFSM was first reported in [8, 9]. The geometric challenge created by the construction of the canopy from "rings" and "sails" with hundreds of ring gaps and sail slits has been addressed with the Homogenized Modeling of Geometric Porosity (HMGP) [8], adaptive HMGP [2] and a new version of the HMGP that is called "HMGP-FG" [10]. These special techniques make the problem tractable. Additional special techniques the T \star AFSM introduced in the context of ringsail parachutes include the FSI Geometric Smoothing Technique (FSI-GST) [3], Separated Stress Projection (SSP) [8], "symmetric FSI" technique [2], a method that accounts for the fluid forces acting on structural components (such as parachute suspension lines) that are not expected to influence the flow [2], and other interface projection techniques [15].

The SCFSI M2C technique was used in [2] and [11] for increasing the accuracy of the membrane and cable structural mechanics solution in parachute FSI computations, and we provide in this paper an overview of those computations.

2 MULTISCALE SCFSI M2C COMPUTATIONS

2.1 Structural mechanics solution for the reefed stage

In [2] the SCFSI M2C technique was used for increasing the accuracy of the structural mechanics solution for the parachute reefed to approximately 13%. During the descent of a spacecraft, the parachute skirt is initially constricted to reduce forces on the parachute structure and the crew, and this is called the reefed stage. The skirt diameter is constrained using a reefing line, with length characterized by the "reefing ratio": $\tau_{REEF} = D_{REEF}/D_0$, where D_{REEF} is the reefed skirt diameter and D_0 is the parachute nominal diameter. Starting with the fully open parachute geometry, which is relatively easier to compute, an incremental shape determination approach based on gradually shortening the reefing line was used in [2] to compute the parachute shape at reefed configurations. Because the objective was just to determine the parachute shape, the symmetric FSI technique was used.

The coarse structure mesh used in the CFSI computation consists of 31,122 nodes and 26,320 four-node quadrilateral membrane elements, 12,441 two-node cable elements, and one payload point mass. The membrane part of the structure forms the structure interface and has 29,600 nodes. More information on the computational conditions, including the homogenized-porosity values, fluid mechanics mesh, time-step size and iteration numbers and computational steps followed, can be found in [2]. Figure 1 shows the structural mechanics solution for the parachute reefed to $\tau_{REEF} = 13\%$ (approximately).



Figure 1: Structural mechanics solution for the parachute reefed to $\tau_{REEF} = 13\%$ (approximately), obtained with the CFSI computation and the coarse structure mesh.

In the SCFSI M2C computation, the interface stresses were extracted from the CFSI computation described above and were used in a structural mechanics computation with a more refined mesh. The interface stress projected to the structure consists of only the pressure component of the interface stress, and the SSP technique is used for the projection. The refined structure mesh has 128,882 nodes and 119,040 four-node quadrilateral membrane elements, 23,001 two-node cable elements and one payload point mass. The membrane part of the structure forms the structure interface and has 127,360 nodes. At this reefed configuration, the interface stresses obtained in the symmetric FSI computation do not have a significantly dynamic nature, and therefore the time-averaged values were used.

As a related technique, a "cable symmetrization" procedure to be applied to the canopy cables during the structural mechanics computation with the more refined mesh was proposed in [2]. In this procedure, it was proposed that for the cable nodes at each latitude, the tangential component of the displacement is set to zero, and the radial and axial components are set to the average values for that latitude. This can be done as frequently as every nonlinear iteration, or as few as just once. In the computation reported in [2], it was done just once and that was during the starting phase of the computation. Also, in the computation reported in [2], the actual symmetrization procedure used was a close approximation to the proposed one. Figure 2 shows the canopy cables before and after symmetrization. In addition to and following that symmetrization, the cable



Figure 2: Structural mechanics solution for the parachute reefed to $\tau_{REEF} = 13\%$ (approximately). Canopy cables before (left) and after (right) symmetrization.

positions are fixed and the computation is continued until the membrane parts of the

canopy structure settle. After that we release all the structural nodes (except for the payload) and compute until the solution settles. Figure 3 shows the structural mechanics solution obtained with the SCFSI M2C computation and the refined mesh. Figure 4



Figure 3: Structural mechanics solution for the parachute reefed to $\tau_{REEF} = 13\%$ (approximately), obtained with the SCFSI M2C computation and the refined structure mesh.

shows the structural mechanics solution obtained with the SCFSI M2C computation and a picture from a NASA drop test.

2.2 Fabric stress computations

It was shown in [11] that the SCFSI M2C technique can be used for computing the fabric stresses more accurately by increasing the structural mesh refinement after the CFSI computation is carried out with a coarse mesh. It was also shown how the SCFSI M2C technique can be used for computing the fabric stress more accurately by adding the "vent hoop" after the FSI computation is carried out without it. The vent hoop is a reinforcement cable placed along the circumference of the vent. Again, we can do this because the structural model without the vent hoop is sufficient for the purpose of FSI computations, and including the vent hoop does not change the FSI results that much. However it makes a large difference in the fabric stresses near the vent.

In the tests carried out in [11] with the SCFSI M2C technique, the interface stresses are extracted from the FSI computation reported in [10] (for the case where the horizontal speed of the payload is instantaneously hiked to 20 ft/s to emulate the swinging motion). The stress projected to the structure consists of only the pressure component of the interface stress, and the SSP technique is used for the projection. Also, to expedite the tests,



Figure 4: Structural mechanics solution for the parachute reefed to $\tau_{REEF} = 13\%$ (approximately). Left: obtained with the SCFSI M2C computation and the refined structure mesh. Right: picture from a NASA drop test.

in [11] a time-averaged, circumferentially symmetric pressure was applied to the structure. The coarse structure mesh for the canopy has 29,200 nodes, 26,000 four-node membrane elements, and 10,920 two-node cable elements. The fine mesh has 115,680 nodes, 108,480 four-node membrane elements, and 21,640 two-node cable elements. Adding the vent hoop increases the number of cable elements by 80. Figures 5 and 6 show the coarse and fine meshes for one gore.

The cases with and without a vent hoop were computed in [11] using both meshes, resulting in a total of four test cases. Additional information on the computational conditions, including the time-step size and iteration numbers and computational steps followed, can be found in [11]. Figures 7 and 8 show the fabric (maximum principal) tension for the coarse and fine meshes with no vent hoop. Figures 9 and 10 show the fabric tension for the coarse and fine meshes with a vent hoop. Figures 11 and 12 show, for the cases without and with a vent hoop, the maximum fabric tension for each ring and sail, computed with the coarse and fine meshes.

3 CONCLUDING REMARKS

We showed that the spatially multiscale SCFSI techniques we have developed, specifically the SCFSI M2C technique, which is spatially multiscale for the structural mechanics part, can be used very effectively for increasing the accuracy of the membrane and cable structural mechanics solution in parachute FSI computations. In the computations reported here, the SCFSI M2C technique is used in conjunction with the SSTFSI tech-



Figure 5: Coarse structure mesh for one gore.



 $\label{eq:Figure 6:Fine structure mesh for one gore.$

nique, which serves as the core numerical technology, and a number of special parachute FSI techniques developed in conjunction with the SSTFSI technique. We presented results from computations where the SCFSI M2C technique is used for increasing the accuracy of the structural mechanics solution for the parachute reefed to approximately 13%, for computing the fabric stresses more accurately for a fully open parachute, and for computing the fabric stress more accurately when a vent hoop is added to the parachute structure.

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Figure 7: Fabric tension for the coarse mesh with no vent hoop.

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Figure 8: Fabric tension for the fine mesh with no vent hoop.

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Figure 11: Maximum fabric tension for each ring and sail for the case with no vent hoop. Coarse mesh results denoted with red diamonds and fine mesh results denoted with blue squares.



Figure 12: Maximum fabric tension for each ring and sail for the case with a vent hoop. Coarse mesh results denoted with red diamonds and fine mesh results denoted with blue squares.

STRUCTURAL DYNAMIC FAÇADE A.P.H.W. HABRAKEN

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Key words: lightweight structures, cable net, membrane structure, pneumatic structures, second skin, dynamics, hyperbolic, sound barrier, water shield, Schiphol, Polderbaan

Summary: Second skin façades are used to improve the building physics of a façade not for structural reasons. This research at the Eindhoven University of Technology focuses on the structural advantages of form-active lightweight second skins façades using their dynamic structures behaviour.

By detaching the form-active façade from the rest of the building, larger deflexions are possible. Their flexible behaviour and the dynamic structural response to (wind)loading reduces the internal forces and optimizes the efficient use of material. The second skin takes a global stabilizing role as a main structural element

1 INTRODUCTION

Façades play a very important role in the quality of a building. It forms the barrier between the internal space and the outside climate. In this role of building physics, second skin façades are designed to make use of the natural energy sources as wind- and solar energy.

Structurally the façade line is the most efficient line for placing the main structural elements. Therefore high-rise building have often a tube structure integrated in the façade. It is interesting to see how a second façade can be active as a structural element.

These two technical aspects must be designed in line with a third very important role of the façade, the architectural quality. The façade is the medium through which the interaction takes place between the activities inside and outside. The image of a building, and therefore also of the users, is reflected through the design of the façade

The danger of the combination of all three - building physics, structure and architecture - will lead to a compromised design instead of fully optimizing all three aspects. Often the technical requirements of the façade will restrict the architectural wish of transparency.

At the Eindhoven University of Technology several studies are done in which the second skin façade is part of the main structural system and takes a global stabilizing role. The function of the façade is focussed on absorbing wind energy. This limitation in function allows us to use form-active lightweight façade structures that reshape under different load patterns. Because of allowing these movements two major advantages can be obtained:

- forces are transferred by axial forces, resulting in higher efficiency and less use of material,
- forces are absorbed in a dynamic way resulting in lower internal forces and therefore less use of material.

After describing both issues, two projects will be presented with a form-active lightweight second skin façade.

2 FORM-ACTIVE FAÇADE STRUCTURE

Membrane structures, pneumatic structures and cable net structures are typical form-active structures. They can only support loads by deformation. Without external loads, the internal prestress forces are balanced by a form-finding process. When the structure is loaded externally the shape must change to find a new balance in forces.

The structural elements are flexible and cannot take bending moments. Internal forces are only axial forces or forces in plane of the membrane. These internal forces will activate and use the full sections of the material efficiently, reducing the amount of material used.

Minimising structural material by increasing its efficiency is the key of the behaviour of lightweight structures. Form-active structures can be very efficient but because they adjust in shape to resist loading there stiffness is often not sufficient for certain usages. Form-active structures are therefore often used where stiffness and the related deformation is less critical, as in large span roof structures.

Looking at this in another way, does it mean that when we allow higher deformations that we can reduce the amount of material? Maybe this is not an overall correct statement but in many cases this could be correct. For instants, compare a concrete floor with a trampoline. A concrete floor is often more than 200mm thick with a heavy weight to provide the comfort in limiting the deflexions and vibrations. A trampoline is a lightweight structure from a few millimetres thick membrane with a high flexibility. To support the load it must deform largely. They both serve their purpose.



Figure 1: Stiff and heavy concrete floor



Figure 2: Flexible, lightweight form-active trampoline⁸

It is therefore most important to find the optimal link between the requirements of a structure and its capacity / behaviour. Depending on the requirements it could be that the solution is not to resolve all issues within one structure. Combining the best characteristic of multiple structural systems can result in a more optimum result.

3 SPLIT OF STRUCTURAL FUNCTIONS

There is often an architectural wish of high transparency in the façade, but with high rise buildings structural requirements often restrict this. To stabilize a building, the façade is the best location to place the construction. This so called "tube structure" build up from façade columns and beams can stabilize the building by moment-stiff connections or introducing bracing between them.

Buildings need to be stabilized in a way that no unpleasant horizontal movements occur. This is not only governed by the amount of movement in millimeters, but mainly by the acceleration a person will feel from the moving floor the person is standing on. This restriction has a high impact on the structural sizing of the stabilizing structural elements in the façade.

The loading on buildings we can split in functional loading (live load) and non-functional loading (wind- and snow load). Live load, as of people standing on a floor slab, need strength and stiffness for the above mentioned comfort reasons. For wind loading strength is needed in a structure but stiffness could be interpreted differently. The wind loading however is transferred via the same stiff structure as the live load to the foundation that could result in an interference of requirements. The wind loading can accelerate the structure that supports functional loading in an uncomfortable way. This way the wind load is a governing load case, requiring to strengthen the structure to meet its primary requirements.

Would it be a good solution to design an independent form-active façade structure for wind loading and allow larger deflections to increase the efficient use of the materials? In this way the independent façade protects the internal space in an efficient way, prevents horizontal wind forces and acceleration to act on the internal structure, making it possible to design the overall stability structure in a different way.

In 1971, a feasibility study called "City in the Antarctica" showed us an air-supported building over a city. The building, a climatic shell spanning 2km, protected the buildings underneath from rain snow and wind. At the same time, it also functioned as a first physical plane.



Figure 3: 'City In Antarctica'; study of Frei Otto, Kenzo Tange en Ove Arup & Partners'

Although this is a futuristic design, imagine a building placed within a protected space, embraced by a structural independent film. The scale is reduced, but the principle stays the same.

The protection against external influences will reduce the structural requirements of the building within, and increase both the efficiency in use and the architectural freedom. Structural requirements have moved towards the embracing structure. Because of the structural independency between the building and the embracing skin, the horizontal movement of the embracing structure is not limited by the value of acceleration that is acceptable to people. Consequently, the allowable deflections and accelerations in the embracing structure can be much higher. This opens a whole new field of structural possibilities, especially within the field of using lightweight structures.



Figure 4: Tube structure ³

Figure 5: Independent second façade structure ³

Limiting the stability requirements of the internal structure will put earthquake resistance in a different perspective. The more flexible the building, the less energy it attracts. The link to earthquake resistance is part of the research at the Eindhoven University of Technology.

3 DYNAMIC BEHAVIOUR

Wind is often calculated as a static load on a static structure. In reality it is a dynamic wind spectrum acting on a dynamic structure. This raises the questions "How is the dynamic load transferred to a static load?" and "What difference in impact does a loading have on a static or a dynamic structure?".

When a constant force is pushing against a fully encastre mast, the properties of the mast does not influence the internal forces and the foundation forces. When a dynamic loading is acting on the mast the properties of the mast does make a difference. A more flexible mast will reduce the internal loading and therefore the foundation forces. You can take grass moving in the wind as a reference. It would break if reality would be static.

This can be explained by hitting your hand against a concrete wall, this hurts. But when you place a pillow between it makes a big difference. In both cases the energy in your movement (kinetic energy) is the same and is transferred to the concrete wall. Without the pillow this energy transfer is done in very short amount of time. With the pillow the time in which your energy is transferred is longer, therefore the force is lower.

Beneath we compare the internal forces, the foundation forces and the deformation of a 10m high mast with different stiffness. At the top of the mast an impact of a mass of 15 kg with a speed of 10m/s is modelled. The following results are found.

Model	Mast A	Mast B
Mast length	10 m	10 m
Mast profile	Round 150x5 steel	Round 250x5 steel
	tube	tube
Section area	2277 mm2	3848 mm2
Moment of Inertia	600 cm4	2900 cm4
Displacement top by 1 kN	271 mm	56 mm
Spring stiffness	3690 N/m	17860 N/mm
Kinetic force load – 15 kg at	750 J	750 J
10 m/s		
Period	0,4 s	0,182 s
Frequency	2,5 Hz	5,49 Hz
Maximum displacement at	640 mm	290 mm
top		
Maximum force at top	2,35 kN	5,18 kN
Maximum bending moment	23,5 kNm	51,8 kNm
in mast		
Maximum stress in mast	294 N/mm2	224 N/mm2
Maximum reaction force	23.5 kNm	51,8 kNm

Table 1 : comparison impact on two different mast profiles

The deformation of model A is larger, allowing to transfer the kinetic energy of the mass in a longer period to the mast, resulting in a lower force impact, a lower internal bending moments and lower foundation forces.

Form-active structures like pneumatic structures have a low stiffness. The impact of their dynamic behaviour can therefore be substantial. The internal forces and the forces on the foundation due to peak wind speed will reduce because of their large deformation and the longer period of kinetic energy transfer.



Figure 6: wind speed measurements with peaks of wind speed ⁶

3 PROJECT 1 – HYPERBOLIC OFFICE TOWER

A hyperbolic office tower is embraced with a structural second façade, that protects the building in a structurally independent way. The second façade is made from a hyperbolic cable net structure with a central mast, clad with pneumatic membrane elements.

The cable-net structure is a form-active structure that will deform when loaded by wind. This deformation will limit the impact of peaks in wind speed. Internal forces are axial forces only. Together with the high material strength of the cables and its dynamic behavior this results in a high transparent façade.

Figure 7 shows impressions of the design of the building. Columns are only located along the inner floor edge, although the main stability structure is located in the façade.



Figure 7: Impressions of the building design ³

The internal space within the cable-net can be used freely for the design of the independent internal building.



Figure 8: Model of a hyperbolic cable net structure ³



Figure 9: Sketch of a hyperbolic shaped building embraced with an independent cable net structure 3 .

The cable net structure can be calculated separately. The structure is prestressed to prevent cables to go slack and to reduce the deformation. In figure 10 the deformation is calculated to be maximum about 900mm, between the second façade and the building flexible connections are made capable of bridging these differences in movement, but to control the air movement within the cleft.



Figure 10: deformation because of windloading ³

Figure 11: Structural second skin façade as a cable net with pneumatic cushions infill 3

Together with the basement and foundation structure the cable net is designed as a structurally closed system. The tension forces in the cable net balance the compression forces in the pylon by connecting them in the basement. This excludes pre-stress forces on the foundation. Figure 3 shows that the internal forces balance in a 3-dimensional way.



Figure 12: Structurally closed system of the embracing hyperbolic cable net skin structure 3

3 PROJECT 2 – PNEUMATIC SOUND BARRIER

Occupants of Hoofddorp in the Netherlands experienced for many years noise nuisance from the low frequent sound produced by planes during their start at Schiphol.

Natrix is a design for a pneumatic sound barrier that symbolizes the particular Dutch relation between land and water. The noise barrier is a fluent arc of water carried by air referring to the high see level and air as the supporting medium of airplanes.

The 1800m long Natrix has the shape of a snake, built-up with an inner and outer skin. The inner skin consists of pneumatic air arches spacing 6m with in between PVC-coated polyester membrane. On top of the inner membrane water tubes with a diameter of 28mm are placed side by side for sound absorption and as medium to store and transport sun energy.



Figure 13: 3D-visualization birds eye



Figure 14: 3D-visualization – section part¹

The outer skin exists from transparent foil called ETFE strengthened by a cable net and stabilized by an overpressure between the ETFE-foil and the PVC-coated polyester membrane. The cable net has a varying grid pattern to change its absorbing frequency along

the length of the runway according to the change of sound frequencies of an airplane during his take off.



Figure 15: Optimized sound absorption by changing the pattern in the diagonal network ¹.

The inner and outer skin are fully disconnected from each other. By the application of the two layers, a cleft is created that acts as an insulation layer. Also the overpressure is only between the outer and inner skin, meaning the inside area is not under overpressure and available for all functions.



The maximum deformation with an internal air pressure of 0.3 kN/m2 and a wind speed of 9 on the scale of Beaufort is 2.5m. The maximum deformation with an internal air pressure of 0.6 kN/m2 and maximum wind force is approximately the same. The construction manages larger deformation, but it has been limited to 2,5m so that the external skin - with sufficient safety – never contacts the internal skin.

The pneumatic build-up of the structure will result in an external membrane that will deform by wind loading, while the internal membrane hardly moves. This is because the outer and inner skins are structurally not connected and deformation differences will be absorbed by air movement in the cleft. Studies at the Eindhoven University of Technology confirm this behaviour.

Because the people inside do not experience the deformation of the outer skin, there is no need for too low deformation limitations, resulting in a higher efficiency of material governed by strength.

Fluctuated wind acts on the outer skin. The outer skin deforms in a dynamic way. The larger the deformation the longer it takes to transfer the energy of a wind gust to the structure, the lower the impact force, the lower the internal and foundation force.



Figure 18:. Deformation of external skin loaded on wind loading ¹.



Figure 19: Outer skin deforms because of wind loading, but inner skin hardly moves ¹.

Because of the independent behaviour of the external and internal skin and the air movement in the cleft the varying external loading has limited influence on the internal skin. The pneumatic arches, supporting the internal skin, therefore are mostly loaded with permanent equal divided loading: self weight, internal skin, the water tubes and the air pressure in the cleft. When the differentiation of the loading pattern on an arch structure is limited, the arch can be designed as an optimum compression arch with limited bending moments. This will make it possible to use slender arch profiles.

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CONCRETE SHELL STRUCTURES REVISITED: INTRODUCING A NEW 'LOW-TECH' CONSTRUCTION METHOD USING VACUUMATICS FORMWORK

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Key words: Vacuumatics, Concrete Shells, Construction Process, Formwork System.

1 INTRODUCTION

Concrete shell structures, often referred to as 'thin shells', have been around since the 1930's. The design of these thin shells was stimulated by the desire to cover wide spans in an economically attractive manner. Typically, the thickness of concrete shells is relatively small compared to the curvature and span. The main reason for concrete shells to be economically feasible (especially from a material point of view), is that shells are structurally efficient in carrying loads acting perpendicular to their surface by in-plane membrane stresses. Bending moments may occur locally to satisfy specific equilibrium or deformation requirements, but are considered relatively small in general. The construction process of concrete shells was considered extremely labour-intensive and time-consuming. From the 1960's the interest in concrete shell structures suddenly decreased. The reason for this was that the biggest motivation for designing concrete shells, reducing material costs, was losing ground to the rapid increase of labour costs.

In the last decade, curved (concrete) structures in general seem to have (re-)gained popularity, supposedly due to the vast developments in digital modelling technology. In contrast to a few decennia ago, nowadays literally all 'thinkable' shapes are easily drawn by Computer Aided Design (CAD) software and even calculated by advanced Finite Element Modelling (FEM) software. Nevertheless, the construction process of concrete shells seems to have lacked the same degree of development as the design and engineering processes. Although Computer Aided Manufacturing (CAM) equipment is available in some industries, the limiting factor at the moment, with respect to the realisation of concrete shell structures, turns out to be the manufacturability and adaptability of the formwork system.

This paper provides a new perspective on the construction process of concrete shell structures and introduces a new cost saving approach for constructing (single curved) concrete shells using Vacuumatics formwork.

DESIGN-BASED CLASSIFICATION OF (CONCRETE) SHELL STRUCTURES 2

The diversity of shell structures is vast. Any surface that is curved in one or more directions can be considered a shell surface. Shell surfaces may be defined by the classification of their curvature, expressed in terms of Gaussian curvature. The Gaussian curvature of a curved surface is the product of the two principle curvatures: $\kappa_g = \kappa_1 \cdot \kappa_2$. A positive Gaussian curvature characterises a clastic surface, whereas a negative Gaussian curvature characterises an anti-clastic surface. Cylindrical surfaces (as well as planes) have a Gaussian curvature of zero (Figure 1).



Figure 1: Gaussian curvatures: clastic (a), anti-clastic (b) and cylindrical (c)

A more comprehensive approach of describing shell surfaces, however, is by concentrating on the way the surface is generated (or designed). In 1980 Heinz Isler¹ identified three types of shells according to this philosophy, referred to as 'Geometric', 'Structural' and 'Sculptural'. In this paper we will further elaborate this classification by re-interpreting Isler's terms in an attempt to clarify the origin of each shell form.

2.1 Analytical Forms ('Geometric')

A blooming period of widespread concrete shell construction took place from the 1930's, where engineers like Felix Candela, Eduardo Torroja, Anton Tedesko and Pier Luigi Nervi managed to design, calculate and construct extremely elegant concrete shells (Figure 2).



Figure 2: Les Manantiales Restaurant in Xochimilco Mexico City by Felix Candela²

As the designs between 1930-1950 were mainly based on mathematical defined geometries, these shell shapes can be referred to as 'Analytical Forms'³. Typical traditional analytically-based surfaces are referred to as 'revolution surfaces', 'translation surfaces' and 'ruled surfaces' (Figure 3).



Figure 3: Analytical Forms: translation surface (a), revolution surface (b) and ruled surface $(c)^4$

Since no digital design and calculating equipment was available in that time, the mathematical formulas were not only essential for drawing and calculating these structures, but also aided the actual construction process. The majority of the thin concrete shell structures were constructed by pouring wet concrete onto a rigid wooden formwork, often assembled from straight elements. This construction process required many skilled craftsmen (Figure 4).



Figure 4: formwork construction by Felix Candela²

2.2 Experimental Forms ('Structural')

In the 1950's, engineer Heinz Isler introduced a slightly different approach for designing thin concrete shells. In the spirit of Antonio Gaudi's hanging models, he successfully applied

several 'natural' phenomena, like air pressure, gravity and material flow, to design thin concrete shells (Figure 5). Due to the experimental character of his approach, these shapes can be referred to as 'Experimental Forms'³. Structural calculations were made by conducting load tests on small-scale models which were interpreted for the design of the full-scale concrete structure.



Figure 5: Isler's Experimental Forms, based on air pressure, gravity and material flow⁵

From structural point of view, in particular Isler's shells based on gravity, behaved superior to the Analytical Forms from the 1930's. The explanation for this is that these shapes obey the laws of nature under their own weight (pure compression or pure tension), whereas Analytical Forms are merely approximations of these 'natural' forms.

As Isler's designs were not easily described analytically, the construction process was considered somewhat more complex than it was the case with Analytical Forms. Nevertheless, Heinz Isler managed to design his formwork is such a clever way (using amongst other things prefabricated curved wooden segments) that he was able to re-use it numerous times, even integrating thermal insulation into his formwork system⁶ (Figure 6).



Figure 6: formwork system by Heinz Isler⁶

2.3 Digital Forms ('Sculptural')

After a sudden decrease in interest in curved (shell) structures from the 1960's, an increased interest arose in the 1990's as rapid developments in digital modelling technology offered new possibilities for architects and engineers. Where in the past the design and engineering of curved (shell) structures were the playing field of a few experts, nowadays a

larger group of designers is able to design and even calculate almost any thinkable shape. The term 'free-form' has become an integral part of modern design, effectively utilising CAD, FEM and even CAM technology (Figure 7). In spirit of Isler's afore mentioned terminology, this third type of shell geometry will be referred to as 'Digital Forms'³. With these types of structures the shell shape is no longer based on structural efficiency (and thus material reduction), but rather derived from aesthetics and spatial functionality.



Figure 7: Kakamigahara Crematorium by Toyo Ito and Matsuro Sasaki⁷

3 (CONCRETE) SHELL STRUCTURES ANALYSED

The aforementioned shell categories might easily be considered as several consecutive steps in the evolution of shell structures. However, this would imply that each successive type is superior to the former. As can be illustrated with a timeline indicating the dominant category of each shell type, this is not the case³ (Figure 8). All three categories are still being realised these days. Furthermore, each of the three categories has had its peaks and low points, often related to the peak of the career of an influential designer or engineer or imposed by a certain bottleneck within the building process. In many cases, his bottleneck appears to be directly related to the degree of knowledge or technology available.

In general, it can be stated that Analytical Forms originate from the need to mathematically describe a curved shell surface in order to calculate and construct them. Experimental Forms, on the other hand, are designed very intuitively and are considered structurally 'pure', yet more difficult to calculate and construct. Digital Forms offer an unprecedented freedom of form and can be calculated with sophisticated digital modelling technology, but are even more challenging to construct. This history of concrete shell structures teaches us that successful examples are, without exception, the result of a highly integrated design and building process. Whereas engineers of the 1930's to 1960's often were considered to be the architect as well as contractor embodied within one person, nowadays this synergy in (shell) building can only be achieved by making architect, structural engineer and construction specialist cooperate from the early stages of the building process³.

When focussing on the biggest bottleneck nowadays, with respect to the successful realisation of concrete shell structures, we can conclude that the construction process in particular requires a new impulse in order to 'keep up' with the vast developments of digital modelling technology (with respect to design as well as engineering). In particular the manufacturability and adaptability of the formwork system (related to construction time and labour costs) seems to be the limiting factor. Focusing on the construction methods used in

the past, we can conclude that the largest number of shell structures is constructed by means of conventional timber formwork. In the last few years several 'new' techniques have been developed (like CNC moulds, fabric formwork, adjustable moulds using independently activated pistons), but only minor success had been reached on replacing timber moulds, mainly due to lack of adaptability or lack of repetition for prefabrication, or due to a large waste production or simply because of too large initial (start-up) costs. Therefore, the need arises for an economically attractive or perhaps 'low-tech' formwork system, that discards the abovementioned disadvantages altogether.



Figure 8: history of concrete shell structures³

3.1 A philosophy for successfully constructing concrete shells

From a theoretical point of view, it would make sense to consider the way shell surfaces are generated as the most effective construction method. Strictly speaking, many of the successful aforementioned concrete shells confirm this philosophy. For instance, Candela's designs are constructed using straight elements to create hyperbolic-paraboloid-shaped (or hypar) shells. Although Isler (as a successful exception) still used conventional timber formwork for the full-scale construction, others however, did follow the 'generation = construction'-philosophy and used an inflatable formwork to successfully construct dome-shaped shells (e.g. Binishells³). With respect to Digital Forms, CAM technology seems to be the way to go.

4 VACUUMATICS FORMWORK

Inspired by the aforementioned philosophy, now Vacuumatics formwork will be discussed with respect to the construction process of (single curved) concrete shell structures. Furthermore, the analogy with the construction process of gridshells will be illustrated. First, a small introduction on Vacuumatics and gridshells in general.

4.1 Vacuumatics

Vacuumatic structures, or Vacuumatics, consist of structural aggregates (particles) that are tightly packed inside a flexible membrane envelope (skin). The structural integrity is obtained by applying a (controllable) negative pressure, or partial vacuum, inside the surrounding skin, hence prestressing and stabilising the particles in their present configuration by means of the atmospheric (air) pressure. This process is referred to as 'vacuum prestressing'. When subjected to bending forces, the particles take up the compressive (contact) forces, whereas the tensile forces are mainly taken up by the membrane envelope (Figure 9). The tensile strength (and even the flexural rigidity) can be substantially enhanced by adding a piece of reinforcement (e.g. a piece of textile) in the tensile zone of the structure (analogues to reinforced concrete).



Figure 9: structural principle of Vacuumatics

The beneficial morphological characteristics of Vacuumatics are their ability to be 'freely' shaped and even to be re-shaped repeatedly to fulfil new geometric requirements. These characteristics provide a promising approach for the design of a temporary, adaptable load-bearing structure and thus a truly flexible and reconfigurable self-supporting formwork system⁸.

As systematic research on the flexural rigidity of vacuumatic structures has illustrated⁹, the structural behaviour of Vacuumatics largely depends on the individual characteristics of its filling particles and surrounding skin, as well as on the interaction of these particles with the enclosing skin and the particles mutually. Typically (when using thin plastic films like LDPE), large deflections tend to occur at relatively low bending forces (Figure 10). This specific characteristic might be used beneficially when applied as a formwork system. That is, the vacuumatic structure will be easily curved and 'only' needs to withstand the concrete mortar pressure until the structure is sufficiently hardened. As Vacuumatics behave substantially better when submitted to compression forces rather than bending forces, this implies that when the vacuumatic structure is shaped into its final shell form, it will be able to withstand an external load very effectively by in-plane membrane stresses. The bending stresses are, like mentioned before, considered relatively small (i.e. dependent on the shell shape).



Figure 10: large bending deformations of Vacuumatics

4.2 Gridshells

An interesting approach for the design of shell structures in general was (re-)introduced by Frei Otto in the 1970's, as he put new life into the structural principle of gridshells¹⁰. (Gridshells, also known as lattice shells, were originally pioneered by the Russian engineer Vladimir Shukhov in 1896). Gridshells, are basically shell structures where material has been removed to create a slender lattice grid pattern. Where in plain shells load paths are available all over the surface, in gridshells the internal forces are transferred via discrete members. Inspired by the suspension models of Antonio Gaudi, Frei Otto designed his gridshells by inverting the form of a suspended soap film or that of a flexible suspending net¹⁰ (Figure 11).



Figure 11: Mannheim gridshells design based on a hanging model¹⁰

From construction point of view, the powerful concept that lies behind gridshells is that the construction starts from a flat surface. The straight members are assembled on ground level as a flat mesh. The final shape of the structure is obtained by locally forcing (i.e. deforming by pushing and pulling) the members perpendicular to the surface and fixing the connections and boundaries once the shell reached it desired (equilibrium) shape (Figure 12). To allow this transformation to take place, the connections of the grid need to be initially 'flexible', enabling scissor motion as well as sliding motion.



Figure 12: Mannheim gridshell construction¹⁰

5 VACUUMATICS FORMWORK IN THEORY

Analogues to gridshells, the simplistic (or rather low-tech) assembly of Vacuumatics enables an intuitive design approach for constructing shell structures by manipulating a flat surface. From design point of view, it can be stated that in some cases a close relationship might be seen with the second category of concrete shells (Experimental Forms), as these forms are also often derived from manipulating a flat surface into its desired (equilibrium) shape (e.g. by inflation or suspension). A big difference, however, is that with Vacuumatics (as is the case with gridshells) some degree of initial flexural rigidity exists that influences the internal equilibrium and therefore the shaping process.

A beneficial characteristic of vacuumatic structures is that the flexural rigidity can be (partially) regulated by simply adjusting the level of vacuum pressure. A higher degree of 'deflation' will lead to a higher amount of structural prestressing and therefore to a larger resulting bending stiffness. Only a marginal amount of vacuum pressure is required in order to keep the particles from shifting inside the membrane envelope due to the gravitational forces as the structure is being deformed. When the intended shape is reached, the level of vacuum pressure can be increased to its maximum (approximately 1 bar), which stabilises the shell shape. After hardening of the concrete (which will most likely be applied like shotcrete), the Vacuumatics formwork can be re-inflated (or 're-flated') and peeled off from the cured concrete surface and even be re-used to create an entirely different curved shell shape. Using this so-called 'flexibility control' (Figure 13), the shaping process of Vacuumatics formwork can be effectively directed⁸.



5.1 Shaping of Vacuumatics formwork: suspension method

One of the most imaginative methods of shaping Vacuumatics is the so-called 'suspensions method' (analogues to Isler's suspended cloth and Frei Otto's cable nets). In initial, flexible (or 'deflated') state Vacuumatics behave like a tensile structure (i.e. under pure tension), where the particles act as the (enhanced) dead-weight of the structure. Preliminary small-scale tests of the suspension method (total length = approximately two meters) show promising results (Figure 14).



Figure 14: preliminary tests of the suspension method using Vacuumatics

When inverted, the Vacuumatics formwork (and with this the intended concrete structure) behaves like a shell structure under pure compression. Self-evidently, the inverting process might be considered a rather delicate operation, in particular in case of relatively large shell structures.

In addition, the deformation of the Vacuumatics structure due to its dead-weight can even be restricted (or rather regulated) if desired by increasing the level of vacuum pressure (and thus the flexural rigidity). In other words, an equilibrium state will be reached at a relatively smaller deflection. Even the opposite is possible, as the derived suspended deformation can be enlarged by locally manipulating the structure (i.e. pulling parts the structure downwards or moving the edges of the structure inwards).

5.2 Shaping of Vacuumatics formwork: lifting method

Another (and perhaps more practical) method, derived from real-time gridshell construction, is to locally lift the structure into its final shape. The deformation can be controlled by jacking up 'internal' supports or even using inflatable devices (Figure 15).



Figure 15: lifting method of gridshell construction¹⁰

It needs to be taken into account, that in this case the gravitational forces in fact initiate the deformation process, which requires the structure to have a certain minimal degree of (initial) flexural rigidity. A big advantage, is that with this method no inversion of the formwork in its the final shell shape is required. A simple example of lifting a piece of paper by its symmetry axis, perfectly illustrates the potential of this method (Figure 16). Once the structure reaches its intended shape, the edge supports need to be fixed in order to stabilise the shell, before the concrete (e.g. shotcrete) can be applied.



Figure 16: potential of lifting method illustrated with a sheet of paper

6 CONCLUSION

The construction process of concrete shell structures appears to be the limiting factor with respect to realisation of concrete shell structures in a economically attractive manner. There for, a new impulse is required to keep up with digital modelling technology used in design and engineering. From a theoretical point of view, it would make sense to consider the way shell surfaces are generated (analytically, experimentally or digitally) as the most effective construction method. Inspired by this philosophy, Vacuumatics formwork provide a relatively intuitive and 'low-tech' approach for constructing efficiently shaped concrete shells, by using principles derived from 'nature' as well as real-time gridshell construction, hence saving time, labour as well as material.

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THE USE OF FABRICS AS FORMWORK FOR CONCRETE STRUCTURES AND ELEMENTS

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Summary. This paper presents and describes a series of studies into the use of flexible fabrics as formwork for concrete structures as alternative to conventional rigid formwork.

1 INTRODUCTION

Concrete is, arguably, the most significant of all construction materials, certainly it is the most ubiquitous of materials and is available in one form or another throughout both the developed and developing world. It has many excellent qualities. It is strong, durable and fire resistant and has good thermal and acoustic properties. Apart from cement its constituents are easily sourced and generally require little processing. In the history of architecture, contemporary concrete is a relatively new material, in effect dating back to the end of the 19th century. During its lifetime, from casting to component it exists in two states, an initial fluid condition and a hardened rigid permanence. During the initial state the concrete has to be held in a formwork that defines the final form. The formwork is responsible for the geometry, the surface, the texture and the overall finished quality of the piece. Despite its extensive use concrete has also received considerable criticism as architects 'let concrete be concrete', allowing the contemporary and often rough finish produced with conventional timber formwork to be expressed. This form of expression became known as 'Brutalism' and unfortunately also became associated with poorly constructed and poorly detailed projects. The poor reputation could also be attributed to the poor quality and execution of the formwork rather than the material itself. The established paradigm for concrete is to use planar timber forms. Perhaps there has been a mindset in builders that concludes that as the formwork is disposable and temporary it requires less care in execution. There are, however, many examples of very high quality concrete in architecture² and generally these examples require particular diligence in both design and execution. Thus a dichotomy exists in our understanding of concrete, between its ubiquitous use and the quality of its construction that is centred around the use of planar timber forms. The engineer Pier Luigi Nervi, described this situation,

'It may be noted that although reinforced concrete has been used for over a hundred years and with increasing interest during the last few decades, few of its properties and potentialities have been fully exploited thus far. Apart from the unconquerable inertia of our minds, which do not seem able to adopt freely new ideas, the main cause of this delay is a trivial technicality: the need to prepare wooden forms '³

Flexible fabrics and textiles make excellent form-work for concrete ^{4,5} The use of fabrics as formwork started initially with Felix Candela and Miquel Fisac ^{6.7} The flexible nature of the fabric allows it to adjust its form in response to the hydrostatic pressure of the wet concrete the resulting in a very efficient use of material. Other characteristics of the fabrics provide additional benefits. The permeable nature of the fabric allows the excess water in the concrete to bleed, reducing the pressure on the formwork, trapped air is also able to escape, resulting in fewer surface blow holes and blemishes and hence a better quality of surface finish. A review of the benefits of permeable formwork has been prepared by Malone.⁸ Fabric formed concrete produces non-planar, often complex surfaces that are more visually interesting than flat surfaces. Thus textiles offer new possibilities for the expression and the pragmatic design of concrete. However, these admirable qualities may also be problematic. In construction accuracy and precision is essential. This paper presents a series of studies into the practical use of fabric formed concrete in the development, application and construction and performance:

- connection and precision
- texture and surface
- the structural behaviour and design of form-active beams
- the construction of shell forms
- the relation between geometry and construction process.

These studies have been undertaken in the Architectural Research Workshop at the University of Edinburgh and in a series of live projects in the UK. Figure 1 shows the walls developed for the Royal Horticultural Show in Chelsea in 2009. These were constructed in collaboration with Alan Chandler from the University of East London and students from both institutions.



Figure 1: Fabric formed concrete panels at the RHS Chelsea 2009

An important aspect of the work is the development of technique through repeated prototyping. An overview of some of these projects is presented to demonstrate the range of elements that can be produced, the effect that different fabrics has on the surface finish of the concrete and the accuracy that can be achieved in components. It concludes that the use of fabrics as formwork presents a new approach to concrete design that offers considerable potential providing the design is based on a clear understanding of the material nature of both the concrete and textiles.

2 METHODOLOGY

The research has been undertaken through a series of workshops with students of architecture, empirical experimental research studies and live projects. The workshops encourage students, working in groups, to explore the potential of the technology and develop new construction forms through design, experimentation and repeated prototyping. Emphasis is placed on both the expressive qualities of the technology and pragmatic issues such as precision, repeatability of form, quality of finishes and sequence of construction. The research studies consider detailed aspects of performance such as structural behaviour and optimisation of form. The live projects are used to test the application of the techniques by interfacing with other disciplines and professions in real-time construction programmes. These effectively remove the research from the workshop and place it in the field. The range of components that can be produced using textiles as formwork is extensive; ranging from simple twodimensional panels to complex three-dimensional wall constructions. An important aspect is to maintain the inherent simplicity of the process. Normally conventional concrete mixes are used, without additives or admixtures. A typical mix uses aggregates (10mm maximum size), sand and cement in relative proportions 10:5:4. The amount of water added is critical to ensure that the hydration of the cement will occur and the concrete has an appropriate degree of workability and can flow easily into the formwork. Typically the added water is 0.55 as a proportion of the weight of cement. Depending on the type of element the fabric is cut, shaped and then restrained by a frame. The fabric can be pre-tensioned to further control the geometry. As the concrete is placed into the formwork the fabric reacts, responding to the hydrostatic pressure, adopting a form-active surface as tension develops. During casting the some of the excess water in the mix will seep through the fabric. The rate of seepage is dependent on the permeability of the fabric. This has three benefits, firstly the hydrostatic pressure on the formwork is reduced, secondly entrapped air also escapes and thirdly the ratio of water to cement reduces and therefore increases the strength and durability of the concrete. During the casting procedure one can determine the degree of compaction and fill of the concrete by applying pressure through the fabric, not possible with conventional rigid formwork, which is largely a blind process once the concrete enters the formwork. After the concrete has hardened the fabric can be stripped. Most tightly woven fabrics can be removed easily. Fabrics with loose fibres or open weave may be more difficult to remove and may leave traces of fibre within the concrete. Very good and consistent results have been obtained with polypropylene geotextile fabrics.

3 EXAMPLES OF FABRIC FORMWORK ELEMENTS

The studies have considered a large variety of different elements and components. A selection of projects is presented to demonstrate the range of application and discuss some particular factors

3.1 Studies in texture and surface

Concrete is made from particles of varying sizes. The smallest particles are the cement grains, typically 15 microns on average. An effective mix is a well-graded blend of particle sizes. The finer cement particles migrate to the surface, defining the interface of the concrete with the formwork. The fineness of the cement is therefore able to replicate the texture of the fabric itself. The simplest fabric cast elements are panels. Fabric is stretched onto a frame, generally incorporating an upstand to contain the concrete around the perimeter. Concrete is cast directly onto the stretched fabric and finished to the level of the upstand. The weight of the panel causes the fabric to stretch and deform. The degree of deformation depends on the stiffness of the fabric, which can then be used to create additional surface undulation. Figure 2 demonstrates the fine surface grain that can be transferred from the fabric to the concrete.





Figure 2 Fine surface texture

Figure 3 Smooth textured fabric

Figure 3 shows a panel using a smooth, coated fabric that creates a sheen on the surface of the panel the pattern has been produced by placing a series of steel rings under the fabric, imprinting the concentric circles into the concrete as the fabric deforms. The variety of effects and textures that can be produced simply by manipulating the type of fabric and patterns is almost limitless.



Figure 4 Rain-screen façade using concrete



Figure 5 Rain-screen panels applied to school wall

In collaboration with a stone engineering contractor a demonstration wall was constructed to illustrate some of the variety of textures and surfaces possible using different textiles, figure 4. The panels were developed from the experience gained through a series of studies into surface and texture. The panels were attached to an aluminium framing system designed for thin stone cladding. The minimum thickness of the panels is 35 mm. The system is known as an open jointed rainscreen, systems of this type are used extensively in contemporary buildings. In a subsequent development the technique was applied to a project for a local primary school in Edinburgh. The pupils developed their own designs for the panels, figure 5.

3.2 Connection and precision

As the wet concrete is poured into the formwork the fabric responds by deforming, adopting a shape dictated by the effects of gravity, the degree of deformation is conditioned by the nature of the fabric, the hydrostatic pressure, the initial restraints applied by the formwork and pre-tension within the fabric. At present there exists little methodology to fully predict the exact geometry of a complex fabric cast form. In conventional formwork systems the use of rigid materials ensures consistent geometry throughout the element. In contemporary construction with its increasing emphasis on off-site operations and pre-fabrication the interfaces between components has become critical. At first consideration therefore it may seem that the self-organising nature of fabric formed concrete might be at odds with the need for precision. However the most important issue is to ensure accuracy where is it needed. Studies of the connections between fabric formed elements have been an integral part the research. In figures 4 and 5 the edge details between the panels was designed to ensure consistency in gap size and alignment between the panels although each panel was quite different.



Figure 6 Accuracy, repeatability and connections

Figure 6 illustrates the development of a system for highly articulated columns consisting

of a series of twisted and perforated components. Each component has a slightly different form and uses a different fabric to create a variation in texture. The components connect to each other using an interlocking joint. The ends of each component were produced using a vacuum formed plastic insert into the formwork that ensured a high level of dimensional accuracy and fit. The components can be assembled in a variety of configurations.

Figure 7 shows a study of a column construction that utilises laser cut profiles to manipulate the surface, creating an undulating surface profile.



Figure 7 Undulated surface in column created by laser cut plywood ribs

The formwork consists of a cylinder of fabric stretched between two points. Normally uniform hydrostatic pressure will produce a column with a circular cross section. A sequence of vertical plywood profiles was positioned around the perimeter of the column to displace the fabric and allow the concrete to bulge in a controlled manner. The plywood strips were arranged in three rings around the column. The upper and lower rings were aligned vertically with each other whilst the middle ring was rotated relative to the other two. The result was an organic, articulated form that demonstrates the precise control over the deformation of the concrete that can be obtained.

3.3 Design of form-active beams

The geometry of a form-active structure follows the principal forces applied to it. Beams represent one of the most common structural elements in architecture and the most commonly occurring loading system (a load distributed uniformly along the length of the beam) for beams produces a parabolic distribution of bending moment; with a maximum at mid-span and diminishing to zero at the supports. The planar nature of conventional formwork generally simplifies the geometry to prismatic beams with a rectangular cross section, and therefore, non form-active shapes for which the formwork is relatively simple to construct but whose geometry is inherently inefficient. Constructing form-active beams using traditional methods is labour intensive and expensive. The use of fabrics to create a form-active geometry is quite straightforward. A study was undertaken to investigate the structural behaviour and design of form-active beams. Figure 8 illustrates the distribution of bending



momnent in a uniformly loaded and figure 9 shows a fabric cast form-active beam.

The initial geometry of the beam is as shown in figure 9. The web of the beam follows the parabolic shape in figure 8. The steel reinforcement also follows the same profile. The projecting flange was developed to create an effective support at the bearings of the beam. Initial structural tests demonstrated that failure of the beam occurred at the bearings, where the flange met the web. The geometry of the cross section was gradually adapted through a series of successive modifications based on the analysis of structural tests. The final form that the process produced is shown in figure 10.



Figure 10 Final shape of concrete beam

The beam still follows the parabolic profile, but the major changes in geometry occur at the support. As the web gets shallower it also gets wider towards the supports. An upward curve has been added to the underside of the flange doubling the thickness of the flange towards the supports. These improvements in geometry have resulted in an increase in structural performance of the beam. By adjusting the geometry to place the concrete where it is most effective the failure mode of the beams changed and the load carrying capacity increased by over 35% for beams with the same amount of steel reinforcement⁹. As can been seen from figure 10 this mode of investigation through successive adaptation would have been difficult using conventional methods of formwork production. Each adaptation would have required significant labour in the construction. The use of fabric facilitated the repeated iterations of test, modify and make. A comparative study between of the embodied energy in the final design with a structurally equivalent prismatic beam indicated a reduction of 25- $40\%^9$.

3.4 Shell structures

During the development of reinforced concrete in the early to mid 20th century one structural typology came to signify both the expressive and structural potential of the material, namely shell structures. Shell structures carry their load predominately by compressive stress. They are similar to traditional constructions such as vaults and domes but differ in a very particular way. Vaults and domes tend to be mass constructions and heavy. Angerer ¹⁰. proposed a new classification to describe these structures, 'surface structures'. The primary difference being the thinness of the shell in relation to the span. Techniques such as graphic statics¹¹ and physical models allows the optimum geometry for efficient transmission of axial compressive stresses to be determined. Designers such as Torroja, Isler and Candela¹¹ used these techniques to produce very efficient and expressive structures. West¹² has constructed shells using fabrics as formwork. The fabric is suspended between supports and concrete applied directly. The fabric deforms to carry the concrete in tension resolving itself into the ideal catenary geometry. Once the concrete has set, the hardened form is inverted and the resulting shell has the optimum form for compressive forces. The most sensitive part of the process is during inversion of the form. A project at the University of Edinburgh also studied the construction of thin catenary shells using fabric, figure 11. The geometry was based on the Gaussian vault developed by Eladio Dieste¹¹. The overall form of the vault was described using two catenary curves of equal span but varying heights. Plywood profiles were made to follow these curves, which were then constructed into a frame. Fabric was stretched between the two profiles. Thin layers of concrete were applied directly to the fabric creating a series of compression curves spanning between the support points of varying heights between the two profiles. A preliminary study was undertaken to match the mix proportions with a variety of fabrics to determine best arrangement for adhesion of the concrete sloping surface of the fabric. A mix in relative proportions 4.5:2.5 coarse sand to cement was used. The shell has a thickness of 35 mm along both the principal curves that define the edge of the shell.





Figure 11 Concrete shell and formwork

3.5 Further studies in geometry and construction

In a recent project a group of students explored the complexity of form that could be produced using fabrics whilst maintaining an effective construction process. They sought to construct a formwork that could create a three-dimensional, highly perforated wall. A detailed series of construction studies investigated the effectiveness of filling a formwork with a complex arrangement of cavities. From these studies they determined parametric relationships concerning the position, shape and size of voids within the wall to fill the formwork effectively and minimise creases in the fabrics and surface defects in the concrete. The final wall, approximately 2.0 metres in height, was constructed as a single continuous formwork. Whereas previous projects has sought to produce assemblies of components by developing particular connections to join already cast pieces together this project sought to put emphasis on the monolithic qualities of concrete and thus all joints were formed within the fabric itself. The fabric was laid out and became the drawing on which the final design was developed, figure 11. Two layers of fabric were stitched together. A sequence of shapes was drawn on the fabric and then stitched through both layers. The shapes prevent concrete entering and create voids when the fabric is removed. The position and orientation of the shapes used information from the earlier parametric study to facilitate the placing and distribution of the concrete.



Figure 11 Layout and marking of formwork

Figure 12 Fabric formwork

The fabric formwork was suspended from a sheet of plywood supported on a light steel frame, figure 12. Sleeves of the formwork were fed through holes in the plywood and used to fill the formwork. The fabric was then stretched and attached to a second sheet of plywood at the base of the frame. The formwork was arranged to describe an enclosed space, notionally with three sides. The group had previously developed techniques to create monolithic junctions in the fabric. The concrete was added in a carefully prepared sequence to ensure that all the branches were filled evenly. Reinforcement was added as the concrete was placed. The

concrete was placed in one continuous pour without interruption. The finished wall is shown in figure 13. There were very few blemishes or creases in the surface of the piece.





Figure 13 Images of complex form

4 CONCLUSIONS

The studies presented in this paper are a sample of the projects undertaken at the University of Edinburgh. They have been selected to demonstrate the range and variety of applications of fabrics as formwork for concrete. The use of fabrics, whilst not a universal panacea, offers benefits over conventional formwork techniques in a number of ways:

- Ease of construction
- A high quality of surface texture and appearance
- An almost limitless variety of finishes
- Accurate and precise in construction
- Efficient structural forms, such as shells and form-active beams
- Great complexity in geometry

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DEVELOPMENT AND EVALUATION OF MOULD FOR DOUBLE CURVED CONCRETE ELEMENTS

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Key words: Double curvature surface, flexible mould, organic architecture, CNC milling.

Summary. The present paper describes a concept for a reconfigurable mould surface which is designed to fit the needs of contemporary architecture. The core of the concept presented is a dynamic surface manipulated into a given shape using a digital signal created directly from the CAD drawing of the design. This happen fast, automatic and without production of waste, and the manipulated surface is fair and robust, eliminating the need for additional, manual treatment. Limitations to the possibilities of the flexible form are limited curvature and limited level of detail, making it especially suited for larger, double curved surfaces like facades or walls, where the curvature of each element is relatively small in comparison to the overall shape.

1 INTRODUCTION

Complex freeform architecture is one of the most striking trends in contemporary architecture. Architecture differs from traditional target industries of CAD/CAM technology in many ways including aesthetics, statics, structural aspects, scale and manufacturing technologies. Designing a piece of freeform architecture in a CAD program is fairly easy, but the translation to a real piece of architecture can be difficult and expensive and as traditional production methods for free-form architecture prove costly, architects and engineers are forced to simplify designs. Today, methods for manufacturing freeform concrete formwork are available, and more are being developed¹⁻⁴. The common way of producing moulds for unique elements today is to manufacture one mould for each unique element using CNC milling in cheaper materials, but since the method is still labor intensive and produces a lot of

waste, research is carried out in several projects to find a solution, where one mould simply rearranges itself into a variety of familiar shapes. Such a concept has natural limitations, but would become a complimentary technology to the existing.

The present paper describes the development of a digitally controlled mould that forms a double curved and fair surface directly from the digital CAD model. The primary motivation for the development of the mould is to reduce the cost of constructing double curved, cast elements for architecture, both in-situ cast and modular. Today, such elements are usually cast in milled formwork that is expensive and produces a lot of waste. Architects are often limited in their freedom of design by the high costs of the existing methods and as a result, the possibilities for drawing and evaluating complex shapes in architecture today, are not reflected in the build architecture.

2 CONCRETE CASTING TECHNIQUES

Today, a number of technologies have emerged, that offers casting methods for a range of purposes. On a large scale, the market is dominated by well known techniques such as precast elements made from standard moulds and in-situ casting in standardized modular systems. On a small scale, new methods for casting and new types of moulds have emerged to meet the rising demand for customization and creation of curved concrete architecture. Some of the methods for double curved moulds which have been investigated related to the present project are mentioned below.

2.1 Milled foam moulds

The milled foam method represents the newest and the most economic version of custom manufactured moulds, historically made by hand and recently milled in different materials using CNC.



Figure 1: Photo of a robot CNC-machine milling in a styropor material.

The advantage of foams in comparison to heavier materials is, that they are cheaper compared on volume, they allow fast milling, and they are easy to manually alter and fair after the milling process, that leaves a grooved surface texture. The main strength of the method is that it can be used for very advanced geometry as long as it is possible to de-mould the casted object. Further there is almost no curvature or detailing level limitations besides that of the milling tool. Another clear advantage for this method is that the entire surface is manufactured to tolerances. The weakness of the method is, that it requires manual fairing and coating to a large extend, if the surface has to be of a perfectly smooth, polished quality. For a large project, the formwork is extensive, and after use it has to be thrown out, creating even more waste than was produced during milling.

2.2 Textile formwork

Casting in membrane formwork or textile moulds have been around for a long time. A common feature for all objects that can be cast in this way is, that they must consist of convex surfaces exclusively, since the method relies on the principle that all textiles must be in tension caused by the viscous pressure. The final shape of the cast piece is a result of the membrane's adaption to the pressure, like the shape of an inflated balloon.

The formwork is inexpensive in comparison to the surface area and totally smooth surfaces can be achieved. Because of the limited use of material, little waste is produced. Being inaccurate, the formwork can be produced relatively fast without the use of advanced equipment. The inability to do anything else than convex forms is a distinct limitation. Also, the only precisely controlled parts of the cast geometry, is where the formwork have been fixed to its supports.



Figure 2: Photo of a textile formwork¹.

2.3 Spray applied concrete

This method has been around for decades, but is still used for curved surfaces today. In short, the concrete paste is mixed with chopped fiberglass in a spraying nozzle, and applied to an underlying form with the reinforcement iron bars bend in place. After application, the concrete surface is manually faired or kept rough. The fibers' added to the concrete serve to add extra strength, but more importantly to keep the newly applied concrete in place. The method is mainly used for in-situ castings, and can form large spans and surfaces in one continuous, structural piece, as the reinforcement is a continuous structure as well.

The method is very labor intensive, as both the bending of reinforcement and the application of concrete is a manual process. It is also very difficult to create perfectly smooth surfaces, as the surface is finished off using hand tools.



Figure 3: Photo showing spray applied concrete.

2.4 System based traditional formwork, PERI

PERI is a German producer of traditional scaffolding systems, but they have expanded their product portfolio to include both flexible single curvature formwork and custom double curved formwork. PERI specialty is that they use standard components for the production of all their form work and both the single curvature flexible form and their custom double curved forms are integrated into a complete and rationalized in-situ system. They have also developed software that can automatically determine what parts are needed based on a given geometry. It is a complete and reliable solution from software to hardware, design to construction. The main weakness is that there is still waste produced in the process of creating the double curved moulds, and that it is only possible to create double curved surfaces of very small curvature. The systems and methods shown above cover each their different aspects of freeform architecture. Whether building scale or curvature is taken into consideration, there

seems to be a gap in scale from the textile and milled moulds to PERI's large scale buildings and the labor intensive, hard to fair spray applied concrete. PERI's boards or plywood sheets forced to create double curvature has a fairly small maximum curvature, and it seems futile to use a precision tool like a CNC milling machine, with its capability to produce very accurate and complex geometries, to create larger modules of relatively small curvature without further detailing. When looking at this curvature scale - smaller buildings created from a larger number of precast elements of familiar scale and curvature, it seems such elements could be generated from a common tool, the curvatures of which could be found between the maximum curvature of the force-bend scaffolding from PERI and the small, complex curvatures possible by milled moulds.



Figure 4: Photo showing PERI's single curvature scaffolding.

A flexible tool could be competitive with foam milling in this area, if it were made, so that no additional, manual treatment of cast elements or surface were needed, no waste produced and production speed in comparison to equipment price were better. A tool for creating modular solutions should come with a software or system to rationalize production and communicate possibilities to architects. At the same time, the direct connection between drawing and machine, as with the CNC miller, should be established, to get an automated process. If a tool can be created to meet the criteria stated above, it could help promote the construction of the freeform architecture that is so commonly seen in digital architecture and competition drawings today, by offering cheaper and more efficient custom building parts. It could help bring the build architecture closer to the digital possibilities.

2.5 Flexible moulds

The most important aspect to consider when designing a flexible form is its limitations. The wider the desired range of possible shapes, the more difficult and advanced the construction will be. As discussed in the previous section, CNC milled foam moulds will at some point of complexity be the most attractive solution, as they are able to mill shapes that would be extremely hard to achieve by any other way of manipulating a surface. It is also clear, that no matter how a surface is manipulated in a flexible form, the very nature of the method results in a specialization in a common family of shapes. For instance, if a flexible mould were to create a perfect box, it may be designed to take different length, width and height, but because it needs specialized geometry like corners, it would be unable to create a sphere with no corners. A flexible mould aiming at the ability to do both, would possibly fail to achieve a perfect result in either case.

It all comes down to the fact, that every point on the surface of a flexible mould does not have the ability to change from continuity to discontinuity, because that would demand an infinitely high number of control points. Without an infinitely high number of control points, the flexible form, therefore, has to aim at creating smooth, continuous surfaces, the complexity of which must simply be governed by the number of control points. It is then left to decide, what the least number of control points is, relative to the properties of the membrane which will result in a mould design capable of achieving the curvatures needed for most freeform building surfaces. The initial motivation for the design of a flexible mould for double curved surfaces, was the encounter of other attempts to come up with a functional design for such a form, and the market potentials described in these projects. The technical difficulties and solutions defined in the projects presented here have been the inspiration for our present design.

2.6 Membrane Mould

The mould concept is to have a flexible membrane manipulated by air filled balloons. The use of balloons solves the problem of creating smooth bulges on a membrane with no stiffness in bending, but it is hard to control the tolerances. The edge conditions are, however, defined relatively precise by linear, stiff interpolators connected to rods and angle control.



Figue 5: Edge control by angle measurements¹.

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Figure. 6: This edge control means that the panels can be joined to create a relatively continuous surface¹.

2.7 North Sails

North Sails in North America produces custom cast sails in a digitally controlled, flexible form that uses a principle, where stiff elements created a smooth surface between points defined by digitally controlled actuators. They simply use what appears to be a thick rubber or silicone membrane which has an even surface, since it is supported by a large number of small, stiff rods placed close together underneath it. The small rods are placed on top of larger rods connected to the actuators. This simple system is possible because of the relatively small curvature in comparison to the mould size. The mould is highly specialized and appears to have been extremely expensive, but it is the best example of a flexible mould concept, that could easily be used to cast concrete panels, and it has been the main inspiration for the principles used in our mould.



Figure 7: North Sails mould with numerous actuators.

3 CONCEPT FOR A DYNAMIC SURFACE SYSTEM

In the proposed dynamic mould system, where only a set of points is defined, a stiff membrane interpolates the surface between those points. Stresses in the deflected membrane will seek to be evenly distributed and therefore it will create a fair curve through the defined points. A stiffer member will have a more equally distributed curvature, while a softer member will tend to have higher peaks of curvature near the defined points. This relation between the physical properties of a stiff member and the mathematical properties of a NURBS curve can be applied to surfaces as well. If a plate interpolator can be made, that has an equal stiffness for bending in all directions, and the freedom to expand in its own plane it would constitute a 3D interpolator parallel to the well known 2D solution. To function as a surface suitable for casting concrete or other substances against without the need for further manual treatment, the membrane should be durable and maintain a perfectly smooth and non-porous surface as well. A membrane with these properties has been developed for this project, and it is the core of the dynamic surface mould invention.

The number of actuators in a row defines the precision and possible complexity of the surface. A smaller number of actuators require a stiffer membrane and less control, a larger number means softer membrane and better control. In this way, the amount of actuators needed depends on the complexity of the surface.



Figure 8: Illustration of a surface deformed by 3, 4 and 5 actuators.

Five actuators in a section have been chosen not only because of the finer control, but also because the coherence between the NURBS surfaces in a CAD drawing and the physical shape of the membrane is better. Smaller leaps between the pistons mean less deflection caused by the viscous pressure, and most important, the edge conditions in a 5x5 configuration is less affected by the deflection elsewhere on the membrane, than they are with the 4x4.

3.1 Functionality and limitations

The mould can take any digitally defined shape within its limitations within one minute from the execution of a program reading 25 surfaces coordinates directly from the CAD design file. Once the actuator pistons have taken their positions, the handles must be adjusted manually to a fixed angle calculated by the program. The main limitation of the mould is its maximum curvature. It is defined by the construction of the membrane, and for the prototype, it is approximately a radius of 1.5m. The system can be scaled to achieve smaller radii. Another limitation is that the surface designed has to fit within the 1.2m x 1.2m x 0.3 m which is the box defined by the pistons. This box is adequate to create a square piece of a sphere as big as the mould, with a radius of 1.5m. For most of the freeform architectural references, these limitations mean, that it is conceivable to produce the main parts of the facades. Control of the actuators via CAD software is programmed using the Arduino platform and Rhino supporting NURBS which is ideal for generating surfaces applicable to the mould system. The information layout is based on the following; Arduino informs Rhino about current positions of all actuators and Rhino issues commands to Arduino which then positions all actuators. Controlling a surface in Rhino generates real-time feedback in the mould system and in-program information about curvature degrees and possible warnings. Typical scenarios of use have been implemented as to help from an early point in a design process. Starting from at double curved wall where the program will issue information about different subdivision possibilities and possible problems relating to manufacturability, to the coordinated virtual and physical handling of the separate elements on and off the mould system.



Figure 9: Casting of a double curved fiber reinforced concrete panel.

The following illustrations explain how the flexible mould can be used from design to production of freeform architecture.



Figure 10: Double curved surface, a subdivision of surface and validation of Subdivision



Figure 10: Positioning of actuators, adjusting edge angles and mounting mould sides.



Figure 11: Pouring filling, single or double sided moulds and of mounting elements

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Figure 12: Surface made up of elements produced with the double sided mould technique.

4 CONCLUSIONS

Complex freeform architecture is one of the most striking trends in contemporary architecture. Today, design and fabrication of such structures are based on digital technologies which have been developed for other industries (automotive, naval, aerospace industry). The present paper has presented traditional production methods available for freeform architecture which force architects and engineers to simplify their designs. Further the paper has described the development of a flexible mould for production of precast thin-shell fiber-reinforced concrete elements which can have a given form. The mould consists of pistons fixing points on a membrane which creates the interpolated surface and is fixed to the form sides in a way that allows it to move up and down. The main focus for the development has been on concrete facade elements, but a flexible, digitally controlled mould can be used in other areas as well. Throughout the project interest has been shown to use the mould for composites as well, and among other ideas are the idea of casting acoustic panels, double curved vacuum formed veneer and even flexible golf courses. The flexible mould concept has the ability to advance modern, free form architecture, as the concept offers cheaper and faster production of custom element.

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FABRIC-FORMED CONCRETE MEMBER DESIGN

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Key words: Fabric-formed, flexible, concrete, form-finding, concrete formwork.

Summary. This paper introduces engineering design and analytical modeling techniques providing the design community with an alternative means for forming concrete structures by using flexible fabric formwork.

1 INTRODUCTION

Since its invention by the Romans, concrete has been cast into all manner of formworks whether temporary or permanent. All-rigid formworks have become the containment form of choice for our modern concretes and an industry standard practice ever since humankind first sought to contain these early forms of mortar and "concrete" in their structures.

The American Concrete Institute's Committee 347^{1} (ACI) formally introduced the first standard guide for the design and construction of formwork in 1963. And, it was only recently (2005) that ACI Committee 334^{2} introduced a standard guide for the construction of shells using inflated forms even though several methods of construction using inflated forms³ have been available since the early 1940's. It can take many years to standardize methods of construction today regarded as experimental.

One such experimental and imaginative means of construction is the use of a flexible formwork that, while not inflated, still gives form to structural members previously cast in only all-rigid formwork. Given the need for a mortar or concrete to set and cure properly the use of a flexible formwork might appear to be rather ill-suited for casting any concrete member yet casting concrete into flexible formworks may in fact be used nearly anywhere a rigid formwork is used.

As a means for forming concrete this versatile way of containing concrete saw some of its first use in civil engineering works such as erosion control and now that strong, inexpensive geotextiles have become available it is also beginning to attract attention throughout the world for architectural and structural applications. Architects and designers in Japan, Korea, Canada and the United States have begun to use flexible fabric formworks made from geotextile fabric to form concrete members for their projects. However, a significant amount of research remains to be done to bring these forming systems into everyday practical use by the construction industry.

Standards and guidelines for using flexible fabric formworks need development in a timely manner so that the design community can take full advantage of this means for forming concrete members. This paper focuses on the engineering aspects involved in designing a precast concrete panel member, just one of numerous concrete member types used in architectural and structural works where fabric formworks may be used.

1.1 Research efforts

The author's first introduction to flexible formwork came from reading an article by Mark West, Director of the Centre for Architectural Structures and Technology (C.A.S.T.) at the University of Manitoba, Canada, published in *Concrete International*⁴. Canada is one of a number of countries with schools of architecture and engineering where students conduct research into this unique means of forming concrete. Other countries include the United States, England, Scotland, Mexico, Chile, Belgium and the Netherlands.

Architectural students at C.A.S.T.^{5, 6} explore the use of flexible formwork using cloth fabric and plaster before creating a full-scale cast of a concrete panel. The cloth fabric, when draped over interior supports and secured at the perimeter, deforms as gravity forms the shape of the panel with the fluid plaster as shown in Figure 1.



Figure 1: Model formwork and completed plaster casts (C.A.S.T. photos)

The casting of a full-scale panel using concrete requires finding a fabric capable of supporting the weight of the wet concrete. Ideally suited for this purpose are geotextile fabrics. These fabrics made of woven polypropylene fibers are low cost and have a high tensile strength component. The flexible fabric material is pre-tensioned in the formwork over interior supports where required to give the panel its desired aesthetic form. Geotextile fabric as a formwork material has a number of advantages including:

- The formation of very complex shapes is possible.
- It is strong, lightweight, inexpensive, reusable and will not propagate a tear.
- Less concrete and reinforcing are required resulting in a conservation of materials.

• Filtering action of the fabric improves the surface finish and member durability.

It also has several disadvantages including:

- Relaxation can occur due to the prestress forces in the membrane. There is the potential for creep in the geotextile material, accelerated by an increase in temperature as might occur during hydration of the concrete as it cures.
- The concrete requires careful placement and the fabric formwork must not be jostled while the concrete is in a plastic state.

The author believes however, the benefits of using geotextiles far outweigh any disadvantages until new fabrics are developed. Among the key benefits are economies of construction, durability of the product and freedom of design expression.

Figure 2 shows the interior supports for a full-scale formwork prior to stretching in the fabric membrane and the resulting completed concrete panel.



Figure 2: Full-scale formwork and completed concrete panel (C.A.S.T. photos)

For illustration purposes, an unreinforced 12'-0" long x 8'-0" wide x $3\frac{1}{2}$ "-thick (3.7 m x 2.4 m x 88.9 mm) wall panel will be designed for self-weight and a ±30 psf (±1.44 kPa) lateral wind load using a concrete strength of 5,000 psi (34.5 MPa).

1.2 A design procedure

Due to the complex structural shapes wall panels can take when formed in this manner the challenge is finding an appropriate method of analysis. Straightforward methods of analysis and design are available for the traditionally cast concrete column, floor beam, and wall or floor panel. Shapes as complex as these require the use of finite element analysis (FEA) software and a procedure to "form-find" and analyze the complex panel shape is required. Prior to a thesis⁷ and a paper⁸ by the author, to introduce a design procedure, analysis methods to predict the deflected shape of a fabric cast panel were unavailable.

We introduce a four-step procedure for analytically modeling a fabric formwork employing

the structural analysis program ADINA^{9, 10} to analyze the formwork and the concrete panel cast into it. The final panel form, function and performance of the fabric membrane and the reinforcement of the panel for design loads all add to the complexities of the panel's analysis and design. The four steps in this procedure are as follows:

- 1. Determine the paths the lateral loads take to the wall panel's anchored points.
- 2. Use the load paths, defined in Step 1, to model the fabric and plastic concrete material as 2-D and 3-D Solid elements, respectively. Arrangement of these elements defines the panel's lines of support.
- **3.** "Form-find" the shape of the panel by incrementally increasing the thickness of the 3-D Solid elements until the supporting fabric formwork reaches equilibrium. The process is iterative and equivalent to achieving a flat surface in the actual concrete panel similar to a ponding problem.
- 4. Analyze and design the panel for strength requirements to resist the lateral live load and self-weight dead load.

If, after a completed analysis of the panel in Step 4, it is found that the panel is either "under-strength" or too far "over-strength", adjustments to the model in Step 2 will be required and Steps 3 and 4 repeated. With this iterative process, it should be possible to obtain an optimal wall panel design. Prior to implementing this four-step procedure, however, the modeling techniques utilized in Steps 2 and 3 above require defining.

2 MATERIAL PROPERTIES AND ANALYSIS METHOD

Efficient modeling plays an essential role in the development of the finite element model. The finite elements making up the supporting fabric formwork and the elements, which eventually make up the final concrete panel shape, are defined in the same model. Once the final concrete panel shape is defined by using an iterative "form-finding" technique, the fabric elements are discarded. The concrete panel elements are then designed for the appropriate lateral loads under the given set of boundary conditions.

The difficulty with combining the two element types required to define the overall model is that they each have their own material properties, which can contribute to the overall strength and stiffness of the model. Initially, the concrete is plastic and considered fluid in nature, similar to a slurry. The slurry will contribute weight to the fabric element portion of the model but cannot contribute stiffness to it. Therefore, an intermediate step is required. In this step, the slurry – characterized as a material that has weight, but no strength or stiffness – is used as the material property for the concrete panel elements while the panel shape is being found.

2.1 Fabric model material properties

The geotextile fabric material, used as the supporting formwork, is anisotropic. The modulus of elasticity is different in the WARP (machine direction, along the length of the roll) and the FILL (cross-machine direction, through the width of the roll) directions. These differences are important when modeling the fabric as well as for securing it to the supporting formwork. Mechanical properties for geotextile fabrics are obtained from stress-strain curves

developed in accordance with the standard test methods of ASTM D4595¹¹.

Stress-strain data for the Amoco 2006 geotextile fabric obtained from Amoco Fabrics and Fibers Company¹² allowed the properties shown in Table 1 for this elastic-orthotropic material to be entered into the ADINA material model. There is little interaction between the two perpendicular directions in a woven fabric and a value of zero for Poisson's Ratio was chosen for this material model¹³.

t = 0.03-in (0.762 mm)	Fabric thickness
$E_{warp} = E_a = 46,667 \text{ psi} (321.8 \text{ MPa})$	Modulus of Elasticity, Machine Direction
$E_{fill} = E_b = 90,000 \text{ psi} (620.4 \text{ MPa})$	Modulus of Elasticity, Cross Machine Direction
G = 23,333 psi (160.6 MPa)	Shear Modulus
v = 0.0	Poisson's Ratio

Table 1: AMOCO 2006 geotextile fabric material properties

Relaxation can occur due to the prestress forces in the membrane and there is the potential for creep in the geotextile material. Geotextile fabrics are temperature sensitive, and as a result, creep as the temperature increases¹⁴. Creep may be more of a factor as the concrete panel cures due to the heat of hydration than initially as the concrete is being poured into the fabric formwork.

The effects of creep in the geotextile fabric are not included in this paper but relaxation will be considered in the modeling of the fabric panel. Loss of prestress due to relaxation of the fabric can exceed 50% after just a 20-minute period depending on the percentage of initial prestress and the direction in which the fabric is prestressed¹⁵.

2.2 Slurry model material properties

The slurry material, as stated above, must not contribute stiffness to the fabric element portion of the computer model. As a result, a very low modulus of elasticity must be used for this elastic-isotropic material. The slurry material will function as the load on the fabric element model using the slurry's density as a mass-proportional load. Table 2 summarizes the slurry material properties used in the ADINA material model.

t = varies-in (mm)	Slurry thickness
$E_{sm} = 2 \text{ psi } (13.79 \text{ kPa})$	Modulus of Elasticity
$v_{sm} = 0.0$	Poisson's Ratio
$D_{sm} = 2.172 \text{ x } 10^{-4} \text{ lb-sec}^2/\text{in}^4 (2,321 \text{ kg/m}^3)$	Density

Table 2: Slurry material properties

2.3 Step 1 – Determination of load paths

In the first step, a FEA study of a uniformly thick panel with various boundary conditions is performed in order to determine the load paths an applied lateral load might take. A distributed unit load is applied to a series of panels using 3-D solid elements and the resulting principal stresses examined. For this study, any uniform material type may be used. Figure 3 shows the results of these panel investigations for a variety of boundary conditions. The double-headed arrows indicate the general direction the maximum principal stresses take.



Figure 3: Panel load paths and anchor locations

Note in Figure 3 that panel anchor locations appear to result in load paths, which fall into one of two cases. Load paths defining Case 1 are parallel to one of the panel's edges as shown in Panel BC1, which has a continuous simple edge support, or Panel BC5 and Panel BC8, which have symmetrical 4-point anchor locations. The load paths in the remaining panels appear to triangulate in their direction between the anchor locations and define Case 2. For illustration purposes, the anchor locations shown in **Panel BC3** are assumed – where the load paths triangulate. This anchor arrangement was also chosen for the interesting shape the final panel design takes.

2.4 Step 2 – Define fabric formwork design

Based on the study of the load paths shown in Figure 3, the formwork is laid out and the interior and perimeter boundary conditions are introduced as shown in Figure 4. A "B" in this figure indicates location of the interior supports.

The fabric in this model will be laid with the cross machine direction spanning the narrow dimension of the panel and the machine direction spanning along the length of the panel. The fabric will deflect between these interior supports creating thicker panel regions – capable of resisting more load than at the supports where it remains at its initial thickness. These deflected regions define the panel's load paths. Increased strength will be provided spanning

the width of the panel along a diagonal path, for a 4-point anchor condition, due to these thickened regions. In addition, "collector" paths are formed along the length of the panel to bring the load to the diagonal load paths.



Figure 4: Combined fabric and slurry model (1-in = 25.4 mm, 1 psi = 6.9 kPa)

Figure 5: "Form-finding" combined fabric and slurry model

2.5 Step 3 – "Form-finding" the panel shape

The ADINA model representing the supporting fabric formwork in combination with the slurry material, which functions as the load on the formwork is shown in Figure 4. For clarity, the fabric and slurry element groups are shown separately.

The computer model representing the supporting fabric formwork uses 9-node, 2-D solid elements. The 2-D solid element uses a 3-D plane stress (membrane) kinematic assumption. A prestress load of 2% is applied to the fabric in the cross machine direction with a 50% reduction in the modulus of elasticity, E_b , due to relaxation, assuming the concrete is poured within one hour of prestressing the fabric. A one-half percent prestress load is applied in the machine direction to keep the fabric taut with no reduction in E_a being taken. Thus, the modulus of elasticity is approximately equal in each direction. The 2-D solid fabric elements use a large displacement/small strain kinematic formulation.

The computer model representing the slurry material will use 27-node, 3-D solid elements. To be consistent with the 2-D fabric elements, the 3-D slurry elements also use the large displacement/small strain kinematic formulation.

Now that the model is defined, "form-finding" of the panel shape may proceed. Initially, the 3-D slurry elements are uniformly $3\frac{1}{2}$ -in-thick (88.9 mm). "Form-finding" the panel shape proceeds as follows:

- **1.** Run the model under slurry gravity loading and determine the interior fabric element node displacements.
- 2. Increase the 3-D element thicknesses at each interior node (e.g., at node 357, Figure 5) by the amount the fabric displaces (e.g., at node 367, Figure 5). The bottom node remains stationary while the top and mid-level nodes are adjusted upward. (The computer model panel is formed in reverse of how it would occur if the slurry were

actually being poured into the fabric formwork.)

- 3. Rerun the model and determine the interior fabric element node displacements.
- **4.** Repeat Steps 2-3 until displacements between the last two runs are within a tolerance of approximately 1%.

Given the hundreds or even thousands of interior nodal locations that will require adjustment, depending on the size and complexity of the model, the task of manually adjusting the nodal locations becomes daunting. Fortunately, ADINA can both output displacement information and input nodal locations using text files, which when used with a spreadsheet program greatly facilitates this "form-finding" task. Still, what would be desirable is a program that can automatically update its nodal locations.

The finite element model in Figure 6 shows the results of "form-finding" the panel shape made up of the slurry material. The boundary conditions that created it were illustrated in Figure 4.

2.6 Step 4 – Panel analysis and design

A strength analysis of the panel will need to be performed before any judgment can be made of whether or not the panel is adequate. The panel design may be optimized, to account for over or under-strength, by adjusting the following list of variables and repeating Steps 2-4 of the design procedure.

- Concrete strength
- Initial panel thickness
- Prestress in fabric formwork and
- Anchor locations



Figure 6: Panel shape after "form-finding"

Figure 7: Panel load cases. (1 psi = 6.9 kPa)

The panel shape defined in Figure 6 may now be analyzed for strength under the ± 30 psf (± 1.44 MPa) design lateral wind load and gravity self-weight. Two lateral load cases are considered, a positive load case and a negative load case as shown in Figure 7. The lateral loads will cause bending in the panel and the gravity loads are in-plane loads that will contribute to membrane action in the vertically oriented panel. The panel will be analyzed using the strength design method for plain concrete and ACI 318-02, Section 22¹⁶.

t = varies-in (mm)	Concrete panel thickness
E _c = 4,074,281 psi (28,091.2 MPa)	Secant Modulus of Elasticity
E _{tc} = 7,129,991 psi (49,159.6 MPa)	Initial Tangent Modulus of Elasticity (Assume 1.75 x Secant Modulus)
f ′ _c = 5,000 psi (34.5 MPa)	Compressive strength of concrete (SIGMAC)
$\varepsilon_{\rm c} = 0.002$	Compressive strain of concrete at SIGMAC
f ′ _{uc} = 4,250 psi (29.3 MPa)	Ultimate compressive strength of concrete (SIGMAU, assumed @ 85% f 'c)
$\varepsilon_{uc} = 0.003$	Ultimate compressive strain of concrete at SIGMAU
$f_r = 5\sqrt{f'_c} = 353.6 \text{ psi} (2.4 \text{ MPa})$	Uniaxial cut-off tensile strength of concrete
$v_{c} = 0.20$	Poisson's Ratio
$D_c = 2.172 \text{ x } 10-4 \text{ lb-sec2/in4} (2,321 \text{ kg/m3})$	Density
$\Phi_{\rm p} = 0.55$	Strength reduction factor for plain concrete

The properties for the slurry material are now replaced with the properties for concrete. Table 3 summarizes the concrete material properties used in the ADINA material model.

Table 3: Concrete material properties

The governing criterion for structural plain concrete design is the uniaxial cut-off strength of the concrete or Modulus of Rupture as stated in Section 22 of ACI 318-02. Maximum principal tensile stresses resulting from positive and negative wind loads combined with gravity loads must fall below this value, which for 5,000 psi (34.5 MPa) concrete is 353.6 psi (2.4 MPa). When the maximum principal tensile stress is greater than the Modulus of Rupture, the ADINA model indicates this point by a "crack" in the panel model. The ADINA Theory and Modeling Guide notes: "...for concrete.... these are true principal stresses only before cracking has occurred. After cracking, the directions are fixed corresponding to the crack directions and these variables are no longer principal stresses"¹⁰. ADINA uses a "smeared crack" approach to model the concrete failure. Following are summary graphic output and results for the panel under investigation.

3 ANALYSIS RESULTS

Figure 8 shows the finite element analysis (FEA) model for the panel under consideration. Positive and negative load cases as shown in Figure 7 are considered. The finite elements are arranged in a pattern that follows the fabric formwork design shown in Figure 4 and are supported with a 4-point anchor arrangement. After "form-finding", the final weight of the panel is 4,941 lbs (21,977 N). Figure 9 shows the deflected shape under the factored positive load case. The maximum service load deflection is 0.0066-in (0.168 mm).



Figure 8: Panel model

Figure 9: Panel deflections

Figures 10 and 11 show the loading conditions under which the panel first cracks. For case two, the negative load case, the first cracks occur at 1.3-times the factored load as shown in Figure 11. For case one, the positive load case, the panel does not crack, within the body of the panel, until 2-times the factored positive load is reached, as shown in Figure 10 – local cracking at the supports being ignored.



Figure 10: First panel cracks, back.

Figure 11: First panel cracks, front.

Figures 12 and 13 show tensile and compressive principal stress at a section cut along the diagonal load path. Figure 12 shows the effect of arching action similar to a strut and tie model under the positive lateral loads, a direct result of the three-dimensional funicular tension curves produced in the fabric as it deformed under the weight of the wet concrete. Compressive forces in these curved panel elements, created under the positive lateral load, allow the panel loads to be steadily increased without the interior of the panel cracking. Conversely, under the negative lateral load case the benefit is not observed, as shown in Figure 13, where the principal stresses are mostly in tension. The benefit of the functular tension curves in the fabric formwork, which produced this panel shape, is evident. Selective reinforcement in the negative moment regions would be required if additional load capacity or



a much thinner panel were desired – preference being given to noncorrosive reinforcement.





Figure 14 shows a maximum principal tensile stress of 193 psi (1.3 MPa) for the positive lateral load case at the factored load. Figure 15 shows the maximum principal tensile stress of 289 psi (2.0 MPa) for the negative lateral load case at the factored load. The double-headed arrows indicate load paths between the supports. This corresponds to the load path for **Panel BC3** shown in Figure 3. This panel has a maximum thickness of 5.89-in (149.6 mm) and an equivalent uniform thickness of 4.26-in (108.2 mm). While this panel has achieved an optimal form, it is slightly "over-strength". Ideally, first panel cracks should occur just as the factored design load is reached.



Figure 14: Panel principal stresses, back

Figure 15: Panel principal stresses, front

4 CONCLUSIONS AND FURTHER RESEARCH

The procedures introduced in this paper provide an efficient method for the analysis and design of a flexible fabric formwork and the resulting complex concrete panel shape thus formed. The slurry material model used with the 3-D solid finite element proves very helpful

in saving FEA modeling time by allowing the panel shape to be formed and then later analyzed by simply substituting a concrete material model for the slurry material model and without remeshing the FEA model. Key among the benefits for forming concrete using flexible fabric formworks are economies of construction, durability of the product and freedom of design expression.

Much work remains to be done including design and modeling verification, investigation of reinforcement types and options, development of new types of formwork fabrics and the development of standards and guidelines for this unique means of forming concrete members.

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DATABASE INTERACTIVE AND ANALYSIS OF THE PNEUMATIC ENVELOPE SYSTEMS, THROUGH THE STUDY OF THEIR MAIN QUALITATIVE PARAMETERS

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Key words: Pneumatic envelope systems, database interactive, proposals compilation **Summary.** The design of pneumatic envelope systems has experienced an important development since the beginning of the 20th century; when military, social and economic necessities required new systems to cover large rooms with easy and quickly technologies.

However, many projects and experiences have been developed in periods without constant compilation and cataloguing studies. The historical review of these technologies has had an intermittent analyse process; existing periods of maximal researches, like the 70's and others with lower publication levels, like the '80s and early '90s.

Because of that, the research had the main objective of developing a new critical and historical review of the evolution of this technology; in order to complement low catalogued periods and also to allow the study of the main conditions that had influenced the use of pneumatic envelopes applied in architecture.

The methodology of study has been focussed on the location and documentation of projects, not only constructed also utopian and non built proposals, which qualitative properties have supposed special innovation in the field of this technology. In this way, it has been studied the following parameters in each of the documented projects: contextual, morphological, functional, constructive, climatic and comfort-energy efficiency.

The organization methodology has been coordinated by the construction of an interactive and open data-base, helping not only the individual study of each proposal, also the comparative analyse of different projects, associated to the different parameters and sub-parameters of characterization.

More than 660 different strategies have been catalogued, since the first patent of F.W. Lanchester in 1917, to the last projects developed in 2010. The proposal of the interactive database also has helped to define a dynamic, non-linear and open study of the results, allowing the comparison between the main pneumatic envelope typologies and the selection of new research strategies in this engineering and architectural field.

1 INTRODUCTION

The pneumatic envelope systems have experienced an important development in the last century. However, this technology has suffered different documentation processes; with high publication periods followed by others with very low volume of articles or books.

By studying the main Spanish and International library database sources in the architectural context, three different periods can be differentiated: The seventies and the first decade of the 20. Century reached the highest publications volume, while in the eighties and early nineties the proposals decreased considerably. The variability of the economic, social and technical conditions on the last 40 years, have favored the discontinuity of the cataloguing and documentation of the pneumatic envelope systems.

The following library database sources have been studied:

REBIUN. Red de Bibliotecas Universitarias / Spanish Academic Library Network.

_Delegaciones de Colegios Oficiales de Arquitectos de España / Regional Libraries of the Official Architects Spanish Associations.

_ILEK Institute. Institute for Lightweight Structures and Conceptual Design (ILEK) at the University of Stuttgart.

_Avery Index to Architectural Periodicals of the Columbia University.

According to the great volume of periodical publications of the Avery Index database source; the following figure (fig. 1) represents the relation between periods with high interest about pneumatic systems and those with low volume of published articles.



Figure 1: Volume of articles compiled in the Avery Index. Diagram: Own elaboration

The type and characteristics of publications have evolved according to each period, from the great compilation of projects of the seventies to the currently diffusion by technical articles. In this sense, it has been observed that while the first books were focused on the compilation of large quantity of different projects and pneumatic typologies, in order to analyze and promote the characteristics of this technology; the current publications are more focused on the study of particular projects and specific constructive systems, than in the comparison between different strategies.

In this way, the Roger Dent (1971)¹ and Thomas Herzog (1976)² publications are still the international reference compilation of pneumatic systems. Also, more recent studies are the Karsten Moritz compilation about ETFE projects (2007)³ or the Wolfgang Naumer doctoral thesis about pneumatic systems (1999)⁴. However, an open source and interactive database with such amount of projects, including different pneumatic typologies and materials, had still not been developed.

2 OBJECTIVES

2.1 Main objective

The following in-depth study has been made with the objective of giving continuity to the proposals documentation that has been developed in the different periods, from the first patents to the current projects. Also, the study purpose is to design a compilation system that allows detailed and qualitative analysis of the pneumatic architectural typologies, by the study of the main parameters which characterize them.

In this way, an interactive and open database has been developed, in order to allow the evolution analysis of this systems and his gradually complementation as new projects arise. The interactive database allows a combined, linear and nonlinear reading of the documented systems; and also the open source character favors the proposals diffusion and the introduction of new pneumatic strategies. These properties complement the static nature of conventional methods, such as databases and networks of traditional academic libraries.

The database has been organized in different parameters in order to help a critical review of the pneumatic systems development, from their beginning to nowadays, through their contextual, morphological, functional, constructive, climatic and energetic conditions.

2.2 Specific objectives

The development of the interactive database has helped also to achieve the following specific objectives:

_Analysis and classification of the different systems, through the isolated study of each analysis parameter (morphological, functional, technical, energetic, social and economic)

_Global study of the pneumatic skin systems development influences, through the comparative relation between the different analysis parameters.

_Identify the main points of inflection of this technology and study of the main grounds of pneumatic systems development.

_Identify the new lines of research. Redefine strategies and approach to new proposals.

3 METHODOLOGY

The compilation methodology has been organized in two types of elements: The open source and interactive database and the summary files.

In order to allow the comparative analysis between projects, different parameters have been considered to define the main characteristics of the pneumatic envelope systems; so the projects have been selected according to these qualitative parameters. In this way, those proposals which are repeatable have only be considered one time; and those which no represent any innovation respect the selected parameters, will be omitted. In this way, a selective and qualitative process has been developed, related to the main parameters that influence on the definition of the different pneumatic typologies.

The reference sources have been the publications (books and technical articles) of the libraries which are described in the introduction; but also, the direct information by architects, engineers, manufacturers and tensile associations, about the pneumatic projects which they have developed.

4 DATABASE

The database has been build according with two main premises: It should be open source and interactive; in order to favor an active and flexible way to compile, organize and consult the documented projects.

The selection of the projects has been made according to qualitative conditions. In this way, the prototypes which are repeated n-times have been only taken one time into account. It has been only selected the projects that represent a qualitative difference between the others, with at least on one of the study parameters.

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Figure 2: Compressed example of the database

The open source property helps the introduction of new projects, while the interactive characteristic allows the organization of the database according to each of the different parameters which have been defined. In this way, projects can be ordered chronological, morphological, climatic, functional, etc.; allowing the study and comparison of different type of projects in relation with independent issues.

The figure 2 shows an example of some projects compiled in the compressed database model. However, the main database is larger and it has been organized in the following 6 main parameters, further subdivided into their respective subcategories.

Contextual parameters

Chronological study Location. City, Country _Type of project. Built, in progress, non built, conceptual Author. Designer, promoter and manufacturer **Functional parameters** Type of building use Type of pneumatic application New construction / Retrofitting / Enlargement Flexibility of use **Morphological parameters** Main dimensions. 1D, 2D, 3D, 4D. Form finding Form of each unit, geometry and number Size of each unit and relation with the total envelope surface Types of morphological modification **Constructive parameters** Pneumatic typology according to the pressure type Substructure typology and materials Type of anchoring _Auxiliary reinforcements Type of membrane. Material and number of layers. Supporting gases **Climatic parameters** Latitude Altitude Climatic zone Main climatic parameters: Temperature, humidity, rainfall, snow, solar radiance **Comfort and energy efficiency** Thermal control Ventilation _Natural lighting / shadowing Acoustic conditioning Solar energy integration Maintenance

4 SUMMARY FILE

In parallel to the main database, a summarized file has been developed in order to improve the particularized study of each project. It has been built a coordinated system of summary files, where the data of each project will be exposed in a more visual and easier representation. The relation between the database and the summary files has been optimized, so it is only necessary to select a project in the database, and their values will be automatically organized in the summary file.

While the database facilitate the catalogue and the comparative analysis between different strategies; the summary files allows the consultation of each project, individually, through the use of a simple interface.

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Figure 3: Summary file

The figure 4 shows a zoom of a summary file, where are showed the different contextual parameters of one of the documented projects.

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Figure 4: Zoom of the summary file

5 CONCLUSIONS

The research has allowed the qualitative definition of the pneumatic envelopes evolution, from the first patents to the present. Against the discontinuity of the publications and the specific thematic of current periodical publications; the selected methodology helps the compilation of great volume of proposals and gives continuity to the pneumatic systems state of the art.

A total amount of 663 projects have been documented, catalogued and analyzed; although the interactive properties of the database allow their increase, according to the approach of new proposals.



Figure 5: Percentage of projects documented, according to their typology

Also, the database has been developed to possibility the organization according to different analysis parameters (contextual, morphological, functional, constructive, climatic and comfort-energy efficiency); which helps the comparison between proposals, and the detailed study related to each parameter. This interrelated system will help to define the most relevant innovations and the new opportunity research areas of development.

In contrast with the static databases, the open source and interactive system favors the comparative and detailed analysis related to each of the typologies and parameters of study. Also, the design of the database allows an easy incorporation of new projects without modifying the previous values of other projects. In this way, it is possible to organize many different parameter classifications and their respective subcategories (contextual, functional, morphological, constructive, climatic and comfort-energy efficiency) in order to favor non linear and comparative analysis.

The relation between summary files and database allows the construction of an equal representation model for all the projects; which optimizes the data transmission and the organization of the consult.

The different projects have been analyzed according to each of the defined parameters, by determining the main problems and trends of this technology. As result, a new research line has been promoted with the objective of developing new strategies that improve this technology in the frame of sustainability.

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GEOMETRY AND STIFFNESS IN THE CASE OF ARCH SUPPORTED TENSILE ROOFS WITH BLOCK AND TACKLE SUSPENSION

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Key words: Cable Net, Nonlinear Analysis, Block and Tackle, Dynamic Relaxation Method.

Summary. The paper presents the comparative analysis of cable net roofs supported by truss arches with block and tackle suspension system. The effect of the friction (between the pulley and its shaft) on the internal forces of the supporting arches and on the displacements of the cable net roofs has been analysed. Structures with different number of supporting arches have been compared.

1 INTRODUCTION

The block and tackle suspension system has been invented by Kolozsváry¹ to minimise the bending moment of the supporting arches of tensile roofs by converting the random meteorological roof loads into nearly uniform, symmetric arch loads, based on the well-known principle of block and tackle. The main idea is to suspend the tensile roof by continuous suspension cables, which pass through series of upper and lower pulleys. Pairs of upper pulleys are secured to the truss arch; the lower pulleys are secured to the ridge cable of the roof (Fig. 1). Since the force in the continuous suspension cable is nearly uniform along the arch, the suspension forces acting on the arch are also nearly uniform. This means that the supporting arch can be designed to correspond to the pressure line of uniform arch loads; and the bending moments of the arch can be decreased radically.

The block and tackle suspension system and the first results of the static analysis based on idealised (frictionless) pulleys have been presented in Hincz^2 . Later the author has developed a Dynamic Relaxation^{3,4} based procedure for the exact analysis of structures with block and tackle suspension, taking into account the friction of the pulleys. The details of the developed numerical method, the main steps of the analysis and the results of the analysis of a single arch supported tensile roof have been presented in Hincz^{5,6}.

The current paper presents the comparative analysis of tensile roofs with different number of supporting arches. Since the displacements of the roof are larger in the case of block and tackle suspension system than in the case of conventional suspension with individual suspension cables, it has been interesting to analyse the effect of the geometry of the roofs on the stiffness. The effect of the friction between the pulley and its shaft has also been analysed on the internal forces of the supporting arches and on the displacements of the roof.



Figure 1: Side view of the block and tackle suspension system

2 THE ANALYSED MODELS

The models of four structures have been analysed. The main parts of the structures are the truss arch(es) of constant curvature, block and tackle suspension system, hyperbolic cable net and the supports. The static analysis of cable net roofs supported by 1, 2, 3 and 4 arches has been completed. Fig. 2. – Fig. 9. show the floor plan and the axonometric view of the models. The length of the diagonal of the covered area is 100 m. The free span of the supporting arches is 107.4 m. The height of the cable net roofs is approximately 18.5 m. The depth of the supporting arch(es) is 3 m, the width is 2.5 m. The arches have one lower and two upper chords and they are supported by universal hinges. The cable net is suspended at 15 points in the case of model 1. In the case of the other three models there are 14 suspension points on every arch. The ratio of the radius of the pulley (*R*) and the radius of its shaft (*r*) is R/r=5, the coefficient of friction (μ) is varied between 0.005 and 0.5.

The ratio of the total weight of the different element types to the covered area have been set to constant in the case of the different models to get comparable results. The ratio of the covered areas in the case of the four structures is 1 : 1 : 1.299 : 1.414. For example in the case of the different models the cross-sectional area of the chord members of the arch(es) is A_1 =500 cm² (model 1), A_1 =250 cm² (model 2), A_1 =216.5 cm² (model 3) and A_1 =176.8 cm² (model 4). The cross-sectional area of the snow cables is A_s =14.85 cm² (model 1), A_s =12 cm² (model 2), A_s =14.68 cm² (model 3) and A_s =15.64 cm² (model 4). The cross-sectional area of the wind cables is A_w =16.98 cm² (model 1), A_w =12 cm² (model 2), A_w =9.00 cm² (model 3) and A_w =6.99 cm² (model 4). The prestress in the continuous suspension cable is P=200 kN (model 1), P=100 kN (model 2), P=86.6 kN (model 3) and P=70.71 kN (model 4).



Figure 2: Floor plan of model 1, cable net supported by a single truss arch



Figure 3: Axonometric view of model 1, cable net supported by a single truss arch


Figure 4: Floor plan of model 2, cable net supported by two truss arches



Figure 5: Axonometric view of model 2, cable net supported by two truss arches



Figure 6: Floor plan of model 3, cable net supported by three truss arches



Figure 7: Axonometric view of model 3, cable net supported by three truss arches



Figure 8: Floor plan of model 4, cable net supported by four truss arches



Figure 9: Axonometric view of model 4, cable net supported by four truss arches

All models have been analysed under six load cases:

- self weight and prestress: the construction shape without any external loads,
- total snow load: 1 kN/m² load on the whole roof,
- partial snow load 1: 1 kN/m² load on the flat part of the roof, where the slope is less than 30°,
- partial snow load 2: 1 kN/m^2 load on the half of the roof, where x > 0,
- wind load x: parallel with direction x, the dynamic pressure is 1 kN/m^2 ,
- wind load xy: the angle between the wind direction and direction x is 45° , the dynamic pressure is 1 kN/m².

During the wind analysis fictitious, simplified pressure coefficients have been taken into account, calculated from the angle (α) between the wind direction and the normal vector of the roof, pointing into the roof, on the basis of the following relations:

0.8 when $\alpha \leq 30^{\circ}$,

-0.6+1.4(75°- α)/45° when 30°< α ≤75°, -0.6 when α >75°.

3 NUMERICAL RESULTS

During the nonlinear analysis the forces in the different elements and the displacements of the joints have been calculated. The normal and shear forces and the bending moments of the arches have been calculated between the suspension points on the basis of the forces in the truss members. All models have been analysed besides different coefficients of friction.

The analysis of the internal forces of the arches due to different load cases shows that the maximum normal force of the arches can be detected in the case of total snow load. The maximum shear force, the maximum bending moment and the maximum displacement of the roof can be detected in the case of partial snow load 2.

Fig. 10 shows the maximum normal force in the arches of model 2 due to different load cases, besides different coefficients of friction. The results show that the smaller coefficient of friction results in larger maximum normal force in the arches, in the case of μ =0.005 the maximum normal force is approximately 15% larger than in the case of μ =0.5.

Fig. 11 and Fig. 12 show the maximum shear force and maximum bending moment in the arches of model 2 due to different load cases. The results show that the smaller coefficient of friction results in significantly smaller maximum shear force and bending moment. In the case of μ =0.005 the maximum shear force is 69%, the maximum bending moment is 82% smaller than in the case of μ =0.5.

Since the aim of the block and tackle suspension system is to decrease the weight of the supporting arches by decreasing the bending moment, one of the most important questions is the effect of the coefficient of friction on the normal stress in the chord members of the truss arches. Fig. 13 presents the maximum normal stress in the chord members under partial snow load 2. The results show that the maximum normal stress is more than 30% smaller in the case of μ =0.005 than in the case of μ =0.5 (for every model).



Figure 10: The maximum normal force in the arches of model 2 due to different load cases



Figure 11: The maximum shear force in the arches of model 2 due to different load cases



Figure 12: The maximum bending moment in the arches of model 2 due to different load cases



Figure 13: The maximum normal stress (compression) in the chord members under partial snow load 2 in the case of the different models

Fig. 14 and Fig. 15 show the maximum displacements of the cable net roof in the case of model 2 and model 3 due to different load cases. The results show that the smaller coefficient of friction results in larger displacements. The displacements of the joints of the cable net

consist of two effects, the displacements of the lower pulleys (the "supports" of the cable net) and the deformation of the snow and wind cables. Since the smaller coefficient of friction results in larger displacements of the pulleys and the ridge cables, it results in larger maximum displacements of the cable net also. The difference between the maximum displacements for μ =0.005 and for μ =0.5 in the case of the different models are 33% (model 1), 39% (model 2), 87% (model 3), 139% (model 4). The results show that there are significant differences in the behaviour of the models.





Figure 14: The maximum displacements of the roof supported by 2 arches due to different load cases

Figure 15: The maximum displacements of the roof supported by 3 arches

Fig. 16 shows the maximum displacements of the different models under partial snow load 2. In the case of large coefficient of friction the motion of the pulleys is less significant than the deformation of the snow cables. Therefore the length of the snow cables is determinant, the longer snow cables result in larger maximum displacements, the smallest maximum displacement has been detected in the case of model 4. In the case of smaller coefficient of friction the effect of the motion of the pulleys is more significant. On the other hand the motion of a pulley depends on the stiffness of the cable net in the suspension point of the ridge cable in radial direction. The increasing of the number of supporting arches results in smaller stiffness, because of the less significant, smoother ridge. In the case of μ =0.005 the motion of the pulleys and the stiffness of the cable net in radial direction at the ridge is determinant, the maximum displacement is detected in the case of model 4.



Figure 16: The maximum displacements of the structure under partial snow load 2 in the case of different number of supporting arches

On the other hand more supporting arches results in shorter snow cables and smaller forces in the snow cables. Fig. 17 presents the maximum stress in the snow cables due to different snow loads in the case of the different models.



Figure 17: The maximum stress in the snow cables due to different load cases

4 CONCLUSIONS

The numerical analysis of cable net roofs supported by different number of truss arches is presented. The results show that the use of block and tackle suspension system can decrease the shear force and the bending moment in the arches significantly. The decreasing of the normal stress in the chord members and the efficiency of the block and tackle suspension system depends on the friction of the pulleys and almost unrelated to the number of supporting arches. On the other hand the results show that the number of supporting arches has a strong effect on the stiffness and the displacements of tensile roofs.

Acknowledgement

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A VERY LARGE DEPLOYABLE SPACE ANTENNA STRUCTURE BASED ON PANTOGRAPH TENSIONED MEMBRANES

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Key words: tensioned membranes, deployable, pantograph, direct radiating array (DRA), space antenna.

Summary. This paper provides investigation results on the membrane technology development of a high accuracy (low deformation and high eigenfrequency) membrane antenna structure and its demonstrator for space application.

1 INTRODUCTION

For future earth observation missions from space such as ice-sounding and biomass monitoring space borne radar systems that operate in P-band (432-438 MHz) are needed ^I. In order to provide the required antenna performance in this relatively low radio frequency band, very large antenna apertures are needed. In this study, antenna structure solutions from 65m² up to 100m² surfaces are investigated based on a foldable membrane technology. This solution consists out of three membranes that are installed in 30mm distance to each other and that are tensioned by a deployable pantograph back structure.

In the first part of the paper it is shown that the antenna can be built out of a very lightweight membrane structure up to 100m² radiating surface. This solution fits the in orbit eigenfrequency and accuracy requirements as well as fits the stiffness and packaging requirements for the small European Vega launcher.

In the second part of the paper, the technology development for the tensioned membrane structure is addressed. Therefore, the results of the technology and accuracy study on the manufacturing and assembly process of the needed thin and flexible membrane laminate (glass fabrics, flexible adhesives, Kapton, copper) are presented in the paper. This includes studies on the manufacturing process of the membranes, the flexible assembly of membrane stripes, the boundary fixation structure and technology, the mechanical and thermo-elastic material property investigations, the transverse electrical connections between the membrane planes, and folding and deployment tests.

Furthermore, a 1.5m by 2.5m demonstrator is discussed. It represents one sub-array of the antenna, which is about 1% of the total antenna surface. Here, the shape optimization results for the tensioned membranes are addressed. Finally, the surface accuracy of the demonstrator is measured by the photogrammetry method and compared to the results of a finite element model.

2 CONCEPT OF THE MEMBRANE DIRECT RADIATING ARRAY ANTENNA

The developed antenna is operating at P-band (432-438 MHz) in low earth orbit (638 km altitude, 97.94 deg inclination) for biomass and ice sounding missions. Its mission details and radio frequency (RF) performance is given in ¹. Electrically, the antenna is a direct radiating array (DRA). This array consists of many radiating elements that are arranged in plane as an array of e.g. 36 elements in azimuth direction and 6 elements in elevation direction. The azimuth direction is split into 9 functional "electrical panels" for better foldability of the structure and optimal electrical feeding of the antenna (figure 1, see also figure 2).



Figure 1: Schematic representation of the concept of the membrane DRA antenna

From structural and mechanical point of view, the antenna is built out of 3 tensioned membranes that are separated by 30mm distance and a tensioning back-structure.

Each membrane has to provide RF properties that are realized by the use of photo-etched copper-clad Kapton polyimide films. For robustness in orbit, the membranes are stiffened by a thin and lightweight glass fabric that is bonded to the membranes by a flexible acrylic adhesive.

The membranes are tensioned by a CFRP back-structure that uses the pantograph principle for deployment in-orbit. Therefore, two identical pantographs are deployed simultaneously starting from the symmetry-axis at the center of the satellite. This system is driven by electrical motors, which transfer a pulling force by deployment-cables to the pantograph levers. This system is shown in figure 2 in a deployed view and in figure 3 - stowed in the European Vega launcher). Investigations in stiffness and packaging has shown that length of the antenna could be increased up to nearly 30m accommodating 13 electrical functional panels



Figure 2: Mechanical design of the DRA antenna; deployed view (one wing shown)



Figure 3: Schematic of the antenna accommodation in the Vega launcher

3 TECHNOLOGY DEVELOPMENT FOR THE TENSIONED MEMBRANE ANTENNA STRUCTURE

AKAFLEX KCL 2-17/50 HT is used as the base material for the membrane antenna surface, which provides the electrical functionality (photo-etched copper clad on a Kapton foil) and the needed mechanical properties. A thin glass-fabric (Interglas 02034) is bonded onto the Akaflex by a thin, flexible acrylic adhesive (DuPont Pyralux LF0100). This lay-up is shown in table 1.

This material is manufactured in an autoclave 2 hours at 190°C and 9,9 bar pressure.

Material	Component	Brand-name	Thickness
Akaflex KCL 2- 17/50 HT	PI substrate	DuPont Kapton VN 50	50 µm
	Adhesive	Modified C-stage epoxy resin	15 μm
	Copper layer	RA (rolled and annealed) copper	17 µm
Adhesive		DuPont Pyralux LF0100	25 µm
Glass fabric		Interglas 02034	27 µm

Table 1 : Membrane material plys

For the shown sheet material, detailed mechanical properties were determined. It was observed that it has the same mechanical properties before and after thermal cycling (shown in figure 4). After the thermal cycling of specimens, no micro cracks were found under the microscope.



Figure 4: Thermal cycling of the membrane material; specimens in liquid nitrogen (left); at 150°C (right)

The laminated membrane material has a mean areal density of 285 g/m² and a mean thickness after curing of 118 μ m. The material shows a bilinear tensile behavior with a Youngs's modulus of 9500 N/mm² in roll direction and 8900 N/mm² in orthogonal direction in the first linear sector (up to 0.5 % of strain). The ultimate strength of the material is at 125 N/mm² for both directions with more than 3% strain at break. The thermo-elastic behavior

was determined by using a standard vertical-push-rod dilatometer with rolled membrane specimens (see figure 5). A CTE of 17.6×10^{-6} /K was determined for the roll direction and a CTE of 18.1×10^{-6} /K in the orthogonal direction.



Figure 5: Dilatometer test with rolled specimens for determining of the CTE values

Additionally, new methods for bonding of membrane strips with electrical connections and electrical connections in out-of-plane direction were investigated in this study. These were used for assembling the demonstrator, discussed in the following.

4 TECHNOLOGY AND FUNCTIONALITY DEMONSTRATOR

For studying of the membrane manufacturing technology, verifying the accuracy prediction (complex FEM models) and studying the foldability of the antenna structure, a demonstrator $(2,5 \times 1,5m)$ was designed, manufactured and tested. Details of the demonstrator design are shown in figure 6. It uses a frame structure for prestressing of the membranes in both directions (x-axis: screw system; y-axis: cable system). Three flat laminated membranes were assembled, positioned, aligned, electrically connected and mounted onto the support frame shown in the figure. The frame provides folding and deployment possibilities as well. Contour shapes of the membranes have been optimized for reducing the out-of-plane deformations.



Figure 6: Definition of a technology demonstrator

Figure 7 shows the laminated and assembled bottom (ground plane) membrane of the antenna demonstrator, equipped with membrane pockets for point-wise steel cable attachment (Y-axis prestressing) and equipped with photogrammetry targets (white/gray dots on figures).



Figure 7: membrane laminate assembly, right: tensioning cable attachment

Figure 8 shows deployment and foldability of the membranes for stowing them in a dense package during the launch of the VEGA launcher.



Figure 8: folding and deployment (figures are not in the same scale)

Both FE-modeling and photogrammetric measurements show a high flatness accuracy of the membrane surfaces. Figure 8 shows a good qualitative comparability of the deformation patterns although the gravity effects causing out-of plane deformations could not be compensated completely. The measured surface deformations were still well below the required flatness accuracy that is a maximum of ± 1 mm deviation. A root mean square (RMS) of the measured deformations is calculated as 0.45mm, while predicted deviations fall in the range of 0.3mm RMS.



Figure 9: comparison between FE-modeling and photogrammetry measurements

4 CONCLUSIONS

A new tensioned membrane DRA antenna concept is presented in this paper. It is characterized with the high stiffness, low mass and high flatness accuracy and can be folded into the European Vega launcher with up to 30 m length of the effective surface (platform still to be defined).

The DRA concept technology and feasibility was demonstrated via manufactured single sub-array supported by the rectangular frame. It includes:

- Three electrical laminated membranes, transverse ribs, and electrical wire-walls which connect electrically the front (with round slots) and the rear (ground plane) membranes and pass through the middle (feeding and radiating) membrane without an electrical contact. Auxiliary support structure includes outer support frame, tensioning bars and deployment guides.
- Good surface flatness quality of 0.45 mm RMS was demonstrated under the gravity conditions (better figure is predicted for 0g conditions).
- It is shown that the membranes could be prestressed in both in-plane directions fitting the stiffness and flatness accuracy requirements.
- Folding of the membrane DRA is confirmed to be feasible into VEGA
- Deployment of the demonstrator was performed, no visual damage of membranes and electrical wall-wires was observed

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(UN)FOLDING THE MEMBRANE IN THE DEPLOYABLE DEMONSTRATOR OF CONTEX-T MARIJKE MOLLAERT^{*}, LARS DE LAET^{*}, JAN ROEKENS^{*} AND NIELS DE TEMMERMAN^{*}

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Key words: Tensile surface structures, Deployable systems, Foldable plates, Adaptable structures

Abstract. The paper presents a system, made out of foldable 'kinked' beams and a membrane skin, based on a concept referring to origami. The anticlastic curvature of the membrane is obtained by transforming flat triangular parts into space. The following research question is considered: can the foldable system be stable in intermediate configurations? To obtain a well-tensioned membrane in the intermediate positions, the belts connecting the membrane to the frame can be released or increased in tension.

A full scale 'demonstrator' has been built within the frame of IP-project Contex- T^8 . Although the deployment - rotating the 'kinked' beams about the central axis - was feasible, the tensioning and structural behaviour of the membrane, attached in the nodes of the frame, was not yet thoroughly examined.

For that reason one single unit has been analysed. Forces and deformations in the membrane are verified for different opening angles using integrated models including the membrane, connecting belts and 'kinked' beams (for the frame).

The results of the experimental investigations and numerical models are compared and occurring discrepancies are clarified.

1 GENERAL CONCEPT

The study of the deployable circular demonstrator is part of a more general research topic to expand the possibilities of high tenacity coated textiles for adaptable lightweight constructions, such as simple retractable canopies and kinematic facades for shading or movable roofs with large free spans.

The concept of the deployable structure with a foldable membrane skin is based on a foldable paper model^{1,2,4}, as shown in Figure 1. The foldable model consists of a series of triangles, connected at their edges by continuous joints, allowing each triangle to rotate relative to its neighbouring triangle. The foldable model can fold into a flat stack and unfold into a predetermined three-dimensional configuration, with a corrugated surface: a circular

dome. The folds moving outwards are called ridge or mountain folds, the folds going inwards are the valley folds.

The demonstrator (which fits in half a sphere with a radius of 4.25m), built in the frame of the IP-project Contex-T *Textile Architecture - Textile Structures and Buildings of the Future*^{5,6,8} is made out of 14 identical triangular panels and 2 boundary panels (see lower part of Figure 1).



Figure 1: Upper: Origami concept (unfolded)¹; Lower left: Folded paper model, Right: Deployed

Two units (consisting of four triangles) are fixed in the unfolded configuration and act as the stiff core for the foldable parts: the apex of the dome is supported by these two units.

2 EXPERIMENTAL VERIFICATIONS

2.1 First prototype: one quarter

A first experimental set up of one quarter of the system was analysed³ and realised at the Vrije Universiteit Brussel (Figure 2). The frame was made of steel profiles with a 50mmx50mmx5mm section. The folding hinges were made by means of hollow tubular axes.



Figure 2: Setup: a. the frame b. the frame with the membrane

The membrane (in this case a PVC-coated polyester membrane from Contex-T partner Sioen) was only attached in the nodes of the steel frame: in the top, at mid height and in the corner points at ground level. Ridge, valley and boundary belts were foreseen to tension the membrane.

The feasibility of folding the frame from 100% open to 30% was approved (see Figure 3).



Figure 3: Top node a. deployed b. folded

2.2 The Contex-T demonstrator

The full scale demonstrator has been built by Contex-T partner IASO with the following simplifications: (i) the system opens along the ridge instead of along the valley line, (ii) only the long diagonals of the membrane units (valley line) can be adjusted to influence the pretension and (iii) the cut-out of the boundary curves was increased.

A standard PVC-coated polyester membrane was used (686gr/m², strength in the warp direction: 322kg/5cm (64.4kN/m) and in the weft direction: 291kg/5cm (58.2kN/m)) reinforced with stitched vectran and polyester belts.

The two fixed units were placed on site and connected to the concrete base platform. The other elements (on wheels) were joined. Once all elements were attached, the position of the

apex of the dome was lower than theoretically foreseen and the system did not appear stable. A connection point was placed in the central position at ground level and all movable elements are attached to this point by means of thin cables (see Figure 5) which control the rotation of the units. When folding, the different units tilt in a different way, depending on the sequence of folding (Figure 4).



Figure 4. Top node. a. closed configuration b. one unit folded



Figure 5. Overall view: a. closed configuration b. one unit folded

The design considered a slightly tensioned membrane (pretension ~ 0.1 kN/m) in the unfolded configuration. When folding the structure, the distance between ridge and valley line decreases and the membrane becomes slack. Thus, a higher pretension is required in the unfolded configuration to keep the membrane tensioned in the different intermediate configurations between completely unfolded and folded.

To obtain a better insight in the structural behaviour of this foldable structure, one unit is investigated in more detail during (un)folding.

3 STUDY OF A SINGLE UNIT

3.1 Numerical Analysis

The form finding of the membrane model is done with the approximation of a cable net and the force density method⁷ (EASY software from Technet⁹). Two triangles with a base of 6m and an apex angle of 120° are joined along their base and placed with an opening angle of 20° . Next, the equilibrium shape is calculated for a pretension of 1kN/m in the two directions of the net (Figure 6).



Figure 6. The single unit: a. the system lines b. tensioned equilibrium shape

Two kinked beams are added to this membrane unit, the model is transferred to the EASYbeam software⁹ and is calculated for different opening angles (20° to 90°, steps of 10°).



Figure 7. The (un)folding: a. variation of the opening angle from 20° to 90° b. superposed models

In the integrated numeric model (membrane, belts and frame), the evolution of the membrane stresses as well as the forces in the belts can be verified (Figure 8). When folding from an opening angle of 90° to 20° the maximum force in the valley belt decreases from 26.4kN to 23.8kN and the maximum membrane stress decreases from 10.8kN/m to 2.1kN/m.



Figure 8. Equilibrium shape of the same membrane for different opening angles

3.2 Fabrication

A PVC-coated polyester membrane (Sioen T2107, 1050gr/m^2 , strength in warp and weft direction: 4000N/5cm (80kN/m)) was applied in the fabrication of one unit. Compensation was neglected since on the one hand it is a small membrane with a pre-tension of only 1kN/m and on the other hand the length of each connection belt is adjustable. The foldable unit is made from two triangles (base 6m, apex 120°, weft-direction perpendicular to the base) from which circular cut-outs were made along the borders (sag 5%). The borders have been reinforced with a double polyester belt (strength 20kN). The fabrication was done by Carpro (Figure 9).



Figure 9. The single unit: a. membrane parts b. assembled

The membrane is placed in a foldable frame made from the elements used for the first experimental model: only the connection elements - where the membrane needs to be attached to - had to be adapted (Figure 10).



Figure 10. Modified connections: a. welded bars and an additional hole in the plate b. ring to attach the belts

3.3 Assembling and testing

The membrane and the frame have been assembled a first time (Figure 11.a. and b.) to check the folding and the possibility to introduce a pre-tension of 1kN/m in the membrane. The system could not be folded up to the angle of 20° due to the thickness of the shackle and the ratchets (Figure 11.b.) and the pre-tensioning was not performed.

After the connections have been simplified (Figure 12.a.), the membrane has again been attached in the frame. The lengths of the links connecting the membrane to the frame were set according to the prescribed values.

This time (un)folding was no problem, but the membrane seemed to be too large for the frame. It was not possible to tension the membrane uniformly in the different configurations (Figure 12.b.), although the frame as well as the membrane were made according to the designed geometry.



Figure 11. a. the base angle b. the apex angle



Figure 12. a. Simplified connection at the base angle b. membrane opened 90°

3.4 Further experimental tests

When fixing the membrane, the belts - connecting the membrane to the corner points of the frame - have to be adjusted to a specific length to obtain a 'well tensioned' equilibrium state without wrinkles. Also the valley belt should be tensioned correctly. The influence of shortening the belts or tensioning the valley line, depends on the real stiffness of the fabric and its reinforcements. The constant stiffness set in the numerical model implies a linear approximation.

To be able to understand the structural behaviour of the system the fabricated membrane is tested on its own. The membrane is folded (opening angle 0°) and attached at its corner points in a test rig (Figure 13).

During the tensioning process the anchorage points at the base remain fixed and the apex point moves upward. The geometry (circumscribed triangle, sag at the borderlines), the forces in the anchorage points and the biaxial strains in the membrane are measured in consecutive steps up to a 'well tensioned' state.



Figure 13. Experimental set-up: a. schematic b. membrane with pattern for the DIC-measurements

With the method of Digital Image Correlation for optic strain field measurements (DIC Vic3D) the process of increasing the tension can be observed in detail. The images have been numbered from 0 to 87.

Since the stiffness in the borders is discontinuous, a complex deformation pattern is obtained especially below the circular cut-out at the apex corner (Figure 14). Similar irregularities had been observed in the cable net model (for the forces).



Figure 14. Strain field measurements a. in y- direction b. in x-direction (red: high, blue: low values)

The experimental results for the 'well tensioned' equilibrium state without wrinkles (reference number 87) will now be summarised. The width of the membrane is 5.54m, the force at the base corners 12.1kN and the sag for the valley belt 34,5cm. Hence, the tension in the membrane in the y-direction (vertical) can roughly be estimated to be 0.3kN/m in each membrane layer (considering equal forces for the valley and the two border belts).

When comparing the experimental results with the numerical simulations the following can be stated: (i) the numerical model gives a force value of 23kN in the valley belt which

corresponds to a membrane stress of 1kN/m, the values are lower in the experiments, (ii) in the numerical model the membrane stresses are more uniformly spread over the surface and (iii) the lower tensions below the cut-out at the apex corner (clearly visible in Figure 14) were only obtained at higher opening angles in the numerical models.



Figure 15. Strain field measurements in y- direction a. image 54 b. image 87 (red: high, blue: low values)

From the comparison between the image with reference number 54, for which the geometry corresponds to the numerical model, and the image with reference number 87, corresponding to the highest displacement of the apex corner, it can be seen that the circumscribed triangle highly deforms. The sag of the base line increases from 27.5cm to 34.5cm (Figure 15). Thus, the height of the circumscribed triangle for the 'well tensioned' state is greater than the height of the foldable frame. The stiffness of the membrane, especially in the weft-direction, has been overestimated.

This experiment is only a first step to further investigate the (un)folding procedure of the tensioned membrane.

4 CONCLUSIONS

Numerical simulations demonstrate the possibility to tension one and the same foldable membrane, tensioned in a foldable frame, into different configurations. The validity of these calculations, using a cable net as model for the membrane, has to be confirmed.

For this reason and to gain insight in the structural behaviour, the experimental stressstrain behaviour of a simple membrane unit tensioned in a system of foldable 'kinked' beams was investigated.

In a first set-up the membrane was fixed to a frame. Here it was not possible to obtain a wrinkle-free membrane since it was too difficult to introduce the high forces manually.

In the next set-up the membrane was fixed between two anchorage points and a hoisting crane. The upward displacement of the apex corner was used (without other adjustments) to pre-tension the membrane. The process was registered by digital images to measure the strain fields. The forces in the anchorage points and the sag of the valley line were read off in each step.

This simple test clearly shows: (i) that although designed to be light and low-tech, the high forces in the belts cannot be introduced manually, (ii) that even in the folded reference state the strain behaviour of the membrane, reinforced with belts, is complex, (iii) that high strains occur in the membrane, especially in the weft direction (iv) and that it is feasible to control the tension by moving the apex corner up or down. The folding will be studied in a similar way by rotating the membrane panels about the base line.

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LIGHTWEIGHT AND TRANSPARENT COVERS

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Summary. There has been a gradual shift in construction of long-span covers, from the use of opaque and heavy materials, to the use of transparent and lightweight ETFE (ethylene tetrafluoroethylene) cushions. This paper presents an overview on contemporary covers constructed with lightweight materials, from the research of Frei Otto and Buckminster Fuller, to the possibility of achieving a large city-scale dome enclosure.

1 INTRODUCTION

An enormous variety of efficient structural forms have developed in nature, over many millennia. As human beings, our future will depend upon the prosperity of nature, and our ability to make construction as adaptable and efficient as structures in nature. Human influences have dominated modern climate change, which is now beyond the bounds of natural variability¹. The primary causes of global warming are carbon dioxide emissions from the burning of fossil fuels. The emissions of carbon dioxide are directly proportional to energy consumption. For that reason, in our choice of building materials we must consider the natural resource base and the effects on the environment of the extraction, manufacture and processing of construction materials². That means that if we are unwilling to change the way we build, rapid climate change and harmful damaging effects will continue. However, the resources of the planet could be optimized by creating lightweight and adaptive structures.

"A Lightweight structure is defined by the optimal use of material to carry external loads or pre-stresses. Material is used optimally within a structural member if the member is subjected to membrane forces rather than bending"³. The more a structure can carry with least weight the better. Furthermore, lightweight materials make construction easier and cheaper than standard designs, especially when vast open spaces have to be covered. Adaptive structures have the ability to evolve or change their properties or behavior in response to the environment around them and use knowledge of past events to improve future performance just as nature does. In recent times, the development of lightweight materials has led to revolutionary changes in different fields of science. In architecture, the new material technology has allowed construction of previously utopian ideas. Frei Otto and Buckminster Fuller's utopian idea of large city-scale enclosures has been only possible with ethylene tetrafluoroethylene (ETFE) cushions. The properties of ETFE which have allowed these improvements in construction are shown when compared to glass; ETFE is 1% its weight, transmits more light and costs 24% to 70% less to install. ETFE is an appropriate technology for building where the volume of space to cover is large and high light levels are needed because it allows reduction of the weight of the structure whilst providing the same level of stability⁴.

2 EVOLUTION OF COVERS SINCE THE LAST 50 YEARS

Throughout history, there has been a gradual shift in the construction methods of long-span structures, from the creation of opaque and heavy constructions, to the creation of more transparent and lightweight structures. This evolution is related to the advance in material technology and the possibility of doing more with less weight, energy and time.

Roofs usually consist of two elements; the roof structure and the roof covering. The first is the structural elements which support the roof and the second is the material which is supported by the structure, and which provides shelter.

Since the 1960s, tensile structures have been built by engineers and architects such as Walter Bird, Eero Saarinen, Frei Otto, Ove Arup and Buro Happold. Tensile structures can achieve a very high level of efficiency because the elements of these constructions carry only tension. Most tensile structures are supported by some form of compression or bending elements, such as masts, compression rings or beams, for example the Millennium Dome and the Olympic Stadium Munich. Tensile structures have historically been used in tents. "The tent is man's oldest dwelling except for the cave. Evidence of mammoth bones and tusks used as supports for animal hides has been found at sites verified to be more than 40,000 years old in the Ukraine region... The tent has been the dwelling in one form or another for most nomadic peoples from the Ice Age to the present"⁵.

People need the shelter that roofs provide, but with the new technologies, more comfortable and adaptive spaces can be created. One of the first architects to introduce deployable structures into architecture was Emilio Perez Piñero. In 1961, Piñero won a prize in the VI International Congress of Architects held in London with the Travelling Theatre, a deployable structure based on the scissors system. In the same year, the roof of the Civic Arena in Pittsburgh became the first long-scale structure with a roof that could be opened and closed⁶.Inspired by the studies of Emilio Perez Piñero, Mamoru Kawaguchi created the "Pantadome" erection system to minimize the construction costs and improve site safety in the construction of large space grids. The first building constructed using this system was the World Memorial Hall in Japan in 1984. Four years later, the Olympic Stadium in Montreal was completed. The cover of this Stadium has a system based on fabric that retracts in a roof

created with rigid trusses and space frames. Escrig and Sánchez did the same in Jaen, but folding the parcel around the ring, and in Algeciras, sliding on a tensegrity structure⁷. The first dome with retractable roof was the Rogers Centre in Toronto, built in 1989. In 1991, the Ariake Colosseum Tennis Court was constructed by utilization of crane technology. In the same year, Chuck Hoberman created the sculpture of the expanding geodesic dome. Since then, Hoberman has created many deployable structures, some of which have been used in architecture, like the retractable spherical roof of the Iris Dome in the Expo 2000 in Hanover. In 2001, the Toyota Stadium was opened. The roof has an air-inflated dual membrane structure, and the roofs are opened and closed by controlling the inflow-outflow of air.

There are two types of pneumatic structures, which depend on the role of air pressure for their static principle. The air-supported structure is a dome shaped membrane with a fixed perimeter and an inner pressure higher than the atmospheric pressure, whilst an inflated structure is a closed inflated pneumatic beam with inner pressure lower than the atmospheric pressure. Using both systems, a variety of shapes can be achieved.

Engineers like Walter Bird and Peter Stromeyer were the pioneers on the acquisition of empirical knowledge and commercial applications of pneumatic structures. However, Frei Otto was the first to undertake academic research, especially about the process of form finding⁸. Many pioneering pneumatic buildings were built in the Osaka Expo in 1970 such as the U.S. Pavilion by David Geiger, one of the first air supported cable roofs.

The first movable ETFE cushion roof was the roof of the Meiderich Theater built in 2003 at the "Landschaftspark Duisburg-Nord" in Germany. The roof rides on wave shaped tracks but it does not utilise the whole potential of the pneumatic structure because the motion is not produced by the same mechanism without any extra motors or cable pullies.

Tensairity is a pneumatic lightweight structural concept that balances tension and compression. It has been patented by a Swiss company called Airlight. A tensairity beam consists of a membrane of cylindrical shape filled with low pressure air, a compression element connected to the airbeam and two cables running in helical form around the airbeam, connected at both ends to the compression element. The air is used to stabilize compression elements against buckling.

For the roof covering or cladding, several materials have been used throughout history, such as the skin of animals, metal sheeting, concrete, timber, fabrics, glass and plastics. There has been a progressive evolution from opaque to translucent and more recently to transparent coverings, and that has been related with material improvement. In history, there are some examples of mobile coverings such as Roman theatres and amphitheatres covered by velum to protect from rain and sun. However, the use of glass in architecture marked the beginning of an era described by Paul Scheerbart in 1914:

"We live for the most part in closed rooms. These form the environment from which our culture grows. Our culture is to a certain extent the product of our architecture. If we want our

culture to rise to a higher level, we are obliged, for better or worse, to change our architecture. And this only becomes possible if we take away the closed character from the rooms in which we live. We can only do that by introducing glass in architecture"⁹.

Transparent materials provide light and view without loss of warmth. Until the appearance of ETFE, glass was the most transparent medium but among the rigid plastics, polycarbonate and Glass Reinforced Plastic (GRP) both provide 85% light transmission.

3 ETFE TECHNOLOGY

ETFE, a modified co-polymer, was the result of a research programme to develop an insulating material for the space industry that was resistant to friction and abrasion, immune to radiation and extremely effective at both high and low temperatures. This material was patented by DuPont in 1940, but it started to be commercialized in the 1970s. In 1982, Vector Foiltec pioneered its architectural use. For almost thirty years, ETFE has been used in numerous buildings and public spaces all over the world.

ETFE cushions are pneumatic structures inflated with low-pressure air, restrained in aluminium extrusions and supported by a lightweight structure. When ETFE is used for cladding, the sheets are usually assembled into cushions which are inflated by compressors. "ETFE cushions are a multi-layer system that consists of two or more layers of ETFE foil (100-200 μ m thick) heat sealed and clamped in a frame. They are inflated using a small pump to a pressure of 250-400 Pa and topped up intermittently"⁴. ETFE cushion systems are inflatable structures because the inner pressure is lower than the atmospheric pressure.

This material does not degrade under ultraviolet light and is unaffected by atmospheric pollutants. Moreover, ETFE combines exceptional light transmission with high insulation which can reduce winter heating costs. Each layer of foil has a transparency of between 90-95%. The amount of solar shading and the transparency of the building envelope can be modified by changing the translucency, density and number of layers. If desired, photovoltaic cells can be integrated in the cushions to create pollution-free electrical energy.

ETFE can deal with large deflections in the support structure because of its toughness, high resistance to tearing and ability to work harder over a 300-400% elongation range. ETFE is acoustically transparent with a mass of less than 1 kg/m2. In the case of fire, ETFE has the property of self-venting the products of combustion to the atmosphere. Due to its synclastic shape, ETFE has the ability to self-cleanse under the action of rain. Furthermore, the raw material is not a petrochemical derivative and many components are fabricated from recycled material.

3 ETFE EXAMPLES

The Eden Project in Cornwall designed by Grimshaw Architects and constructed by Vector Foiltec is a lightweight structure made with ETFE cushions. The weight of the construction is less than that of the air enclosed. It has a series of intersecting geodesic domes with structural spans of up to 124 meters. The dome structure is divided into two layers, the outer skin formed by hexagons and the inner layer by a triangular and hexagonal grid.

The National Space Center also called the Rocket Tower is the first tower built with ETFE cushions. The building was designed by Nicholas Grimshaw in 2001 and is located in Leicester, England. In the Olympics of 2008, the ETFE was used to cover the Beijing National Aquatics Centre, the Water Cube and the Beijing National Stadium.

While early ETFE clad buildings were located in temperate regions, recent projects have been located in places with harsh climates. One example is the Khan Shatyry Entertainment Center in Astana from Foster and Partners that was finished in 2010. An ETFE cushion envelope encloses multiple buildings and an urban-scale tropical landscape. It encloses an area of 100,000 square meters. The asymmetrical anticlastic conical form of the biaxial cable net is supported at its apex by a 20 meter high inverted cone that is balanced on a 70 meter tripod mast. The circumferential steel cables resist suction and the radial cables resist positive wind pressure. The ETFE cushions change their form as the structure deflects. The net is anchored to a perimeter concrete ring beam.

The utopian ideas of a city scale dome enclosure proposed by Buckminster Fuller with the Midtown Manhatan Bubble in 1962 and Frei Otto with the City in the Artic, 58 Degrees North in 1971, can be nowadays constructed with the new lightweight materials. The Walker Lake Dome Project in Westwood, California is an oblate ellipsoidal and geodesic dome made of ETFE that encloses the town. It is 6.4 kilometers (4 miles) in diameter, 183 meters (600 feet) tall at the center with a surface area of 120,774 square meters (1.3 million square feet). This project can be used to provide a ground for technology to enclose and protect cities prone to weather related disasters.



Figure 1. Buckminster Fuller. Midtown Manhattan bubble. NY, USA. 1962



Figure 2. ETFE Dome. Walker Lake Dome Project. Westwood, California. 2010

3 CONCLUSIONS

There has been a big evolution during the last years in materials used for cladding such as ETFE. Since the structure of the building is as important as the covering both fields should be improved in the same extent. Innovative ideas in architecture have been related to situations

with very high expectations, under pressure and in critical and competitive circumstances. The climate change provides a very good pretext for innovation. However, architecture continues to focus on immobile structures while scientific fields like aeronautics and the car industry invest more in new materials and started investigating flexible lightweight structures a long time ago.

The evolution of roofs has changed between compression, tension and the balance of both. Moreover, it has been accompanied by the development of covering materials. The dream of architects and engineers to build long-span constructions has become reality with the possibility of building large city-scale dome enclosures.

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Membrane restrained columns

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Key words: membrane, tensile restrained system, hybrid, buckling, non-linear

Summary. This paper describes the structural beha viour of membrane restrained columns. Initial experiments on tensile restrained columns are presented and discussed. FE-analysis has been carried out and the structural behaviour of the hybrid systems is analysed. The influence of several design parameters is research.

1 INTRODUCTION

The use of restraining systems is a very effect ive way to stabilise structures that are under axial forces and/or bending moments. The restraining system reduces the deformation and the bending stress of the structure. For structural elements under axial stress the buckling length is reduced by the restraining system. As rest raining system cables as linear elements or membranes as two-dimensional elements can be used. As cables have the advantage of high stiffness and strength, membranes offer the opportunity of a continuous contact between the stabilised structure and the restraining system. This le ads to a f urther reduction of the buckling-length and avoids peaks in the internal forces as it occur s with cable r estrained structures at the connection-node.

Compression elements are endangered by buck ling. If compression elem ents are very slender the strength of the material can not be used fully, because the buckling load b ecomes determining for the di mensioning. By use of a restraining system the buckling load can be increased so much, that the material of the compression element can be used to its yield limit.

The use of membrane restrained columns is efficient for very slender columns under small loads. They can be used in mobile and rapid deployable constructions as well as for roof and façade systems ranging from small to moderate size.

2 TEST OF TENSILE RESTRAINED COLUMNS

First ideas for tensile re strained columns have been tested by the LSU [1] in physical models. The tests were thought to get an idea of the potential and structural behaviour of tensile restrained columns. Except for the vertical load no d eformations, strains or stresses have been measured. Figure 1 shows the four different system s that have been tested. All systems were flat and had a height of about 1m, the systems s I, II a nd III were built of

rectangular balsa-stripes and cord, system IV was build of circular GRP profiles and paper inbetween.



System I Strut properties: 3 x 25mm Balsa Single Lateral Restraint Weight = 55g Load = 5kg



System II Strut properties: 3 x 25mm Balsa Three Lateral Restraint Weight = 55g Load = 5kg



System III Strut properties: 3 x 25mm Balsa Multiple Lateral Restraint Weight = 55g Load = 14kg



System IV Strut properties: Ø 6mm GRP Paper Web Restraint Weight = 110g Load = 255kg

Figure 1: tests of flat, tensile restraint struts, deformed systems I - IV under vertical load [1]

The systems I and II had the sam e buckling load, the two addition al lateral restrains do therefore not increase the buckling capacity. Sy stem III is restrained by diagonal cords, and the buckling load is nearly three times higher compared to system I and II. This result was quite surprising, as the horizontal connections w ere expected to be most important due to the deformation of the strip es to the ou ter side. The reason for a higher buckling load of the diagonal restrained colum n results from the existing im perfections. This subject will be explained more in detail in chapter 4. System IV is the stiffest system, nevertheless it needs to be stated, that the stiffness of the bars in system IV is higher compared to the systems I, II and III. The failure shape of system IV shows that the paper is torn, which indica tes a material failure and the buckling load is not reached yet. However the paper m embrane in-between seems to be very potent, as the bars are rest rained continuously and the membrane forces can set up freely in the membrane surface.

3 GEOMETRY AND FE-MODELL

To get a more detailed understanding of the structural behaviour of the m embrane restrained columns FEM–analysis was used. T he analysed structure is 3m high, the width varied between 0.1m and 0.8m. The colum n consists of three bars, between which a membrane runs. The bars are rotated by 120° related to the m iddle axis. The bars are orientated in that way, that the local z-axis of the cross-section is in the plane which is defined
by the profile axis and the m iddle axis. The bars are single curved with a constant curvature. The bars are bent warm so they are free of stress. The geom etry of the membrane is double curved, although the curvature of the membrane between the bars is very small.



Figure 2: structural system of the column and local coordinate system of one bar in side view (left), plan view (lower right) and perspective

The bars are connected to each other in the following way:

- related to the local y- and x-axis with a hinge
- related to the local z-axis fixed.

The lower end of the colum n is simple supported and fixed ag ainst torsion, at the upper end the column is simple supported in both horizontal directions but free in vertical direction and fixed against torsion as well. The connection between the membrane and the bars is solved by membrane pockets, where the bars can be pushed in. This procedure would allow an easy and quick assembling-process.

For the bars the cross-sections were defined by $20\text{mm} \times 20\text{mm}$ hollow steel-sections with a thickness of 1.2mm. The membrane was calculated with a sim ilar E-Modulus in warp- and weft direction of 600 kN/m and a G-Modulus of 25 kN/m. The calculations were perform ed with the FE-program SOFISTIK. Geometric Nonlinearities were taken into account. For the membrane the orthotrop ic, linear-elastic material model from Münsch-Reinhard was used. Only tension stiffness was considered for the membrane, whereas for the bars linear material behaviour was set without considering the yield-point of the material.

4 STRUCTURAL BEHAVIOUR

In the FE-calculations the structure has be en analysed with p erfect and imperfect geometry. To find the most unpropitious imperfections the eigenforms of the column were examined. There are two basic kinds of eigenforms for the columns: The first is a twisted one (symmetric to the m iddle-axis), the second is a bended one looking like an "S". Both eigenforms have been u sed independently from each other as imperfection. The eigenforms were scaled to a m aximum deformation of 15mm. This equates a preflexure of 1/200 as requested in the German steel-code DIN 18800-1.

The structural behaviour of the columns is decisive depending on the curvature of the bars and therefore of the width in the m iddle of the columns. In the figures 3, 4 and 5 the loaddisplacement curves are presented for different column-geometries under a vertical load. As displacement the vertical deformation of the upper support is used. For a small column with a width of 10cm the perfect sy stem shows a lin ear load-displacement curve (figure 3). Two different membrane orientations have been analysed, 0° (warp or weft-direction is parallel to the middle-axis) and 45° (membrane is rotated by 45°). For the structural behaviour of the perfect system the membrane orientation has no influence. If the imperfection of the twisted eigenform is applied, the structural behaviour is identical with the perfect system for small loads and slightly weaker for higher loads. In contrast to that an imperfection of the bended eigenform shows a different and much weaker behaviour. The buckling load is only about one third compared to the system with an im perfection of the twisted e igenform. The loaddisplacement curves are identical for both imperfect systems; however the buck ling load is smaller with a membrane orientation of 0°.



Figure 3: load-displacement-curves for a column 0,10m wide, right: perfect geometry and eigenforms, the deformation of the eigenforms have been scaled and applied as imperfections.

Looking at a colum n with a width of 30c m (figure 4) the perfect system shows again a linear structural behaviour. In contrast to the smaller column the load -displacement curve varies depending on the m embrane orientation and an orientation of 0° is stiffer than an orientation of 45°. This results from the fact that in the perfect system the membrane is only stressed in horizon tal direction and the membrane stiffness is higher in warp- and weft direction than in any other di rection. This effect gets more and more significant when the columns are wider, as the length o f the membrane between the bars increas es. When the imperfections of the twisted eigenform are applied, again the structural behaviour is similar to the perfect system for sm all loads and weaker for higher loads. W hereas for the perfect structure a membrane orientation of 0° results in a stiffer behaviour, for the im perfect system the system with a membrane orientation of 45° is stiffer. For the column with an imperfection of the bended eigenform the load-displacement curve is much weaker compared to the perfect and first imperfect sy stem. However there is a significant difference in the structural behaviour depending on the m embrane orientation. The column with an orientation of 45° is stiffer, so a pparently the asymmetric deformations caused by the imperfections can be restrained more efficiently by a membrane orientated at 45°.



Figure 4: load-displacement-curves for a column 0,30m wide, right: perfect geometry and eigenforms.

For a wide column with a width of 60cm the structural behaviour of the perfect system is still linear. The difference in the load disp lacement curves depending on the mem brane orientation is more significant than for s maller columns. Applying the im perfection of the twisted eigenform the structural behaviour is similar to smaller columns: For small loads the behaviour is like the perfect system, for higher loads the imperfect system is weaker. With an increasing load the influence of the m embrane orientation plays a more and more important role: The load-displacement curves are likely the same for loads up to 25kN, but as the system with an orientation of 0° has only a buckling load of 32kN, the system with an orientation of

 45° has a more than the double sized buckling capacity. Applying an imperfection of the bended eigenform to the system the structural behaviour is still the weakest system, nevertheless the difference to the perfect system and the "twisted" imperfect system is lesser compared to smaller columns. The system with a membrane orientation of 45° is again much stiffer compared to an orientation of 0° .



Figure 5: load-displacement-curves for a column 0,60m wide, right: perfect geometry and eigenforms.

The comparison of m embrane restrained columns shows, that for perfect systems the structural behaviour gets weaker when the columns are wider (figure 6). Taking imperfection ("bended eigenform") into account it is the other way round and the structural behaviour gets stiffer when the columns are wider. The buckling capacity, which is predicted by the load-displacement curve of the im perfect, "bended" system is increasing with the width of the columns. As the load-displacement curves of perfect and imperfect system appear each other with the increase of the width, this means, that the structures get less prone to imperfections.



Figure 6: left: load-displacement-curves for a different columns (all membrane orientation 45°), right: vectors of principle membrane forces

Figure 6 shows as well the principle m embrane forces under load. In the perfect systems the membrane forces run horizontal between the bars, in the imperfect system they mostly run diagonal.

In figure 7 the buckling capacity is shown in dependency on the width of the columns and the membrane orientation. For a m embrane orientation of 0° the buckling capacity is increasing quite linear with the width and the buckling capacity reaches a maximum of 24 kN. If the membrane is orientated 45° , the buckling capacity is similar to the 0° orientated membrane for the two smallest columns. As the load-displacement curves were already nearly identical for these two columns the membrane orientation plays obviously only a subordinate role for the structural behaviour. The reason for this behaviour is in the small curvature of the bars, so that under the imperfection of the bended eigenform the bars deform quite parallel to each other. Therefore the membrane can not restrain one bar to the o ther, as both bars have analogue deformations. With an increase of the width of the columns the deformations of the bars differ more (figures 3-5) and this enables the membrane to restrain one bar to another. The restraining effect can be described by the principle stresses in the membrane. For all imperfect columns the principle stresses do not run horizontal between the bars but mostly rotated between 30° and 60°. A m embrane orientation of 45° offe rs therefore a higher restraining effect and leads to a stiffer structural behaviour and higher buckling capacity.



Figure 7: buckling-load (left) and design loads (right) for maximum stress of 214 N/mm² in the bars

Whereas the buckling capacity is determined independently from the stresses in the bar, for the design of the structure the stresses need to be taken into account. In figure 7 the design-loads for a stress of 214 N/ mm² (S 235, safety-factor 1.1) are shown. For the perfect system the design loads decrease when the width of the columns increases. This results from the fact that with an increasing width the distance between the middle of the bars and the middle axis gets higher. This distance causes higher bending moments in the bars and subsequently higher stresses. The design load for a colu mn with a membrane orientation of 0° is m inimal higher compared to a membrane orientation of 45° . It needs to be mentioned, that the design-loads for the perfect system are only theoretical as they lie above the buckling capacity.

Surprisingly the design loads for the imperfect system increase with the width of the system. Although for the imperfect system the same effect occurs, that a higher width causes higher bending moments, a second effect is more significant. This second effect results from the imperfections applied to the s tructure. The stresses in the imperfect structure are by far higher than the stresses in the perfect system. That means for the membrane, that it is not only her task to restrain the bars horizontal to each other but even more to restrain the bars against the asymmetric deformations caused by the imperfections. The load -displacement curves showed already, that this restraining effect in creases with the width of the columns. This increase of the restraining effect is higher than the increase of the bending moment caused by the width and therefore the desi gn loads increase with the width. The design-loads for the imperfect columns with a width of 10 or 20c m are equal for both mem brane orientations.

Regarding the wider colum ns a membrane orientation of 45° offers higher design loads compared to a membrane orientation of 0° - for the same reason as described for the buckling load. For the small columns the design load is only slightly under the buckling load, but with an increasing width of the colum ns the distance between buckling load and design load gets higher. This makes the structures more redundant and allows the use of higher-quality steel.

5 COMPARISON WITH A SINGLE COLUMN

Of course the question comes up under which static conditions a membrane restrained column is more effective com pared to a simple strut with a circular hollow cross section. In figure 8 the buckling- and design-load of a membrane restrained column with a width of 30cm and a membrane orientation of 45° is shown for r different cross-sections. The buckling- and design loads increase with the cross section area, but the in crease gets smaller as the cross section gets "closer". This results from the stresses in the bars, which are dom inantly caused by bending moments and only little by axial f orces and therefore hollow sections are m ore efficient than full profiles.

Beside quadratic also rectangul ar cross sections have been analysed, but as the profiles usually are stressed by two-axial bending, the structural behaviour is very similar. Regarding the buckling- and design-load they offer only a small option of optimisation.



Figure 8: buckling- and design-loads for a membrane restrained column (b=0,30m) and a circular hollow

The compared circular hollow strut is defined by the sam e mass as the sum of the three bars of the membrane restrained column. The thickness is set to one tenth of the diameter and as material steel is used as well. With the increase of the cross section area the diameter of the circular hollow section increases as well and consequently the buckling- and design load increases progressively. For small cross-section areas the membrane restrained columns have higher buckling- and design loads, but using full cross sections for the bars is not very efficient and has less potential as a simple strut. The slenderness of the circular strut is 144 for the cross section area equivalent t to cross section 5. As the cross section area decreases the slenderness increases from 240 (equivalent cross section 4) to 367 (equivalent cross section 1).

6 CONCLUSIONS

The use of m embrane restrained columns is recommend for structures with a s lenderness higher than 200. To use the full potential of m embrane restrained columns the width should be at least 10% of the height and the m embrane should be orientated by 45°. Hollow cross sections for the bars are more effective in relation to the weight than full profiles. As materials steel and aluminium can be used.

As the use of the columns is especially thought for tem porary structures, the transport of the columns becomes an important issue: The bars could be easily divided in two or three (or even more) sections, which are con nected by a plug-in connection or a screwed joint. The membrane can be folded and so both elem ents can be tran sported easily. As the connected bars only need to be pushed into membrane pockets, the column can as well be assembled and disassembled very fast.

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SOFT.SPACES _ new strategies for membrane architecture

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Summary. The desire for new mostly fluent "soft spaces" in architecture cannot be overseen any more. Therefore new tools and approaches are tried out to create architecture with special spatial qualities. Spatially curved membrane structures and especially anticlastic Minimal Surfaces offer one possible approach to this topic. This paper presents the overview of the research on spatially curved Minimal Surfaces that considers the infinite possibilities of membrane forms as elements in architecture in combination with common buildingtechnologies and shows the manifold possibilities of this approach and technology¹. Further on this paper partly reveals new correlations between Minimal Surface and boundary conditions and so far unknown rules in its self organizing processes. Case studies document new capabilities in designing and creating space in architecture. Latest approaches are dealing with alternative boundary-conditions and with software implementation in terms of scripting found rules².

1 INTRODUCTION

On the basis of the research of Frei Otto and his team at IL (University of Stuttgart) and the resulting exceptional pioneer constructions, building with textiles as an alternative to traditional materials like wood, stone, steel, glass, and concrete was rediscovered during the last decades. Deriving from self organizing forms of Minimal Surfaces, prestressed, spatially curved Membrane Structures were up to today mainly used for wide span, lightweight-structures. For this reason membrane structures tend to be seen from a structural or material point of view only. In contrast to our right-angled, conventionally built environment the desire for fluent "soft" spaces in architecture cannot be ignored any more. The possibility to create light and fluent spaces as a symbiosis of form and structure offers new qualities and chances in the design of residential or office buildings.

2 SUBJECT

This paper presents the overview of the research on spatially curved Minimal Surfaces that considers the infinite possibilities of membrane forms as elements in architecture in combination with common building-technologies and shows new capabilities in designing and creating spaces. Seen as an element in the design of architecture these anticlastic, fluent forms caused by structural conditions, follow the rules of FORMFINDING in its initially (by Frei

Otto) defined sense. Very often we misuse the term "formfinding". What Architects mostly mean and do is a man controlled process of SHAPING - a process that happens on a consciously controllable and formal level. In contrast to the man-controlled process of shaping, forms that are arising from self organizing processes can only be influenced by the design of their boundaries. The form itself can only be found and represents the result which cannot be manipulated. The architect finds himself in the unusual position of a creative "formfinder" instead of the "shaper". The fluent forms of Minimal-Surfaces are fascinating by their variety, structural performance, reduction to the minimal in terms of material use and resources and their special fashion-resistant aesthetics. Together these parameters represent the common basis of a potential design or design concept and characterize its grade of sustainability.

3 OBJECTIVES

Since self organizing processes follow precise rules and contain optimization by their nature descriptions and especially in architecture illustrations of these rules can be used as design tools. To find out about the chances for an architecture between "hard" and "soft" morphology, basic research on the systematic determination of very different boundaries - the interface between membranes and common construction technologies - enables the opportunity to analyze anticlastic Minimal Surfaces regarding form and curvature. Vice versa we get an idea of the correlation between 3d-curvature, deflection and determined boundary and further on an idea of formal and structural behavior. In this context the assessment and visualization of the Gaussian curvature, which were adapted especially to this research, played an important role.

4 SPECIAL SPECIFICATIONS

4.1 MINIMAL SURFACE

All experiments are restricted to forms that can be derived from the results of soapfilm models – the Minimal Surface. As long as boundary conditions are not changed, Minimal Surfaces can be arranged as a unity arbitrarily in space without changing its form/geometry.

4.2 INTERFACE

Linear, maximal 2dimmensionally curved, bending resistant, line supported boundaries turned out to be the ideal interface between membranes and common construction technologies. All further experiments were restricted to boundaries of that kind.

4.3 MEMBRANES AS AN INTEGRATIVE ELEMENT

Membranes are seen as an integrative component of architecture and are directly connected to other elements of common construction methods. In terms of structural effectiveness the surfaces themselves are considered to be highly efficient by their spatial curvature but not to be load bearing elements for other structural members although newest approaches in the author's research are dealing with this possibility.

5 INVESTIGATION

The range of exploration covers wall-like elements, T-shaped connections, solutions for vertical, horizontal and free corners and tubular entities – the so called Catenoid.

6 METHODS

Besides physical (Fig. 01) and soapfilm models (Fig. 02) mainly digital experiments (Fig. 03) were used for the interpretation and the verification of results. Soapfilm models were mainly considered to fulfill a control function. Digital models were essential for the analysis and evaluation of forms (section curves, their diagrammatic overview, analysis of angles in space,...) of Minimal Surfaces. In this context the assessment and visualization of the Gaussian curvature (Fig. 04), which were adapted especially to this research, made it possible to compare and to draw one's conclusions on different forms and their structural behavior.



Fig. 01 Physical study model showing catenoids between shifted circular rings



Fig. 02 Sequence of Soapfilm Models





Fig. 03 Digital model of a catenoid between circular rings and Visualization of Gaussian Curvature

Fig. 04 Special unification of assessment and visualization of the Gaussian curvature for this research

7 RESULTS OF INVESTIGATION

The results of physical, soapfilm and mainly digital experiments show surprising and partly new correlations between form and boundary proportions and so far unknown rules of the self organizing processes of Minimal Surfaces – especially in the field of the Catenoid. The overview and the comparison of the results as well as the possibility of a targeted selection can now be the basis for creative applications.

7.1 MINIMAL SURFACES BETWEEN STRAIGHT LINES AND BOUNDARIES CONSISTING OF SEGMENTS OF A CIRCLE

All experiments related to this series (Fig. 05) show, that for this boundary condition it is not possible to find a fully anticlastic curved Minimal Surface. Those surfaces which show few flat areas are generated within a relatively small spectrum of boundary conditions. They concentrate on boundary conditions consisting of semicircles with a diameter that corresponds to the distance of the boundaries. Independent of the amplitude of the curved boundary Minimal Surfaces tend to be flat in the near of the straight line boundary. Experiments show that in average up to 96% of the horizontal deflection that was given by the curved boundary

is disappearing halfway between the upper and lower boundary. Horizontally shifted boundaries (Fig. 06) can be interesting from the architectural point of view. But in terms of anticlastic Gaussian Curvature this always means a further increase of flat areas.





Fig. 05 Minimal Surfaces between straight lines and boundaries consisting of segments of a circle

Fig. 06 Horizontally shifted boundaries

7.2 MINIMAL SURFACES BETWEEN BOUNDARIES CONSISTING OF SEGMENTS OF A CIRCLE

In this case the boundaries of wall like Minimal Surfaces can have the **same direction** or they can be **arranged inversely**. Horizontally shifted boundaries represent special cases and show interesting architectural effects. The horizontal offset can be in longitudinal, cross or diagonal direction.

7.2.1 Minimal Surfaces between boundaries consisting of segments of a circle in the same direction

Boundaries that are curved in the same direction (Fig. 07) generally effect strong anticlastic curvature of Minimal Surfaces. Boundary conditions consisting of semicircles with a diameter that equals the distance of the boundaries can be qualified as 100% spatially curved. Section lines show the smallest circle of curvature exactly on half height and harmonic development of the surfaces (Fig. 08).



Fig. 07 Boundary configurations consisting of segments of circles having the "same direction"



Fig. 08 Vertical section of digital models and their circles of curvature

7.2.2 Minimal Surfaces between boundaries consisting of inversely arranged segments of a circle

Curved and inversely arranged boundary conditions effect anticlastic curvature covering most of the surface, even if the boundaries have little oscillation from the longitudinal axis. The mostly curved surface can be developed with boundaries consisting of semicircles with a diameter of 2/3 of the distance of the boundaries (Fig. 09).



Fig. 09 EM KK 2/3 _ 1,00HK ggs Fig. 10 EM KK 2/1 _ 0,50HK ggs Fig. 11 EM KK 1/2 _ 1,00HK ggs

Areas with little spatial curvature can first of all be found exactly at the maxima of boundary curvature and on half height. Starting from the ideal case these flat areas increase with increasing as well as with decreasing diameters of the base-circles. Surfaces arising from boundary conditions with base-circles bigger than the height show flattened vertical stripes (Fig. 10) whereas flattened horizontal stripes (Fig. 11) appear with boundaries consisting of segments of circles with less than the height.

7.3 MEMBRANE CORNERS

Regarding corner solutions, boundaries can be arranged horizontally (Fig. 12) or vertically (Fig. 13). The free corner (Fig. 14) describes a special case and will not be described in this paper.



Fig. 12 Horizontal Corner

Fig. 13 Vertical Corner

Fig. 14 Free Corner

7.3.1 Horizontal Membrane Corner

All executed experiments with horizontal right-angled corners show almost constant surface curvature (Fig. 15) and deflection in the area of the corner (Fig. 16). This happens independently form the leg length and from being arranged symmetrically or asymmetrically. The section lines of digital models are congruent (Fig. 17). Leg length being shorter than the height cause surfaces with little anticlastic curvature. Surfaces of maximum spatial curvature in all areas can be achieved with a ratio 1/1 to 3/2 of leg length/height. Increased leg length causes areas with little anticlastic curvature at the end of the legs.



Fig. 15 Horizontal corner with ratio of 1 1 1 (leg/leg/height)



Fig. 16 Horizontal corner with ratio of 111 Gaussian Curvature



Fig. 17 Horizontal section lines "Horizontal Membrane Corner" – in comparison

7.3.2 Vertical Membrane Corner

The configuration of the vertical, right-angled corner can be used to explore different element length [EL] or different wing length [FL] and their effect on the spatial curvature of the surface. The analysis of section lines, circles of curvature and Gaussian curvature illustrates the interrelationship of surface and boundary proportions. For predominantly curved surfaces these proportions can be located at a ratio of 1/1/1 (element length/height/wing length) whereby even distribution of curvature and harmonic, fluent transitions of surface curvature can be achieved (Fig. 18 to Fig. 20).



Fig. 18 EV 1 1 1 FL var

Fig. 19 EV 1 1 1 FL var



7.3.2.1 Vertical Membrane Corner – variable wing length

Variable wing length (Fig. 21) cause change of form of Minimal Surfaces until the wing length is 1,5 times longer than height. From this point the Minimal Surface stays constant in terms of form and curvature. Further increasing of wing length leads to flattened areas at the end of the wing. Wing lengths which are shorter than the height generate strong anticlastic curvature in the area of the corner but the vertical part of the surface looses spatial curvature at the same time.

7.3.2.2 Vertical Membrane Corner – variable element length

Elongating the element length (Fig. 22) means a decrease of curvature in the midspan of the element, while the strong anticlastic curvature in the corner region stays unchanged.



A square geometry in plan, meaning that the element length equals the wing length, offer the possibility to attain larger areas with sufficient anticlastic curvature. These curvatures are characterized by soft transitions and even distribution of curvature.

7.4 T-CONNECTION

Surfaces meeting in a T-connected boundary (Fig. 23) generate a Y-intersection (Fig. 24). This happens independently from the angle of the boundary connection. The 3 different parts of the Minimal Surface meet with 120° and form an arch-like intersection. This arch is less curved at its angular point and more curved the closer it is to the T-connection of the boundary. *"In very special cases only, a circular intersection can be formed."*³ These special cases were used to form pressure resistant arches for real structures.



Fig. 23 Geometry of right angled T-connection

Fig. 24 Minimal Surface generated from a right angled T-connection

7.4.1 Right-angled T-connections

In terms of right-angled configurations the leg length of H (Fig. 23) has no influence on the form of the generated Minimal Surface as long as it is longer than the deflection of the Y-

intersection. This happens to be the same, independently from the wings being arranged symmetrically or asymmetrically.

7.4.1.1 symmetric wing length [FL]

For symmetric wing length [FL] one can determine that the magnitude of the Y-intersection is directly connected to the ratio of wing length and element length. For all boundary conditions with $FL \ge EL/2$ the magnitude of the Y-intersection equals 20,6% of the element length. For wing length shorter than the element length, a nonlinear behavior of the Y-intersection can be determined. So the boundary condition FL=EL/2 represents the borderline between linear and nonlinear development of displacement in the direction of H (Fig. 25).



Fig. 25 Displacement of vertical section lines from right angled T-connection with symmetric wing length and different element length [EL]



Fig. 26 T-connection with square boundary geometry FL = EL/2

Fig. 27 linear increase of displacement at increasing element length up to EL/2=FL, then nonlinear

A square geometry in plan causes evenly distributed curvature in the surface (Fig. 26). The curved Y-intersection is similar to a basket arch (Fig. 27). Starting from a square geometry in plan increased wing length results in the generation of insufficiently curved areas at the ends of the wings. On the other hand there are no effects on the form, radii of curvature of the Minimal Surface and the transitional zone with anticlastic curvature to insufficiently curved areas does not move. The enlargement of the element length which corresponds

proportionally to a reduction of the wing length causes insufficiently curved areas which are merged together in the element middle. Strong anticlastic curvature is limited to the areas of the T-connection of the boundary.

7.4.1.2 asymmetric wing length [FL]

Spatially curved Y-intersections and spatially curvature of all partial areas are generated by asymmetric wing length. The horizontal component of the deflection always occurs in direction of the larger wing.

7.4.2 Non-right-angled T-connections

When using T-connected boundaries with angles different from 90° the surface of H (Fig. 28), which is totally flat for the 90° case, will be spatially curved too. Increasing deviation of 90° goes along with increasing anticlastic curvature of surface H (Fig. 29).



The formally interesting Minimal Surfaces which develop as a result of a T-connection with a not at right angles deviating surface H show spatially curved intersection lines. The more the angle differs from 90° the more the anticlastic curvature of H increases. At the same time the vertical deflection of the former horizontal parts decreases.



Fig. 30 vertical section lines T30°, T45°, T60°, T75°



7.5 CATENOID

The shape of the catenoid is basically generated by a catenary that rotates around a longitudinal axis. It is the only rotational body that can be minimal surface at the same time. As we know from SFB230 the maximum attainable height of a catenoid spanning two circular rings is *approximately 1,3 times the radius of a ring.*⁴ For conceptual designs in architecture, boundaries different from two identical circles but with different diameters, not being

arranged in one axis and/or not being symmetrically arranged are needed. So the maximum attainable heights of catenoids with different boundary geometries and arrangements were examined. New rules could be found for major boundary configurations. The resulting diagrams can be scaled at will.

7.5.1 Catenoids between circular rings of different diameters

Starting from the extreme of 1,3 times the radius of a ring the maximum height of a catenoid is decreasing if one of the rings diameter is decreasing (Fig. 32). Fig. 32 also illustrates that upper rings smaller than 1/5 (upper ring /lower ring) effect very little maximum attainable height and surfaces with little Gaussian Curvature at the same time.



Fig. 32 Change of form and change of Gaussian Curvature of a Catenoid with decreasing diameter of upper ring and therefore decreasing height





Fig. 33 3dimensional diagram for catenoids between circular rings of different diameters

Fig. 34 Soapfilm model and diagram for catenoids between circular rings of different diameters

Several experiments showed that all the attainable maxima in dependence from the given diameters are located on a common circle - the extreme value circle. This circle again is in direct proportion to the circular base ring. (Fig. 33) The developed diagram allows a determination of the maximal attainable height when the diameters of the two rings are given. The other way round the maximal diameter of the upper ring can be found by predefining the desired height and the diameter of the base ring. (Fig. 34)



Fig. 35 Case-Study A

Case-Study A

A catenoid is perforating several floors and creates a courtyard situation. Its position is chosen the way that the ground floor gets a spatial incision whilst the other floors are still connected by a catwalk between catenoid and facade.



Fig. 36 Case-Study G

Case-Study G

The form of the catenoid is intersected with a rectangular building. In this case the catenoid was tilted in the direction from (left) and towards (right) the building. For this reason the opening in the façade opens on top and narrows to the sky inside the courtyard (left) and vice versa (right).

7.5.2 Catenoids between shifted circular rings

A displacement of the boundary rings effects lower maximum heights of catenoids. (Fig. 37) This correlation also follows precise rules. The interrelation of displacement of boundary rings and maximal attainable height of the catenoid can be found on circular movements defined by the center of the base ring and the diameter of the rings. (Fig. 38) At the same time we can observe that a displacement of more than ³/₄ of the diameter of the rings effects areas with little Gaussian Curvature. Strongly curved areas can always be found at half height of the catenoid. A displacement of 1 diameter of the boundary rings cannot be attained with a catenoid but forms two separated surfaces within the rings.





Fig. 37 3d view of overlayed catenoids between shifted F circular rings showing the circular movement of the shift upper ring and the dependence of horizontal displacement of the rings and the loss of height.



Fig. 39 Case Study K

Fig. 38 Side view of overlayed catenoids between shifted circular rings showing the circular movement of the upper ring and the dependence of horizontal displacement of the rings and the loss of height.

Case-Study K

A horizontal displacement of one of the boundaries of the catenoid enables a spatial movement. For the fact that the base rings of displaced catenoids have equal diameter they can act like swivel plates. This way vertical connections or orientation to natural light can be solved.

7.5.3 Catenoids between square rings of the same side length

Compared to catenoids generated by two circular rings, catenoids between two equal square rings (Fig. 40) are having their maximum height at 1,44times of the side length of the

square (Fig. 41) In analogy to catenoids between circular rings the maximal attainable height or the smallest possible upper square can be found on a common extreme value circle too.



Fig. 40 3dimensional diagram for catenoids between Fig. 41 diagram for catenoids between square rings of square rings of the same side length the same side length

7.5.4 Catenoids between a square and a circular ring

Catenoids between a square and a circular ring don't follow an extreme value circle but a catenarylike line starting from the center of the square and going through the quadrant of the upper circular boundary. The maximal attainable height equals 1,39times the radius of the inscribed circle of the square respectively half of its side length. This is valid for configurations where the circle is the incircle at the most.

7.5.5 Congruent cut-outs from Minimal Surfaces of Catenoids

All executed investigations have shown that each randomly selected cut-out from a Minimal Surface of a catenoid will be a Minimal Surface with equal position in space and equal curvature of the surface itself. This can be explained by the absolute identical stresses in all directions of Minimal Surfaces. The example in Fig. 42 is showing a randomly selected closed curve that is projected on the surface of a catenoid. For this reason this curve is exactly matching the surface of the initial catenoid. By defining this curve as a new boundary line the new surface within this boundary is also matching the surface of the initial catenoid.



Fig. 42 Congruent cut-outs from Minimal Surfaces of Catenoids

Case-Study M2

intersection line.

The intersection of several catenoids is possible without changing of form of the different parts. This way 3dimensionally curved ridges are developed by the



Fig. 43 Case Study M2

The definition of the new boundary can be found as described before, but it can also be found by intersecting different independent catenoids or by intersection with other forms. As shown in case-study M2 (Fig. 43) catenoids even don't need to have the same position in space or the same size. This way a lot of possibilities are open for a potential design in architecture.

8 CONCLUSION ON RESEARCH, CASE-STUDIES AND EXPERIMENTAL STRUCTURES

The characteristics that Minimal Surfaces can be proportional scaled and that a predefined cut-out of minimal surface keeps unchanged multiplies the possibilities for the design. Using the found rules case studies give an idea of the infinite possibilities that are open to create very special "soft spaces", with new architectural qualities like shown in case studies and experimental structures (Fig. 44 to Fig. 46) and furthermore.



Fig. 44 "Cube of Clouds" experimental structure in model and in scale 1/1 exhibited and published at Premierentage 2005, Best of 2005 and Ziviltechnikertagung 2005



Fig. 45 "Cut.enoid.tower" - experimental structure with a height of about 13 meters in scale 1/1. The distorted appearance is generated by the interaction of pin-joint columns, which work on compression and different versions of prestressed catenoids.

Fig. 46 "minimal T"– structure shows the possibility to deflect surfaces that were flat before being assembled by using special geometries in arrangement

9 PERSPECTIVE

Latest approaches are dealing with alternative boundary-conditions and with software implementation in terms of scripting found rules (Fig. 47). An investigation on the correlation of self organizing forms, their close relation to nature and their aesthetic values also seems to be interesting questions for the future.



Fig. 47 Grasshopper script "Catenoids between horizontally shifted circular rings"

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Direct Minimization Approaches on Static Problems of Membranes M. MIKI*, K. KAWAGUCHI†

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Abstract. Within this work, direct minimization approaches on static problems of membranes are discussed. In the first half, standard direct minimization methods are discussed. Some form-finding analyses of tension structures are also illustrated as simple direct minimization approaches. In the second half, the principle of virtual works for cables, membranes, and 3-dimensional bodies are examined and they are approximated in a common way by using Galerkin method. Finally, some examples that direct minimization approaches can solve are reported.

1 INTRODUCTION

It is widely known that direct minimization approaches are sometimes very effective for solving simple static problems [2], such as minimal surface problem. This work aims to propose a common framework that can solve various types of static problems by direct minimization approaches. In this framework, not only simple problems but also general problems of continuum bodies can be solved by direct minimization approaches.

In chapter 2, standard direct minimization methods are discussed. The dual estimate is also introduced to include constraint conditions into direct minimization approaches. In chapter 3, the principle of virtual works for cables, membranes, and 3-dimensional bodies are examined and they are approximated in a common way by Galerkin method. In chapter 5, it is pointed out that some principle of virtual works can be solved by direct minimization methods even though their objective functions that are to be minimized are not clear.

2. DIRECT MINIMIZATION APPROACHES

2.1 Three Term Method without Constraint Conditions

For form-finding of cable-net structures, let us discuss the following stationary problem:

$$\Pi(\boldsymbol{x}) = \sum_{j=1}^{m} w_j L_j^{2}(\boldsymbol{x}) \to \text{stationary}, \qquad (1)$$

where w_j, L_j are the weight coefficient and the length of the *j*-th cable respectively. In addition,

$$\mathbf{x} \equiv \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix}^r$$
, and $\nabla f \equiv \begin{bmatrix} \frac{\partial f}{\partial x_1} & \cdots & \frac{\partial f}{\partial x_n} \end{bmatrix}$, (2)

where $\{x_1, \dots, x_n\}$ is a set of unknown variables and ∇f is the gradient of a function f.

It has been reported by the authors [1] that Eq. (1) is the problem that simply represents the Force Density Method [3]. While making use of inverse matrix is proposed in the original article, direct minimization approach can also solve the same problem.

In the following discussion, suppose $\{x_1, \dots, x_n\}$ represents the Cartesian coordinates of the free nodes and remark that those of the fixed nodes are eliminated beforehand and directly substituted into each $L_i(\mathbf{x})$.

The stationary condition (to be solved) of Eq. (1) is expressed as follows:

$$\nabla \Pi = \sum_{j} 2w_{j} L_{j} \nabla L_{j} = \boldsymbol{0} .$$
(3)

When this problem is solved by direct minimization methods, $\nabla \Pi$ is used as the standard search direction. The simplest direct minimization recursive strategy is given by

$$\boldsymbol{r}_{Current} = \frac{\nabla \Pi^{T}}{\left|\nabla \Pi\right|} \circ (\boldsymbol{x} = \boldsymbol{x}_{Current}),$$

$$\boldsymbol{x}_{Next} = \boldsymbol{x}_{Current} - \boldsymbol{O}\boldsymbol{r}_{Current},$$
(4)

which is usually called the steepest decent method; however, in the relation with the following discussion, we shall call Eq. (4) as **2-term method**. In Eq. (4), $r_{Current}$ is normalized because too large $r_{Current}$ or too small $r_{Current}$ sometimes causes trouble. In addition, α is a unique parameter which can be adjusted manually for controlling the step-size.

By the way, the following remedy of Eq. (4) sometimes provides a remarkable improvement of global convergence efficiency:

$$\boldsymbol{r}_{Current} = \frac{\nabla \Pi^{T}}{|\nabla \Pi|} \circ (\boldsymbol{x} = \boldsymbol{x}_{Current}),$$

$$\boldsymbol{q}_{Next} = 0.98 \boldsymbol{q}_{Current} - \alpha \boldsymbol{r}_{Current},$$

$$\boldsymbol{x}_{Next} = \boldsymbol{x}_{Current} + \alpha \boldsymbol{q}_{Next},$$
(5)

which gives the simplest 3-term method.

Fig. 1(a) shows a numerical example for verification which can be solved by either **2-term method** or **3-term method**. The model consists of 220 cables and 5 fixed nodes. The first author has tested giving random numbers for initial configuration of $\{x_1 \ \cdots \ x_n\}$, which were ranging from -2.5 to 2.5 as shown in Fig. 1(b). Then, when $\{w_1 \cdots w_m\} = \{1 \cdots 1\}$, Fig.1 (c) was obtained and the corresponding minimum value of Π was 160.214. When 4 times greater weight coefficients were assigned onto the boundary cables, Fig. 1 (d) was obtained and the corresponding minimum value was 188.09.

The history of Π in the **3-term method** corresponding to Fig. 1 (c) is shown by Fig. 2 (α

was constantly set as 0.2). When α was gradually decreased manually, $|\nabla \Pi|$ also decreased gradually as shown in Fig.3.

It is important to note that Eq. (5) has a close relation with the family methods with three term recursion formulae. In 1982, M. Papadrakakis stated that two popular direc minimization methods, the Dynamic Relaxation method and the Conjugate Gradient Method, can be classified under one category, the family methods with three term recursion formulae [4], which was first proposed by M. Engeli et. al [3] in 1959.



By using either 2-term method or 3-term method, both Fig. 1(c) and (d) can be obtained. However, the behavior of the **3-term method** is always very impressive, smooth, and seems to be the best method for solving various types of static problems by direct minimization approaches. Therefore, the scope of the 3-term method must be examined and every numerical example illustrated below was solved by the 3-term method.

2.2 Three Term Method with Constraint Conditions



Figure 4: Form-Finding of Simplex Tensegrity

For form-finding of Simplex Tensegrities that consist of 3 struts (compression) and 9 cables (tension), let us discuss

$$\Pi(\boldsymbol{x},\boldsymbol{\lambda}) = \sum_{j=1}^{m} w_j L_j^{4}(\boldsymbol{x}) + \sum_{k=1}^{r} \lambda_k (L_{m+k}(\boldsymbol{x}) - \overline{L}_{m+k}) \to \text{stationary}, \qquad (6)$$

where the first sum is taken for all the length of the cables and the second sum is taken for all

the length of the struts. The connections over the members are shown by Fig. 4(a). Such a modified functional is obtained by applying the *Lagrange* multiplier method to a minimization problem with equality constraint conditions. The supplemented multipliers are denoted by λ_k and the prescribed length of the struts are denoted by \overline{L}_{m+k} . When $\{w_1, \dots, w_9, \overline{L}_{10}, \dots, \overline{L}_{12}\} = \{1, \dots, 1, 10, \dots, 10\}$, Fig. 4 (b) was obtained as the unique solution and the corresponding minimum value of $\sum_{i=1}^{m} w_j L_j^4(\mathbf{x})$ was 18000. (See Ref. [1] for further detail)

To solve such a problems, let

$$\boldsymbol{x} = \begin{bmatrix} x_1 & \cdots & x_n \end{bmatrix}^T , \quad \frac{\partial f(\boldsymbol{x})}{\partial \boldsymbol{x}} = \begin{bmatrix} \frac{\partial f}{\partial x_1} & \cdots & \frac{\partial f}{\partial x_n} \end{bmatrix}, \quad \boldsymbol{\lambda} = \begin{bmatrix} \lambda_1 & \cdots & \lambda_r \end{bmatrix}, \quad \frac{\partial f(\boldsymbol{\lambda})}{\partial \boldsymbol{\lambda}} = \begin{bmatrix} \frac{\partial f}{\partial \lambda_1} & \cdots & \frac{\partial f}{\partial \lambda_r} \end{bmatrix}^T, \text{ and } (7)$$

$$\nabla f \equiv \frac{\partial f(\mathbf{x})}{\partial \mathbf{x}}.$$
(8)

Then, the stationary condition of Eq. (6) is represented by

$$\nabla \Pi = \boldsymbol{\theta} \,\&\, \frac{\partial \Pi}{\partial \lambda} = \boldsymbol{\theta} \,. \tag{9}$$

Let us discuss the first condition. Due to the supplemented multipliers $\{\lambda_1 \dots \lambda_r\}$, $\nabla \Pi$ is still unknown even if x is given. The simplest idea to determine $\nabla \Pi$ uniquely is making use of general inverse matrix. Let us rewrite $\nabla \Pi = 0$ into the following form:

$$\nabla \Pi = \nabla \Pi_{w}(\mathbf{x}) + \lambda \mathbf{J}_{\lambda} = \mathbf{0} \iff \lambda \mathbf{J}_{\lambda} = -\nabla \Pi_{w}, \tag{10}$$

For example, in this case,

$$\Pi_{w}(\boldsymbol{x}) = \sum_{j} w_{j} L_{j}^{4}(\boldsymbol{x}), \ \nabla \Pi_{w} = \sum_{j} 4 w_{j} L_{j}^{3} \nabla L_{j}, \text{ and } \boldsymbol{J}_{\lambda} = \begin{bmatrix} \nabla L_{m+1} \\ \vdots \\ \nabla L_{m+r} \end{bmatrix}.$$
(11)

Eq. (10) is not satisfied unless a solution is given as x. Nevertheless, by using *Moore-Penrose* type pseudo inverse matrix J_{λ}^{+} , λ can be determined by

$$\boldsymbol{\lambda} = -\nabla \Pi_{\boldsymbol{w}}(\boldsymbol{x}) \cdot \boldsymbol{J}_{\boldsymbol{\lambda}}^{+}, \qquad (12)$$

which provides basically a least squared solution of Eq. (10). This strategy seems rather rough but it works fine because when Eq. (12) turns to a least norm solution, Eq. (10) is simultaneously satisfied and vice versa. A popular numerical environments such as Matlab®, Scilab®, and Octave® provide pinv() for this purpose. A popular linear algebra package Lapack® provides direct least squared solvers such as dgels().

As the result, 1 to 1 mapping between x and $\nabla \Pi$ can be defined by

$$\nabla \Pi = \nabla \Pi_{w}(\boldsymbol{x}) + \lambda \boldsymbol{J}_{\lambda} \circ (\lambda = -\nabla \Pi_{w}(\boldsymbol{x}) \cdot \boldsymbol{J}_{\lambda}^{+})$$
⁽¹³⁾

which enables **2-term method** and **3-term method** feasible. This strategy is often called the **dual estimate** in the context of linear programming [7]. Here, one might notice that Eq. (13)

immediately reduces to the **projected gradient**, in which λ vanishes; however, λ is always calculated explicitly in this work because Eq. (13) simply represents the **composition of forces** (see Fig. 5 (a)).

Let us discuss the second condition in Eq. (9). For example, in this case, it expands as

$$\frac{\partial \Pi}{\partial \lambda} = \boldsymbol{\theta} \Leftrightarrow \begin{cases} (L_{m+1}(\boldsymbol{x}) - \overline{L}_{m+1}) = 0, \\ \vdots \\ (L_{m+r}(\boldsymbol{x}) - \overline{L}_{m+r}) = 0. \end{cases}$$
(14)

The simplest idea to satisfy Eq. (14) is to solve

$$\boldsymbol{J}_{\lambda} \cdot \Delta \boldsymbol{x} = -\boldsymbol{r} \text{, where } \boldsymbol{J}_{\lambda} = \begin{bmatrix} \nabla L_{m+1} \\ \vdots \\ \nabla L_{m+r} \end{bmatrix}, \boldsymbol{r} = \begin{bmatrix} L_{m+1} - \overline{L}_{m+1} \\ \vdots \\ L_{m+r} - \overline{L}_{m+r} \end{bmatrix}.$$
(15)

Again, J_{λ}^{+} plays an important role as follows

$$\Delta \boldsymbol{x} = -\boldsymbol{J}_{\lambda}^{+} \cdot \boldsymbol{r} , \qquad (16)$$

which gives the least norm solution of Eq. (15). It is highly recommended to scale Δx when it is substituted into $x_{current}$, e.g.

$$\boldsymbol{x}_{Current} := \boldsymbol{x}_{Current} + 0.5\Delta \boldsymbol{x} \,. \tag{17}$$

Fig. 5 (b) shows an over-view of above mentioned strategies. In each step, if Eq. (17) is performed once just before Eq. (4) or Eq. (5) is performed, $x_{current}$ would gradually approaches to the hyper-surface on which the prescribed constraint conditions are satisfied. By using either **2-term** or **3-term method**, Fig.4 (b) can be obtained. It was even giving random numbers for the initial configuration of $\{x_1 \ \cdots \ x_n\}$ as well as the previous example. In the analysis α was always set as 0.2.



Fig. 6 shows another example, which illustrates a form-finding analysis of a tension structure that consists of cables, membranes, struts and fixed points. In this analysis, the

following stationary problem was solved by abovementioned strategies:

$$\Pi(\boldsymbol{x},\boldsymbol{\lambda}) = \sum_{j} w_{j} L_{j}^{4}(\boldsymbol{x}) + \sum_{k} w_{k} S_{k}^{2}(\boldsymbol{x}) + \sum_{l} \lambda_{l} (L_{l}(\boldsymbol{x}) - \overline{L}_{l}) \rightarrow \text{stationary}, \qquad (18)$$

where the first sum is taken for all the line elements, the second is for all the triangle elements, and the third is for all the struts. In addition, S_k is a function to give the area of *k*-th triangle element. Fig. 6 (a) shows the initial configuration of *x* and it was given by random numbers as well as the previous examples.

It is important to note that Fig.6 (b) and (c) were obtained by only **3-term method** and it was almost impossible to solve the same problem by **2-term method**. It is because of that **2-term method** always traces "bumpy" objective function precisely whereas **3-term method** does roughly. By using the **3-term method**, unless the process terminated, the process can continue forever and the form can be varied at any moment by varying $w_{i}, w_{k}, \overline{L_{i}}$.

Pseudo codes of 3-term method are presented in Appendix A.



3. PRINCIPLE OF VIRTUAL WORK

3.1 Principle of Virtual Work for General Membranes

In the following discussion, *Einstein* summation convention and standard notations of tensor algebra are used. Let us discuss the surface area of a surface, which is given by

$$a = \int da , da \equiv \sqrt{\det g_{ij}} d\theta^1 d\theta^2 \ (1 \le i, j \le 2) , \tag{19}$$

where $g_{ij}, \theta^1, \theta^2$ represents the *Riemannian* metric and the local coordinates defined on the surface respectively.

Using $\delta \sqrt{\det g_{ij}} = 1/2g^{ij}\delta g_{ij}\sqrt{\det g_{ij}}$ where $g^{ij} \equiv (g_{ij})^{-1}$, the minimal surface problem is expressed as

$$\delta a = \frac{1}{2} \int_{a} g^{ij} \delta g_{ij} da = 0.$$
⁽²⁰⁾

By the way, let us discuss a self-equilibrium membrane whose boundary is fixed. The **principle of virtual work** for such a membrane is expressed as

$$\delta w = \frac{1}{2} \int_{a} t \sigma^{ij} \delta g_{ij} \mathrm{da} = 0 \quad (1 \le i, j \le 2), \qquad (21)$$

where t, σ^{ij} represents the thickness and the Cauchy stress tensor respectively. Here, $\delta_{g_{ij}}$ is

used instead of the variation of strain tensor δe_{ij} , due to the essential identity of them.

Using $\sigma^{ij} = \sigma^{i}_{k} g^{kj}$, the **principle of virtual work** for such a membrane is rewritten as

$$\delta w = \frac{1}{2} \int_{a} t \sigma^{i}{}_{k} g^{kj} \delta g_{ij} da = 0 \quad (1 \le i, j, k \le 2) .$$

$$\tag{22}$$

When a new stress tensor T_k^i is defined by $T_k^i \equiv t \sigma_k^i$, we have

$$\delta w = \frac{1}{2} \int_{a} T^{i}{}_{k} g^{kj} \delta g_{ij} da = 0 \quad (1 \le i, j, k \le 2).$$
⁽²³⁾

On the other hand, Eq. (20) can be rewritten as

$$\delta a = \frac{1}{2} \int_{a} \delta^{i} g^{kj} \delta g_{ij} da = 0 \quad (1 \le i, j, k \le 2) , \qquad (24)$$

which can be a simple demonstration of the essential identity of minimal surface and uniform stress surface.

3.2 Principle of Virtual Work for N-Dimensional Riemannian Manifolds

The length of a curve, the area of a surface, and the volume of a body are respectively given by:

$$l = \int_{l} \mathrm{d}l$$
, $a = \int_{a} \mathrm{d}a$, $v = \int_{v} \mathrm{d}v$, where (25)

$$dl = \sqrt{\det g_{ij}} d\theta^{1}(i, j = 1), \ da = \sqrt{\det g_{ij}} d\theta^{1} d\theta^{2} (1 \le i, j \le 2), \ dv = \sqrt{\det g_{ij}} d\theta^{1} d\theta^{2} d\theta^{3} (1 \le i, j \le 3)$$
(26)

By using the concept of *N*-dimensional *Riemannian* manifold, they can be unified. The volume element, the volume, and the variation of the volume for an *N*-dimensional *Riemannian* manifold *M* are respectively given by

$$\mathrm{d}\mathbf{v}^{\scriptscriptstyle N} \equiv \sqrt{\det g_{\scriptscriptstyle ij}} \, d\theta^{\scriptscriptstyle 1} \cdots d\theta^{\scriptscriptstyle N} \,, \, \mathbf{v}^{\scriptscriptstyle N} \equiv \int_{\scriptscriptstyle M} \mathrm{d}\mathbf{v}^{\scriptscriptstyle N} \,, \, \mathrm{and} \, \, \delta \mathbf{v}^{\scriptscriptstyle N} = \frac{1}{2} \int_{\scriptscriptstyle M} g^{\scriptscriptstyle ij} \, \delta g_{\scriptscriptstyle ij} \, \mathrm{d}\mathbf{v}^{\scriptscriptstyle N} \quad (1 \le i, j \le N) \,.$$

Then, the variational problem of the volume of *M* is defined by

$$\delta v^{N} = \frac{1}{2} \int_{M} g^{ij} \delta g_{ij} \mathrm{d} v^{N} = 0 \Leftrightarrow \frac{1}{2} \int_{M} \delta^{i}_{,k} g^{kj} \delta g_{ij} \mathrm{d} v^{N} = 0 \quad (1 \le i, j, k \le N) \,.$$
⁽²⁸⁾

By the way, the **principle of virtual works** for self-equilibrium cables, membranes, deformable bodies are expressed as follows:

$$\delta w^{1} = \frac{1}{2} \int_{I} A \sigma^{i}{}_{k} g^{kj} \delta g_{ij} dl = 0 \quad (i, j, k = 1) , \qquad (29)$$

$$\delta w^2 = \frac{1}{2} \int_a^{a} t \sigma^{i} g^{kj} \delta g_{ij} \mathrm{da} = 0 \quad (1 \le i, j, k \le 2), \qquad (30)$$

$$\delta w^{3} = \frac{1}{2} \int_{v} \sigma^{i} g^{kj} \delta g_{ij} dv = 0 \quad (1 \le i, j, k \le 3),$$
(31)

where A,t respectively denote the sectional area of the cable and the thickness of the membrane. Then, when a new stress tensor T^{i}_{*} is defined for each dimension separately by

$$T^{i}{}_{k} \equiv A\sigma^{i}{}_{k}(N=1), \ T^{i}{}_{k} \equiv t\sigma^{i}{}_{k}(N=2), \ \text{and} \ T^{i}{}_{k} \equiv \sigma^{i}{}_{k}(N=3),$$
(32)

a common form of Eq. (29), (30), (31) is found and it is expressed as

$$\delta w^{N} = \frac{1}{2} \int_{M} T^{i}{}_{k} g^{kj} \delta g_{ij} \mathrm{d} v^{N} = 0 \ (1 \le i, j, k \le N) , \qquad (33)$$

which is the **principle of virtual work** for *N*-dimensional *Riemannian* manifold *M*. This can be naturally read that $T^{ii} = T^{i}_{k}g^{ki}$ is a general force which act within *M* and tend to produce small change of g_{ii} [7]. Additionally, Eq. (28) is a special case of Eq. (33) such that $T^{i}_{k} = \delta^{i}_{k}$.

3.3 Galerkin Method

Suppose a form of a curve, a surface, or a body is represented by *n* independent parameters such as $\mathbf{x} = [x_1 \ \cdots \ x_n]^T$. Let us define gradient vector of a function *f* with respect to \mathbf{x} by

$$\nabla f \equiv \left[\frac{\partial f}{\partial x_1} \quad \cdots \quad \frac{\partial f}{\partial x_n}\right]. \tag{34}$$

In such a case, at most *n* independent δg_{ij} can satisfy Eq. (33). Such a set of δg_{ij} is provided by weighted residual method family. The Galerkin method is one of them. Using the Galerkin method, δg_{ij} is altered into

$$\delta \widetilde{g}_{ij} \equiv \nabla g_{ij} \cdot \delta x . \tag{35}$$

where δx is the variation of x, or just an arbitrary column vector, namely $\delta x = [\delta x^1 \cdots \delta x^n]^T$.

Then, the discrete stationary condition is deduced as follows:

$$\delta \widetilde{w}^{N} = \frac{1}{2} \int_{M} \left(T^{i}{}_{k} g^{kj} \delta \widetilde{g}_{ij} \right) \mathrm{d} \mathbf{v}^{N} = 0 \iff \delta w^{N} = \left(\frac{1}{2} \int_{M} T^{i}{}_{k} g^{kj} \nabla g_{ij} \mathrm{d} \mathbf{v}^{N} \right) \cdot \delta \mathbf{x} = 0$$
(36)

$$\Leftrightarrow \boldsymbol{\omega} = \frac{1}{2} \int_{M} T^{i}{}_{k} g^{kj} \nabla g_{ij} \mathrm{d} \mathbf{v}^{N} = \boldsymbol{0} .$$
(37)

For general problems of statics, the following can be used:

$$\left(\frac{1}{2}\int_{M}T^{i}{}_{*}g^{kj}\nabla g_{ij}\mathrm{d}\mathbf{v}^{N}\right)\cdot\delta\mathbf{x} = \mathbf{p}\cdot\delta\mathbf{x} \Leftrightarrow \boldsymbol{\omega} = \frac{1}{2}\int_{M}T^{i}{}_{*}g^{kj}\nabla g_{ij}\mathrm{d}\mathbf{v}^{N} - \mathbf{p} = \mathbf{0}, \qquad (38)$$

where p represents the nodal forces.

3.4 N-dimensional Simplex Elements

When the form of a structure is represented by m independent elements such as line elements, triangle elements, and tetrahedron elements, Eq. (37) expands to

$$\boldsymbol{\omega} = \sum_{j} \frac{1}{2} \int_{j} T^{\alpha} g^{j\beta} \nabla g_{\alpha\beta} \mathrm{d} \mathbf{v}^{N} = \boldsymbol{0} , \qquad (39)$$

where the sum is taken for all the elements and integrals are calculated separately within each element. Let us define

$$\boldsymbol{\omega}_{j}^{N}(T^{\alpha}{}_{\gamma}) \equiv \frac{1}{2} \int_{j} T^{\alpha}{}_{\gamma} g^{\beta\beta} \nabla g_{\alpha\beta} \mathrm{d} \mathrm{v}^{N} , \qquad (40)$$

so that the discrete stationary condition can be simply represented by

$$\boldsymbol{\omega} = \sum_{j} \boldsymbol{\omega}_{j}^{N} = \boldsymbol{\theta} \text{ or } \boldsymbol{\omega} = \sum_{j} \boldsymbol{\omega}_{j}^{N} - \boldsymbol{p} = \boldsymbol{\theta} .$$
(41)

The simplest idea to calculate Eq. (40) is making use *N*-dimensional Simplex elements which are shown by Fig. 7. Within each dimensional element, the base vectors $g_1 \cdots g_N$ are constant and they are given by

$$g_i = p_i - p_{i+1} \quad (1 \le i \le N),$$
 (42)

where p_i represents the global Cartesian coordinate of *i*-th node. Then,

$$g_{ij} = \boldsymbol{g}_i \cdot \boldsymbol{g}_j \,. \tag{43}$$

In almost cases, T_{**}^{i} is given by a function of *Riemannian* metric, then T_{**}^{i} is also constant within each element. Therefore,

$$\boldsymbol{\omega}_{j}^{1}(T^{\alpha}{}_{,\gamma}) = \frac{1}{2}L_{j}T^{1}{}_{,1}g^{11}\nabla g_{11}\Big|_{j}, \quad \boldsymbol{\omega}_{j}^{2}(T^{\alpha}{}_{,\gamma}) = \frac{1}{2}S_{j}\left(\begin{bmatrix}T^{1}{}_{,1} & T^{1}{}_{,2}\\T^{2}{}_{,1} & T^{2}{}_{,2}\end{bmatrix}\begin{bmatrix}g^{11}{}&g^{12}\\g^{21}{}&g^{22}\end{bmatrix}\right) : \begin{bmatrix}\nabla g_{11}{}&\nabla g_{12}\\\nabla g_{21}{}&\nabla g_{22}\end{bmatrix}\Big|_{j},$$

$$\boldsymbol{\omega}_{j}^{3}(T^{\alpha}{}_{,\gamma}) = \frac{1}{2}V_{j}\left(\begin{bmatrix}T^{1}{}_{,1} & T^{1}{}_{,2} & T^{1}{}_{,3}\\T^{2}{}_{,1} & T^{2}{}_{,2} & T^{2}{}_{,3}\\T^{3}{}_{,1} & T^{3}{}_{,2} & T^{3}{}_{,3}\end{bmatrix}\begin{bmatrix}g^{11}{}&g^{12}{}&g^{13}\\g^{21}{}&g^{22}{}&g^{23}\\g^{31}{}&g^{32}{}&g^{33}\end{bmatrix}\right) : \begin{bmatrix}\nabla g_{11}{}&\nabla g_{12}{}&\nabla g_{13}\\\nabla g_{21}{}&\nabla g_{22}{}&\nabla g_{23}\\\nabla g_{31}{}&\nabla g_{32}{}&\nabla g_{33}\end{bmatrix}\Big|_{j},$$

$$(44)$$

where ':' symbol is defined by $a^{ij}: b_{\alpha\beta} \equiv a^{ij}b_{ij}$. In addition, L_j , S_j , V_j denote the length, the area, and the volume of each element, namely

To conclude this section, it is important to note that

$$\boldsymbol{\omega}_{j}^{1}(\boldsymbol{\delta}_{\gamma}^{\alpha}) = \nabla L_{j}, \ \boldsymbol{\omega}_{j}^{2}(\boldsymbol{\delta}_{\gamma}^{\alpha}) = \nabla S_{j}, \text{ and } \ \boldsymbol{\omega}_{j}^{3}(\boldsymbol{\delta}_{\gamma}^{\alpha}) = \nabla V_{j}.$$

$$(46)$$

Therefore, the examples illustrated in chapter 2 are the special and simple cases of Eq. (41) such that each $T^{i}{}_{*}$ is given as just a scalar multiple of $\delta^{i}{}_{*}$. It seems obvious that Such simple cases can be solved by direct minimization methods. Conversely, if such special cases can be solved by direct minimization methods, it may be also possible that general cases such that $T^{i}{}_{*}$ are not a scalar multiple of $\delta^{i}{}_{*}$ can be solved by direct minimization methods. Since ω is just a mixture of gradient vectors, Eq. (4) or Eq. (5) may still feasible when $\nabla\Pi$ is altered into ω .

5 ST. VENANT BODY

To solve the **principle of virtual works** by direct minimization approaches, an explicit expression of T_k^i that provides 1 to 1 mapping between g_{ij} and T_k^i must be prescribed, which is called constitutive law. Let us examine

$$T_{k}^{i} = Eg^{il}(g_{lk} - \overline{g}_{lk}), \qquad (47)$$

which gives a simplest one (St. Venant body when Poisson ratio is 0). Here, note that \overline{g}_{ik} is the *Riemannian* metric on the undeformed state which is measured in advance. In the analyses illustrated below, unlike the above discussed examples, the initial configurations of $\{x_1 \ \cdots \ x_n\}$ were given by just the undeformed state and were not given by random numbers.

Fig. 8 and 9 show two natural forms of handkerchief under gravity whose dimensions are 8.0-8.0. For both results,

$$\boldsymbol{\omega} = \sum_{j} \boldsymbol{\omega}_{j}^{2} (\boldsymbol{T}_{\cdot k}^{i}) - \boldsymbol{p} = \boldsymbol{0}$$
(49)

was solved by the **3-term method**, in which $\nabla \Pi$ is altered into ω . The prescribed E, p, α for both results were 50, $\begin{bmatrix} 0 & 0 & -0.1 & 0 & 0 & -0.1 \end{bmatrix}$, and 0.2.



Fig. 10 shows a large deformation analysis of a cantilever whose dimension is 2.0-2.0-12.0. Fig. 11 also shows a large deformation of the same body after bucking. For obtaining both the results,

$$\boldsymbol{\omega} = \sum_{j} \boldsymbol{\omega}_{j}^{3} \left(\boldsymbol{T}_{k}^{i} \right) - \boldsymbol{p} = \boldsymbol{0}$$
(50)

was solved by the **3-term method**. The prescribed $E_{and}\alpha$ were 50 and 0.2. In Fig. 10, p represents the values which were set to the components of p that represents the *z*-components of all the nodal forces. In Fig 11, p denotes the values which were set to *z*-components of only 9 nodes and they place on top of the body.

For the body shown by Fig. 11, the Euler's buckling load is $p_{cr} = 1.14$, then its division by 9 is 0.126 and note that it places just between Fig. 11 (a) and (b).



6. CONCLUSIONS

2-term method and **3-term method** were described for not only simple minimization problems but also for minimization problems under constraint conditions. The **principle of virtual work** for *N*-dimensional *Riemannian* manifolds was also formulated. Finally, some **principle of virtual works** that can be solved by the **3-term method** were shwon, which imply the potential ability of the **3-term method** for solving various types of static problems.

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Appendix A. Pseudo Codes

(a) 3-term method

 $\begin{aligned} \mathbf{x}_{1} &\coloneqq \text{random numbers;} \quad \mathbf{p}_{1} \coloneqq \mathbf{0}; \quad \mathbf{q}_{1} \coloneqq \mathbf{0}; \\ Current &\coloneqq 1; \quad Next \coloneqq 2; \\ \text{while}(true) \{ \\ \text{read } \alpha, \{w_{1}, \cdots, w_{m}\} \text{ from GUI;} \\ \nabla \Pi &\coloneqq \sum_{j} 2w_{j}L_{j}(\mathbf{x}_{Current}) \nabla L_{j}; \\ \mathbf{p}_{Current} &\coloneqq \frac{\nabla \Pi}{|\nabla \Pi|}^{T}; \\ \mathbf{q}_{Next} &\coloneqq 0.98 \mathbf{q}_{Current} - c \mathbf{p}_{Current}; \\ \mathbf{x}_{Next} &\coloneqq \mathbf{x}_{Current} + \alpha \mathbf{q}_{Next}; \\ Current = Current + 1; \quad Next = Next + 1; \\ \text{visualize}(\mathbf{x}_{Current}); \end{aligned}$

(b) 3-term method under constraint conditions

$$\begin{aligned} \mathbf{x}_{1} &:= \text{random numbers; } \mathbf{p}_{1} := \mathbf{0}; \quad \mathbf{q}_{1} := \mathbf{0}; \\ Current := 1; \quad Next := 2; \\ \text{while}(true) \{ \\ \text{read } \alpha, \{w_{1}, \cdots, w_{m}\}, \{\overline{L}_{1}, \cdots, \overline{L}_{r}\} \text{ from GUI;} \\ \mathbf{J}_{\lambda} := \begin{bmatrix} \nabla L_{m+1} \\ \vdots \\ \nabla L_{m+r} \end{bmatrix}; \quad \mathbf{r} := \begin{bmatrix} L_{m+1}(\mathbf{x}_{Current}) - \overline{L}_{m+1} \\ \vdots \\ L_{m+r}(\mathbf{x}_{Current}) - \overline{L}_{m+r} \end{bmatrix}; \\ \Delta \mathbf{x} &= -\mathbf{J}_{\lambda}^{+} \cdot \mathbf{r}; \\ \mathbf{x}_{Current} := \mathbf{x}_{Current} + 0.5\Delta \mathbf{x}; \\ \nabla \Pi_{w} := \sum_{j=1}^{m} 4w_{j}L_{j}^{-3}(\mathbf{x}_{Current})\nabla L_{j}; \quad \mathbf{J}_{\lambda} := \begin{bmatrix} \nabla L_{m+1} \\ \vdots \\ \nabla L_{m+r} \end{bmatrix}; \\ \lambda := -\nabla \Pi_{w} \cdot \mathbf{J}_{\lambda}^{+}; \\ \nabla \Pi := \nabla \Pi_{w} + \lambda \mathbf{J}_{\lambda}; \\ \mathbf{p}_{Current} := \frac{\nabla \Pi^{T}}{|\nabla \Pi|}; \\ \mathbf{q}_{Nexxt} := \mathbf{0.98q}_{Current} - cp_{Current}; \\ \mathbf{x}_{Next} := \mathbf{x}_{Current} + ca\mathbf{q}_{Next}; \\ Current = Current + 1; \quad Next = Next + 1; \\ \text{visualize}(\mathbf{x}_{Current}); \end{aligned}$$

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REFRAMING TEXTILES INTO ARCHITECTURAL SYSTEMS; CONSTRUCTION OF A MEMBRANE SHELL BY PATCHWORK

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Key words: Reframing, Shells, Membranes

Summary. In this paper different qualities of architectural textile techniques and tectonics are explored. By using a reframing strategy micro techniques and tectonics as used in fashion and textile design are evolved into an architectural scale. To reveal the quality of this reframing procedure a case scenario has been executed. By reframing patchwork techniques into a formactive shell geometry, a design is made. To reflect upon the case scenario a structural analysis by using Abaqus software has been performed.

1 AIMS AND MOTIVATIONS

With a broader knowledge on formfinding and membrane engineering, for the last decade form-active structures settled into the architectural vocabulary more and better. The structures proved to be, for instance, light weight, easy retractable, durable and recyclable. Most of these properties go well with nowadays conscious design culture. Because of a rich and notable form language of fluid lines and natural shapes, integrated climate control and media capabilities, the structures are often used in avant-garde architecture to positively brand certain events and companies. Given this development, these structures proved to be of big influence to concepts of forthcoming designers and architects. Because of a rather unconventional design strategy, structural membranes don't adapt very well to normal given architectural situation. Sometimes the introduction of these techniques late into the architectural process proves to be more than a challenge. To provide textile design with a more diverse vocabulary, the researchers started working on a broader view of textile techniques in architecture. By a reframing strategy micro tectonics as used in fashion and textile craft are scaled to architectural dimension, specific textile qualities were innovated into an architectural variant. In some cases using the same techniques but changing its materials suffices. Other techniques demanded a different design approach.

2 REFRAMING TEXTILE SYSTEMS

Since halfway nineteen hundreds scientific research as a base for professional practice has excelled rapidly. Research with a positivist background supported practicing designer and engineers with applied mathematics and science-based technology to contribute to a society based on science. In the positivist approach, applied to this research, research and design was practiced as a rational process. With this design methodology being more influenced by theory of technical systems than by designers and design problems, positivism didn't apply well to all fields of research. In reaction to the positivist research, constructionist methods in research and design techniques were explored. These constructionist methods were based on reflection-in-action; learning by doing. With the general positivist problem solving design methodologies on one side, constructionist theories defined research problems as unique. In general, constructionist methods were opted by arts and the social sciences. [4] In particular the work of ILEK has been a well known example of reflection-in-action. [10]

Professional practice is a process of problem solving. In scientific research problem setting is a significative point and well described in recognized research methodologies. Designerly research on the other hand, generally cannot describe the research process as accurate as its scientific counterpart. With ill-structured, more emotional described tasks the design process has developed differently from the scientific approach. In research by design, ill-defined problems are framed instead of set. By framing the problem a solution space is defined. By converging the framed research description through reframing strategies to an editable research description, the solution-space is narrowed down simultaneously. [2]

Design and technology have always been in close contact. Both share a history of doing and making. Both involve knowledge from both sciences and the humanities to be more than just applied sciences. [2] In the balance of skills and science, doing and making is mostly ahead of understanding. [1] Like the early ILEK research, contemporary geometrical research requires physical feedback to progress in unknown territory. For contemporary digitally driven research, material reflection is as necessary as before for the research to remain constructionist instead of positivist.

2.1 Reframing Strategy

In constructionist research, reframing is used to discover pleasant nooks, views and soft back areas to evoke a potential new coherence. [1] By reframing a problem from the existing frame into another it is partially released from former prejudice and conventions. Some new defined
frames may fit the contours of the former framework better than other. A bad fit for instance, can generate interesting reframing alternatives. A good fit on the other hand can generate interesting outlooks and research solutions.

With an interest in architectural structures and textile design, complementing frameworks were generated. For textile design a framework of textile techniques and textile tectonics is made. For structural design the framework of Structure Systems by Heino Engel [5] is used. By framing an instance from one framework into the other, textile-structure combinations originate. Bad fitted combination will be rated with a low success rate, well fitted combinations will be rated with a high success rate. A bad fit mostly originates from a misfit in textile geometry to the structural description. A successful fit is generated when the textile geometry complements the structural behavior. When a successful fit has been obtained, the potential of the reflective conversation continues. [1]

2.2 Textile Framework; Textile Techniques and Tectonics

To be able to frame and reframe textile-structure combinations efficiently, frame analysis is performed. By describing the content of independent frames, quality of combinations can be rated in advanced.

Form-Active	Surface-Active	Section-Active	Vector-Active
Systems	Systems	Systems	Systems
▼	▼	▼	▼
Ruffling			
Techniques			
\checkmark			
Form-Active	Surface-Active	Section-Active	Vector-Active
Ruffling	Ruffling	Ruffling	Ruffling
(+)	(++)	(+)	(+/-)

Table 1: A reframing schedule of Structural Systems and Ruffling Techniques

The framework of textiles is divided into three groups of which two are used in the reframing procedure. The first group consists of all textile tectonics. Textile tectonics or textile fabrication are conceived as ways with which textiles are constructed. Crafts like braiding, weaving, knotting and knitting belong to this division.

The second group consists of textile techniques. Textile techniques or textile processing are conceived as ways with which textile tectonics are processed. Crafts like folding, ruffling, pleating and patterning belong to this division.

The third group consists of textile products. Textile products or textile application consists of elements constructed with textile tectonics, textile techniques or a combination of both. This division will be regarded to in the form of case scenarios.

2.3 Structural Framework; Structure Systems

For the structural framework, the classifications as described in Structure Systems by Heino Engel is applied. From the six systems described in Structure Systems, four apply well as a

framework for the reframing strategy. The four groups contain Form-active, Vector-active, Section-active and Surface-active systems. In the context of this paper only Form-active systems are used. With a big understanding of form-active behavior in the International Conference on Textile Composites and Inflatable Structures, properties of this structure group will not be discussed in this paper.

3 MEMBRANE SHELL GEOMETRY

In developing efficient structural concepts for architectural use, both engineer and designer have to be willing to work in an interdisciplinary or Mode 2 [8] approach. In architectural design in general, on the one hand architects give importance to form. On the other hand, the engineer will give importance to efficient use of structural elements and techniques. In the application of efficient structural concept, the attention of both parties has to be in equilibrium to result in a satisfying structure or architecture. [7]

In the use of efficient structural concepts like form-active systems, the design process will behave more like a form-finding than a form-giving procedure. Unlike formalist design results like for instance the Guggenheim Museum in Bilbao, in interdisciplinary design, acting players have to cooperate in a heterarchical and transient process without compromising quality. Instead of transposing the structural elaboration to the end of the design process, all parts have to be of same importance during the process with each acting player employing a different type of quality control. [7, 8]

The case scenario as described in this paper is the result of a cooperation of three disciplines, being architecture, civil-engineering and architectural engineering. During the process, each design step was reflected to following steps to come. With the expertise of a vast array of disciplines, mistakes in steps to come could be avoided in advance. In addition to the reframing process performed in this case scenario, every framework was monitored by a matching professional.

3.1 Reframing Strategy; Form-active Systems / Patchwork Techniques

In de case scenario as described in this paper, a reframing procedure has been performed combining form-active systems and patchwork techniques. Patchwork is conceived as a technique or craft where mostly square patches are composed to a greater cloth. Reframing anticlastic form-active systems into a patchwork framework results in a double curve geometry by smaller square elements. To broaden form-active possibilities, the use of stiff panel materialization was chosen.

3.2 Qualities Membrane-Shell Combination

With the use of bending resistant panels in a form-active anticlastic structure, shell qualities are introduced. Compared to a form-active pre-tensioned membrane, bending resistant membrane shells yield great potential. In construction, border conditions are less demanding. Since pre-tensioning is less present, boarder details like construction plates and cable details are superfluous. With less pre-tension forces present, supporting structures can be constructed more slender. In materialization, transparent structures in plastics and glass are made possible.

Without complying to efficient material use, flat sheet material can be arranged into formactive systems; surface-active structures evolved into form-active behavior.

4 CASE SCENARIO; BUILDING A MEMBRANE SHELL

To take advantage of the qualities of reflection-in-action, a case scenario is erected to reflect on the theories as described in chapter 3 and 4.

4.1 Geometry

The geometry as described in chapter 3 is materialized in 4 mm wooden multiplex sheet. With the inability of multiplex to bend easily, the double curved had to be converted into a geometry composed by flat wooden plates.

In membrane engineering double curved surfaces are described by a mesh for the minimal surface to be calculated. The mesh used in, for instance, the Force Density Method consists of a non-planar quad subdivision. For a planar subdivision triangles are generated within the mesh. [9, 6]

Using the triangle to describe a double curved surface has great qualities digitally. A vast array of surface distribution is possible and the procedure itself is relatively easy. In fabrication, triangles are hard to deal with. The vertices have a relatively high valence; connection points consist of up to 6 panels. Supporting beam structures consist of a tri-axial geometry. Planar quads on the other hand, have a valence up to four panels with a supporting structure of a bi-axial geometry. [6]



Figure 1: (a); Planar Quad generation by a cone of revolution, (b); Planar Quad generation by a general cylinder surface (Images [6])

Designing with planar quads leaves less flexibility in surface distribution. In [6], two methods are distinguished; Planar Quad generation by a cone of revolution and Planar Quad generation by a general cylinder. The first surface, presented in figure 1a, consists of sections of a triangle, coupled with the connecting surface edge. The latter surface, presented in figure 1b, consist of a strip, built of parallel lines.



Figure 2: Left; Initial surface design, Right; Design after quad generation

Based on the membrane / shell qualities, described in chapter 3, a design for a canopy was made. Processing this surface with a script running in Rhino3D, a planar quad distribution was generated according to the concept of generation by a cone of revelation. To generate a structure with a balance in an equal surface distribution and a workable tile size, the surface had to be adjusted to meet these needs. To avoid a decrease in panel size in high curvature surface areas, the border conditions of the initial surface design was remodeled to generate a sufficient working surface.

4.2 Materialization

The surface is built from 4 mm wooden multiplex. Wood was chosen because of its accessibility in processing. For the internal connection of the panels several techniques were discussed. With the ambition to create a water tight geometry, two techniques were evaluated for feasibility.

The first technique is a connection by overlap. Layering techniques as used with copper shingles proved to be an interesting surface tectonic. Data like surface layering and angle behavior in this geometry were hard to describe digitally. More research is needed to generate a fluent tectonic by these conditions.

The second technique is a taped connection. By connecting the panels with a tape or adhesive strip, the surface geometry is generated. With the surface geometry remaining intact, minor changes in translation to materialization are notable.

4.3 Composite Tape Connections

A tape connection should be able to transfer the internal membrane forces from one panel to the next. Together with the requirement of water tight connection a composite tape connection can be considered. Research has been done on developing this type of connection using Polypropylene (PP), also known as polypropene, is a thermoplastic polymer [11]. Polypropylene has good properties in respect to durability, strength, stiffness and fatigue. It can be woven to form sheets, which in strips connect the panels. The strips are glued to the panels and form a water tight connection.



Figure 3: Woven PP sheet, internal membrane forces

4.3 Building Strategy



Figure 4: Panels and Work in Progress

The canopy design consists of an arched wooden boarder structure with panels in between. Like with patterns in membrane engineering, the panels were nested digitally and milled by a CNC machine. Connecting the panels in a fixed sequence resulted in the given geometry.



Figure 5: Membrane Shell Geometry

After finishing the panel assembly, the border geometry was met. The given outline of the supporting structure lined up with the membrane geometry. The form-active behavior was noticeable after closing the shell. Deformation in the shell geometry by loading it locally was the result of bending in the border structure instead of a misfit in geometry.



Figure 6: Connections and Surface Tectonics

To tension the structure internally, all panels were interconnected with four tie-rips vertically and four horizontally. The taped connection was not applied because more research in the structural use of taped or adhesive connections is needed.

5 STRUCTURAL ANALYSIS

Describing a textile membrane with a shell structure consisting of flat panels, results in a change in structural behavior of the geometry. To reflect the difference in structural behavior of the form-active geometry in different "discretization" stages, a brief Finite Element study is performed. The three different discretization stages are:

- Continuous shell form-active shape
- Continuous shell discretized with planar quads
- Discountinuous shell discretized with planar quads interconnected in the corners with hinges.

The study focuses on the structural response of the different structural systems.

5.1 Description of models

-Y

A numerical simulation of the above described models is performed with the finite element software Abaqus (version V6.10.EF). The models consist of quadratic quadrilateral elements with a edge size of approx. 50 mm.



Figure 7: Model plot with mesh of the three models

To stay close to the geometry as built, the material used in the model is the equivalent of wood with a modulus of elasticity of 9000 N/mm² and a shell thickness of 4 mm. Instead of simulating the exact behavior of the individual wooden panels, an idea of the differences in global behavior of the three structural concepts was obtained. Herewith no material non-linearity's have been included. As with the built scenario, the three translational degrees of freedom along the edges of the shell models are set to zero.

For the structural response study three different load cases have been reviewed:

- Symmetrical: gravitational load (9.81 m/s²)
- Symmetrical: vertical concentrated load in the middle of the shell structure (100 N).
- Asymmetrical: horizontal pressure load on one side of the shell (100 N/m^2) .

5.3 Discussion of results

As to be expected, both continuous shells react quite differently compared to the discontinuous shell. The capability of the continuous shells to transfer the applied loads more evenly to the boundary conditions makes both shells more rigid.



Figure 8: Overview of deformations of the three models ; gravitational loads (in m)

In figure 7 the differences of the three structural concepts are shown for the gravitation load. The deformations of the discontinuous shell is 4 times higher than the continuous shell with planar quads.

The continuous shell has a ideal shape. This geometry combined with the bending stiffness of the material make it a very stiff structural geometry.

The deviation of this ideal shape by discretizing it with planar quads, still results in a stiff shape. The folding lines in between the quads are areas with more stiffness and attract more loads. This clearly shows on the stress plots of the structure under gravitational load as presented in figure 8.



Figure 9: Stress plot of continuous shell with planar quads with gravitational load (in N/m²)

The discontinuous shell acts like a "net"; coupled plates, kept in shape by its boundary conditions. Globally the surface is has no bending stiffness and acts like a textile membrane. Due to the local bending stiffness of the panels and the hinged connections, the panels start rotating out of plane when loaded asymmetrically. This somewhat instable behavior is shown figure 10 (scaled deformations).



Figure 10: Deformation of discontinuous shell loaded with a concentrated load in the middle (in m)

In both the shells materialized with planar quads, the quads will experience local bending in between the folding lines, when loaded with a surface pressure (figure 10; plots with different deformation scales).



Figure 11: Local bending of planar surfaces (deformation in m)

From this brief study on the structural behavior of different discretizations steps, it can be concluded that further study of form-active shells consisting of planar quads should involve the development of edge connections between the quads to be able to obtain a shell behavior. Given this connection the geometry should behave more like the continuous shell with planar quads.

12 CONCLUSIONS & RECOMMENDATIONS

Reframing textile techniques into form-active systems yields great qualities for both structural use as material application. The construction of a membrane bending-resistant anticlastic shell proved to be an interesting addition to membrane structures. In materialization, broader possibilities in surface tectonic are provided. In structural use, efficient materialization can be obtained with bending resistant flat sheets. More research is needed in the sheet connections. To obtain a durable watertight surface, research has to be done in tape or sealant. To obtain a hybrid structural integrity, research has to be done in for instance composite tapes or textile adhesives. As a start this mesh layout was chosen. In further research the mapping of a quad mesh over the desired shape by conforming techniques could be investigated.

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STEP BY STEP COST ESTIMATION TOOL FOR FORM-ACTIVE STRUCTURES¹

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Key words: Membrane Architecture, Innovative Design, Formfinder Software Tool, Cost Estimation, Value Finder.

Summary: It is a fact that the available budget is one of the main driving factors for the architectural design. Therefore it is beneficial for designers to know, how to control the costs with the help of their design. Traditionally cost estimations are done by specialists with the knowledge and the experience necessary to achieve an accurate result. But according to decision makers even rough accuracy is mainly sufficient for deciding whether a project can be executed.

So for the designers' intentions and ideas to be realized they already need to have the expected costs and the economical influence of various design parameters at the very beginning of the project.

A step by step cost estimation tool could provide knowledge for a more economic design process and should encourage designers to follow their inspiration. This motivated us for developing an interactive tool to be integrated into the "Formfinder²" – a design software for form-active structures. The resulting cost estimations are based on expert knowledge and a project database (pre- and post-cost calculations of realized projects) developed and provided by the architect Horst Dürr. He had been the technical director of Stromeyer Konstanz and had been CEO of IPF from 1973 to 1980. Since 1980 Horst Dürr has been partner of the IF Group.

This paper presents a description of our software tool and gives an overview about which possiblities for cost optimization the designer has.

^{1 [}Engl97] p. 57, Heino Engel determinates: 1 form-active, 2 vector-active, 3 section-active, 4 surface-active, 5 height-active and 6 hybrid

² [RRoith9] Formfinder - concept for a software-tool to assist architects in the preliminary design of form-active structures

1 INTRODUCTION

Due to the fact cost estimation is often neglected in the design stage, time and money is spent although the realization of a building project is not finally decided at this stage. In many cases economic aspects are reason for cancelation of projects. Architectural design is often influenced by the available budget. Therefore the knowledge about the parameters for cost optimization is beneficial for designers. Rigid structures may excuse unfavorable design through local strengthening measures. The sensibility of non-rigid, flexible, form-active structures in contrary is one of the main cost driving factors. Small adjustments and modifications on the design may significantly influence the total costs of the whole project.

The more accuracy is required from cost estimations the higher its level of complexity gets. The number of parameters to be considered increases potentially. Every single factor's potential cost impact is to be evaluated.

At this point two types of parameters need to be distinguished; parameters able to be determined and such which only can be valued either by chance or by experience.



Figure 1: Examples for determinable and indeterminable parameters for cost optimization

Examples for "Determinable Factors"

- Determinable architectural design concept and intention (facts that can be described by designers e.g. by sketch, by reference projects or verbally)
- Geometry described with physical models or CAD tools and readability of the structure (e.g. four-point sail, ridge-valley sail, high-point sail, etc.)
- Construction components ("element catalogue", materials, available budget)
- Limitations (building law; external loads like wind load, snow load, and dynamic excitability; general climate conditions like weather, temperature, and local climate characteristics)

Examples for "Indeterminable Factors"

- Indeterminable architectural perception and requirements that are not part of the available design concept
 ("Design philosophy"; consideration only possible with the help of pictures of
 - ("Design philosophy"; consideration only possible with the help of pictures of reference projects)
- Special design intents (new materials or techniques, first of a kind-project)
- Feasibility of the statical concept
- Building site and infrastructure (construction site, accessibility of site, infrastructure, soil situation)

As a consequence designers are only able to vary the material and production costs of a project. Not determinable parameters, however, are not in the sphere of influence of the designer but still can be controlled up to a certain extent at the very beginning, right after the project idea. So the design's impact on delivery costs, installation costs, and overhead costs on the other hand is minor but not to be neglected.

Within the group of determinable factors substituting parameters can be defined. This subgroup contains alternative (design) solutions which can be chosen for geometric or even economic reasons without having an impact on the overall design intention.

Especially on form active structures an elaborated design is the most important parameter to optimize the production costs.

Anticlastic surfaces with a homogeneous and well-proportionate curvature k $(k = (1/r_1) * (1/r_2))$ give higher mechanical stiffness to the membrane and further to the whole structure. As a consequence the weave needs less pretension and the deformations due to external loading are less. This results in smaller internal stresses and the material fatigues slower. Furthermore due to values of lower stress and smaller reaction forces in the fixation points, an optimization of the adjacent structural elements and wider spans of the membrane are possible.

Our consideration is focusing on the membrane and its primary structure only. As a necessity the foundations are excluded from our consideration as the final soil situation is not known before the detail design and possibly even not before the realization of the construction. Especially on extensive projects different foundation concepts may be required on individual anchor points due to differences in the local soil configuration.

2 THE TOOL

Goal of this tool is to visualise cost factors and to support designers to keep their own design by providing a set of positive arguments to convince the decision makers of the design intention and the design concept.

In general the process of designing consists of two steps: qualification (more ideas) and quantification (reduction of ideas) [Höller, 1999]. Following this approach it is necessary to decide over the final design up to a certain depth before first cost calculations can be performed. Precise cost estimation is a highly time-consuming and complex task. For conventional rigid constructions a detailed structural analysis under consideration of various aspects such as aesthetics, shape, materials, legal restrictions and requirements, etc. is required. For non-rigid structures this complexity maximizes, since additional factors induced by the flexible nature of the structure occur; factors like more restrictions at selecting materials, more complex aerodynamic characteristics, or influence of dominating weather conditions.

Therefore this task calls for knowledge of a domain specialist capable of handling all of these cost parameters. This however decouples the designer from the process chain. For this reason the lack of this valuable information often leads to initial designs being distorted significantly. In the past many projects failed due to this simple fact known in many engineering domains. The later design mistakes are detected in the process chain, the more expensive it may get to correct them eventually.

Our cost estimation tool is addressed in particular to designers. Surveys have shown most designers consider estimation deviations of 30-40 % for project costs to be acceptable. This fact allows a significant simplification of the calculation process. Reduced complexity in return increases understanding for and usability of the provided cost estimation tool to the designer.

Our tool is an intelligent, self-learning tool based on databases and algorithms found on basis of theoretical design studies and of comparisons of pre- and post-cost calculations of real projects considering factors like shape, size, curvature, and material among others.

Nevertheless the user has to be aware of the limitations of this cost estimation tool. According to the parameters mentioned above the value given just reflects the production and material costs of a membrane structure. Factors like the foundations, accessibility of the building site, legal requirements, installation and geographical location only can be measured up to certain extent and therefore are excluded from our calculation model. Anyway the corresponding values of these parameters can be added manually by the user.

The given values, respectively the costs, are transferred to $[\ell / m^2 / year]$.

In this context the time factor is crucial for evaluating the costs as for temporary/single-use roofs the time of usage differs from permanent roofs significantly as the latter remain installed over the whole material life expectancy. Therefore in terms of accounting the amortization of the whole structure runs over a longer period of time.

In a first step our model is applicable to four base shapes:

- four-point sail
- high-point sail
- beam supported sail
- ridge-valley sail

The default settings of this cost estimation tool only allow a low accuracy of the actual value. These settings reflect the most common design parameters of the reference projects used for finding the calculation algorithms. It is up to the user to narrow down the degree of detailing in the project specifications. Additional information on the design can be added. The more data is provided the more accurate the estimation gets.

In our (unreleased) software package a control panel is located on the right hand side of the screen. It contains various sliders which can be used to adjust the importance of individual factors.

On basis of a product database design parameters and material properties can be preset; involving decisions on the membrane material, whether cables or belts are used at the edges or corner details are made of standard products or individual design solutions.

Based on the geographical location and the Eurocodes the wind and snow loads for certain countries can be added automatically. In a first step loading can be generated for simple standard shapes only.

Besides the steadily growing database of integrated codes and standards, products and materials individual calculations outside these presettings are allowed for.



3 CONCLUSION

We have presented a cost estimation assistant that targets designers of form active structures. It can be used either as a pure calculation tool approximating a step by step behaviour or if desired expose educational facets to submit better understanding of the involved cost factors and dependencies to the designer. Even while kept simple, we believe the estimation mechanism to be powerful enough to present estimations within a range of accuracy of 20-30 %. Due to its ability to extract knowledge from existing/finished projects and to feed this knowledge back to the project database we further believe estimations to become more accurate by time, when more projects are calculated and so more data of cost estimations and the corresponding real project costs is available.

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IMPROVEMENT OF THE SYSTEM OF MODULAR INFLATED SHELLS BY MEANS OF PHYSICAL MODELLING

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Key words: Modular inflated shells, Physical modelling.

Summary. The paper presents results of experimental research of modular inflated shells. Experiments carried out in the first phase showed that proposed connection between cushion modules has led to the creation of articulated joints and resulted in local instability. Thus, a new type of connection has been developed and new series of experiments have been carried out.

1 INTRODUCTION

In the previous works^{1,2,4}, author introduced idea to form a large and complex pneumatic structure as a set of repeatable, modular inflated units with tensioned cables and with additional bars or without them. The core idea was to eliminate some disadvantages of traditional pneumatic structures.

Modular inflated shells present a class of spatially curved surface girders – single and double curved shells. They are composed of relatively small inflated cushions combined with cables and cross-braces. Their rigidity and loading ability are strongly related to the shape. Spatial curvature itself, is not a permanent, generic feature of the structure, but is achieved and maintained by means of post-tensioning, causing very large initial deformation. Post-tensioning is a way of integrating small elements as well as shaping the structure. Coupling of these operations raises the efficiency of solution. Thus, the final shape and properties of the structure are function of initial configuration and the course of post-tensioning process.

Presented structures can be applied for any type of building where large clear span is a challenge for designers as well as short time and low cost of erection. Self-erection is a convenient method of shaping both permanent and temporary, rapidly assembled covers for industrial buildings, warehouses, sport facilities, exhibition halls, field hospitals, temporary covers of building sites or for military applications such as deployable bridges.

Typical layout of modular inflated shell composed of cushions, cables and cross-braces is presented on Fig. 1.



Figure 1: Typical layout of modular inflated shell

2 INITIAL EXPERIMENTAL RESEARCH

The presented system was tested using physical models³. The main purpose of carried out experiment was to verify realizability of modular inflated shells. It was assumed that the experiment would be mostly of qualitative not quantitative character. It was also assumed that the subject of the survey would be relation of elevation and shape to the shortening of distance between supporting points.

2.1 Description of the models

The modular inflated elements were prepared with use of inflated cushions widely used in popular sport equipment. Each cushion forms a single modular element. These cushions are made of 0.30 mm PVC foil. Nominal dimensions are: 89.5×89.5×21.5 cm. Cushions are equipped with internal diaphragms, reducing deformation after inflation. Diaphragms do not obstruct airflow inside cushion. Air valve is placed on side face of the cushion. The cushions have been adapted for the purpose of experiment by means of addition of connectors, i.e. fabric belts, touch fastener strips and connectors for tensioned members and cross-braces. Figure 2 presents details of inflated cushion used in experiment.



Figure 2: Structure of inflated cushion used in experiment

2.2 Examined configurations of shells

It was assumed that the model would represent a section of cylindrical shell with and without cross-braces. Initial length of the shell should be at least ten times longer that the length of single modular element.

In the initial arrangement, the model presented a string of eleven modular elements. Two models were prepared: M-3 (shell without cross-braces) and M-4 (shell with cross-braces).

Model M-3 was composed of eleven cushions and tensioned cable, without cross-braces while model M-4 was composed of eleven cushions with cross-braces and tensioned cable. Figure 3 presents geometry of the model M-3 before erection and its expected geometry after erection and Fig. 4 – geometry of model M-4 before and after erection.



Figure 3: Geometry of model M-3 before and after erection (expected)



Figure 4: Geometry of model M-4 before and after erection (expected)

2.3 Course of experiment

Experiment was performed in three stages. At the first stage, models M-3 and M-4 were assembled and fitted with tensioned cable and cross-braces. A 6 mm polyethylene fiber rope was used as tensioned member and 20/1 mm PVC pipes were used as cross-braces.

Models were erected up to approx. 50% reduction of initial span. Additionally, model M-4 after erection was loaded at the central point by force Q=0.03 kN.

Survey was executed by means of infrared rangefinder (Zeiss Elta S20). A measuring point – the target for measuring equipment, made of holographic foil, was placed on cushion's side face, Fig. 5. Survey covered initial and final geometry of models at every stage of experiment. An angle and distance to the measuring base was recorded.



Figure 5: Measuring point on the cushion's side face

2.4 Results of surveying

Recorded results of surveying are presented below. Figure 6 presents comparison of shapes of models M-3 and M-4 after erection and deflected shape of loaded model M-4.



Figure 6: Curves of deformation of models M-3 and M-4

2.5 Conclusions from the initial experiments

The study confirmed the validity and usefulness of the overall design of the system. However, several errors were noted in the experiment. Most of them resulted from rough technical means used for preparation of models, but also a significant problem with the details of the adopted technical solutions has been noted. According to assumptions, the top layer of the modular inflated shell must be able to sustain bending moment. This requires that the upper cord is continuous, and the side surfaces of the cushions are fully touching. An outline of transmission of forces between cushions is shown on Fig. 7.



Figure 7: Schematic diagram of transmission of forces between cushions

Tested connections assured only continuity of the upper layer of fabric, while side surfaces – deformed by the air pressure – constituted hinge connections. This, in addition to the fact that the connection between the cable at the lower cord and the cross braces is sliding connection caused instability of the tested model. It's clearly visible on Fig. 8.



Figure 8: "Hinge" contact of side surfaces

3 INITIAL EXPERIMENTAL RESEARCH

To resolve problem of hinge connection between cushions, new models have been developed and tested. New solution forced full contact of the side surfaces of the cushions by means of using rigid panels. Also, a continuity of the upper cord was increased by use of screw connectors in the rigid panels instead of velcro-tape connectors used previously.

3.1 Description of the improved models

Unlike before, the cushions were made specifically for the purpose of this experiment. Dimensions of each cushion are $100 \times 100 \times 40$ cm. They are made of 0.30 mm PVC foil, bonded by means of high frequency welding. One internal diaphragm is placed in the center of the cushion. The most important modification is the use of additional flat pockets on the cushion side surfaces. A rigid panel is inserted into these pockets, which ensures the flatness of the surface. Cushions are equipped with an air valve placed on its bottom face.

Rigid panels (10 mm plywood) inserted into the pockets are screwed together by standard screws M12. Figure 9 presents modified connection of two cushions.



Figure 9: Modified connection of two cushions

3.2 Course of experiments

Experiment with modified cushions was related to the previous experiment. Two models were tested, named respectively M-3' and M-4'. Models were erected up to approx. 50% reduction of initial span. No additional loading was applied. Other fittings, cables, cross-braces etc. were the same as previously.

3.3 Results of surveying

Behavior of the inflated shells composed of modified cushions has significantly changed comparing to the shells composed of unmodified cushions. No hinge joints were observed and no local instabilities occurred. Curvature of both tested models was smooth and almost symmetrical. Recorded results of surveying are presented on Fig. 10.

4 CONCLUSIONS

Modification of the connection between cushion has significantly affected the behavior of the shell. Although the test models were still very simple and require further improvement of



production technology, results of the experiments can be used for further analyze the modular inflated structures.

Figure 10: Curves of deformation of models M-3', and M-4'

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HOMOGENIZATION AND MODELING OF FIBER STRUCTURED MATERIALS

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Key words: Technical Textile, Fiber Structured Materials, Homogenization

Abstract. For the mechanical modeling and simulation of the heterogeneous composition of a fiber structured material, the material properties at the micro level and the contact between the fibers have to be taken into account. The material behavior is strongly influenced by the material properties of the fibers, but also by their geometrical arrangement. In consideration of the different length scales the problem involves, it is necessary to introduce a multi scale approach based on the concept of a representative volume element (RVE). For planar structures like technical textiles the macromodel is discretized by shell elements. In contrast the microscopic RVE is modeled with three dimensional elements to account for the contact between the fibers. The macro-micro scale transition requires a method to impose the deformation at a macroscopic point onto the RVE by suited boundary conditions. The reversing scale transition, based on the Hill-Mandel condition, requires the equality of the macroscopic average of the variation of work on the RVE and the local variation of the work on the macroscale. For the micromacro transition the averaged forces and the resulting moments have to be extracted by a homogenization scheme. From these results an effective constitutive law can be derived.

1 INTRODUCTION

The sector of technical textiles is expanding because of a variety of applications of recently developed fiber shapes, materials and structures. Innovative potential and economic growth make technical textiles to an important research area. Examining textiles it is obvious that on the macro scale textiles show inhomogeneous material properties, which are different from the underlying fiber material. These nonlinearities rely on interactions in between the fibers on the micro level and the friction in the contact areas. The determination of phenomenological constitutive laws for this material group by classical material testing procedures is very time and cost intensive. Therefore numerical methods to determine the material characteristics of textiles are developed. Apparently discretization of the whole macro structures modeled in micro scale element dimensions leads to systems with too many degrees of freedom. So for the macro- and microscopic consideration multi scale methods, so called FE²-methods are introduced. Hence each macroscopic material point is assigned a microscopic, heterogeneous Representative Volume Element (RVE). For a simultaneous calculation a relation between microscopic and macroscopic scale has to be derived. From the deformations on macro scale boundary conditions for the micro calculation were developed. For the reversing scale transition a homogenization scheme is introduced.

2 TOWARDS FIBER STRUCTURED MATERIALS

Technical textiles are considered to be all textile products with an application in the technical sector. Thereby they can be used in architecture, agriculture or engineering. Textiles are flexible materials, which consist of particular fibers where a fiber has a high length to diameter ratio. In this paper, without loss of generality, a fiber is assumed to consist of homogeneous material and to be characterized by a circular cross section.



Figure 1: Towards a FE²-scheme

The mechanical properties are closely linked to the manufactured materials. Generally one can differentiate between natural and chemical fibers. Polymers as well as glass, ceramic, carbon and metal fibers belong to the last-mentioned ones. Because of growing ecological awareness there is a trend towards natural fibers like hemp and jute. They find application in architecture and as geological and agricultural textiles. For textile behavior material properties and volume fraction are as important as the assembly of the fibers [1]. On the one hand there are periodic assemblies like woven or knitted structures, on the other hand there are random ones like felt.

In the following, technical textiles are examined in a multi scale homogenization scheme, which is different to the classical first order approach. A requirement of the first order homogenization is that the macroscopic and the microscopic length scales differ in dimension [2]. This is achieved for textiles for the in plane direction, but in thickness direction the lengths are considered the like. For that reason no classical homogenization can be accomplished in this direction. Textiles are rather assumed to be shells on the macroscopic level. Intrinsically, shell problems are second order homogenization problems because beside stretching and shearing, bending and twisting of the shell are considered.

3 SHELL-KINEMATICS

For the description of a shell continuum \mathcal{B}_0 in the material configuration and \mathcal{B}_t in the spatial configuration in a three dimensional space, two coordinate systems are introduced [5]. One cartesian system x_i with the orthonormal vector \mathbf{E}_i and one curvilinear, convective system θ_i that is connected to the middle surface \mathcal{M}_0 and \mathcal{M}_t of the shell. For the notations it is introduced that Latin indices range from 1 to 3 and Greek indices range from 1 to 2. A description of a finite deformation shell is given by the equations

$$\begin{aligned} \boldsymbol{X}(\theta^{i}) &= \bar{\boldsymbol{X}}(\theta^{\alpha}) + \theta^{3} \boldsymbol{D}(\theta^{\alpha}) \quad \text{and} \\ \boldsymbol{\varphi}(\theta^{i}) &= \bar{\boldsymbol{\varphi}}(\theta^{\alpha}) + \theta^{3} \lambda(\theta^{\alpha}) \boldsymbol{d}(\theta^{\alpha}) , \end{aligned}$$
(1)

where X is the position vector of a material point in the undeformed shell and φ is the position vector of a material point in the deformed shell. Streching in thickness direction is neglected $\lambda(\theta^{\alpha}) = 1$.

The vectors \bar{X} and $\bar{\varphi}$ provide a parametric representation of the middle surface of the shell in the reference and the current state. The parameter $\theta^3 \in [\frac{-h_0}{2}, \frac{h_0}{2}]$ determines the position of a point normal to the middle surface in the undeformed state.

All kinematic values can be calculated, if the shell geometry in the material and the spatial configuration are known. For this the covariant basis vectors on the middle surface of the shell can be derived from the partial derivative of the material vector \bar{X} and the spatial vector $\bar{\varphi}$ with respect to the curvilinear coordinates

$$\boldsymbol{A}_{\alpha} = \frac{\partial \bar{\boldsymbol{X}}}{\partial \theta^{\alpha}} = \bar{\boldsymbol{X}}_{,\alpha} \quad , \qquad \boldsymbol{A}_{3} = \boldsymbol{D} \; , \\
\boldsymbol{a}_{\alpha} = \frac{\partial \bar{\boldsymbol{\varphi}}}{\partial \theta^{\alpha}} = \bar{\boldsymbol{\varphi}}_{,\alpha} \quad \text{and} \quad \boldsymbol{a}_{3} = \boldsymbol{d} \; .$$
(2)

The unit director D, which is normal to the shell in the material configuration is given by S. Fillep and P. Steinmann



Figure 2: Towards a FE²-scheme

$$\boldsymbol{D} = \frac{\boldsymbol{A}_1 \times \boldsymbol{A}_2}{|\boldsymbol{A}_1 \times \boldsymbol{A}_2|} \,. \tag{3}$$

For the displacement $\boldsymbol{u}(\theta^i)$ of a point from the reference to the current state it reads

$$\boldsymbol{u}(\theta^{i}) = \bar{\boldsymbol{\varphi}}(\theta^{\alpha}) - \bar{\boldsymbol{X}}(\theta^{\alpha}) + \theta^{3}(\boldsymbol{a}_{3} - \boldsymbol{A}_{3}).$$
(4)

For the macro to micro scale transition the shell formulation has to be extended to a three dimensional formulation [6]. From equation (1) covariant basis vectors for the description of the shell body can be derived to

$$\boldsymbol{G}_{i} = \frac{\partial \boldsymbol{X}}{\partial \theta^{i}} \quad \text{and} \quad \boldsymbol{g}_{i} = \frac{\partial \boldsymbol{x}}{\partial \theta^{i}}, \quad (5)$$

with the connection to the covariant shell basis vectors

$$G_{\alpha} = \frac{\partial \mathbf{X}}{\partial \theta^{\alpha}} = \frac{\partial \bar{\mathbf{X}}}{\partial \theta^{\alpha}} + \theta^{3} \frac{\partial \mathbf{D}}{\partial \theta^{\alpha}} = \mathbf{A}_{\alpha} + \theta^{3} \mathbf{A}_{3,\alpha} , \\
 G_{3} = \frac{\partial \mathbf{X}}{\partial \theta^{3}} = \mathbf{A}_{3} , \\
 g_{\alpha} = \frac{\partial \varphi}{\partial \theta^{\alpha}} = \frac{\partial \bar{\varphi}}{\partial \theta^{\alpha}} + \theta^{3} \frac{\partial \mathbf{a}_{3}}{\partial \theta^{\alpha}} = \mathbf{a}_{\alpha} + \theta^{3} \mathbf{a}_{3,\alpha} , \\
 g_{3} = \frac{\partial \varphi}{\partial \theta^{3}} = \mathbf{a}_{3} .$$
(6)

The contravariant basis vectors result from the relation

$$\boldsymbol{G}^{i} \cdot \boldsymbol{G}_{j} = \delta^{i}_{j} \quad \text{and} \quad \boldsymbol{g}^{i} \cdot \boldsymbol{g}_{j} = \delta^{i}_{j} ,$$
 (7)

with the Kronecker delta δ^i_j . For the deformation map the deformation gradient F is introduced

$$\boldsymbol{F} = \frac{\partial \boldsymbol{\varphi}}{\partial \boldsymbol{X}} = \frac{\partial \boldsymbol{\varphi}}{\partial \theta^{i}} \otimes \frac{\partial \theta^{i}}{\partial \boldsymbol{X}} = \boldsymbol{g}_{i} \otimes \boldsymbol{G}^{i} .$$
(8)

Due to the shell kinematics in equation (1) reads

$$\boldsymbol{F} = [\boldsymbol{a}_{\alpha} + \theta^3 \boldsymbol{a}_{3,\alpha}] \otimes \boldsymbol{G}^{\alpha} + \boldsymbol{a}_3 \otimes \boldsymbol{G}^3 .$$
(9)

For a Kirchhoff-Love shell the director d in the current state is also normal to the middle surface

$$\boldsymbol{d} = \frac{\boldsymbol{a}_1 \times \boldsymbol{a}_2}{|\boldsymbol{a}_1 \times \boldsymbol{a}_2|} \,. \tag{10}$$

Therefore, the contribution of the transverse shear is neglected. Based on that assumptions only the in plane components of the deformation gradient are necessary to be taken into account, with the projection on the middle surface

$$\widehat{\boldsymbol{a}} = \boldsymbol{a} \cdot (\boldsymbol{I} - \boldsymbol{a}_3 \otimes \boldsymbol{a}_3) , \qquad (11)$$

with the second order unit tensor I. With that the in plane deformation gradient \vec{F} reads

$$\widehat{\boldsymbol{F}} = \widehat{\boldsymbol{H}} + \theta^3 \widehat{\boldsymbol{K}} , \qquad (12)$$

with

$$\widehat{H} = a_{\alpha} \otimes G^{\alpha}$$
 and $\widehat{K} = a_{3,\alpha} \otimes G^{\alpha}$. (13)

4 SCALE-TRANSITIONS

The approach to create a boundary value problem for a mircostructural RVE requires to consider the shell deformation gradient derived in equation (12) to be the macroscopic gradient \hat{F}_M of the multi scale analysis [3]. Furthermore the macroscopic deformation gradient has to be equal to the volume average of the microscopic deformation gradient \hat{F}_m . The index X_m is connected to quantities on the micro structure and the index X_M is assigned to quantities on the macro structure. For the homogenization the position of a point in the microscopic RVE is given by

$$\widehat{\boldsymbol{\varphi}}_m = \widehat{\boldsymbol{F}}_M \cdot \boldsymbol{X}_m + \widehat{\boldsymbol{\omega}}(\boldsymbol{X}_m) , \qquad (14)$$

where

$$\widehat{\boldsymbol{F}}_{M} = \frac{1}{V_{0}} \int_{\mathcal{B}_{0m}} \widehat{\boldsymbol{F}}_{m} dV_{m} = \frac{1}{V_{0}} \int_{\partial \mathcal{B}_{0m}} \widehat{\boldsymbol{\varphi}}_{m} \otimes \boldsymbol{N}_{m} dA_{m} , \qquad (15)$$

with the micro fluctuation $\widehat{\boldsymbol{\omega}}(\boldsymbol{X}_m)$ and the outward normal in the material configuration \boldsymbol{N}_m . Inserting equation (14) in (15) leads to the condition for the micro fluctuation field

$$\int_{\partial \mathcal{B}_{0m}} \widehat{\boldsymbol{\omega}}(\boldsymbol{X}_m) \otimes \boldsymbol{N}_m dA_m = \boldsymbol{0} .$$
(16)

This equation is a requirement for the boundary conditions of the RVE and a possibility for the realization are periodic ones. For this the boundary has to be split in three parts $\partial \mathcal{B}_0 = \partial \mathcal{B}_0^+ \cup \partial \mathcal{B}_0^- \cup \partial \mathcal{B}_{0t}$. The parts $\partial \mathcal{B}_0^+$ and $\partial \mathcal{B}_0^-$ are on opposite sides on the RVE faces normal to the plane direction. On this faces periodic boundary conditions are applied with the condition to the fluctuation

$$\widehat{\boldsymbol{\omega}}^+ = \widehat{\boldsymbol{\omega}}^- \,, \tag{17}$$

as pointed out in [2] and antiperiodic tractions

$$\widehat{\boldsymbol{t}}_0^+ = \widehat{\boldsymbol{t}}_0^- \,. \tag{18}$$

On the faces $\partial \mathcal{B}_{0t}$, parallel to the textile plane zero traction boundary conditions are applied. The reversing scale transition is based on an averaging of the microscopic stresses. An energy averaging theorem, which requires the equality of the microscopic average of the virtual work δW_m on the RVE and the virtual work on the macro scale δW_M is the Hill-Mandel condition. The Hill-Mandel condition can be expressed for a volume RVE

$$\delta W_m = \frac{1}{A_{0m}} \int_{\mathcal{B}_{0m}} \boldsymbol{P}_m : \delta \boldsymbol{F}_m dV_m = \widehat{\boldsymbol{N}}_M : \delta \widehat{\boldsymbol{H}}_M + \delta \widehat{\boldsymbol{M}}_M : \delta \widehat{\boldsymbol{K}}_M = \delta W_M , \qquad (19)$$

where δa is the variation of a and the coefficients of the stress resultants are given by

$$\widehat{\boldsymbol{N}}_{M} = \int_{H} \left[\frac{1}{A_{0m}} \int_{\mathcal{M}_{0m}} \widehat{\boldsymbol{P}}_{m} dA_{0m} \right] d\theta^{3} = \frac{1}{A_{0m}} \int_{\partial \mathcal{B}_{0m}^{+} \cup \partial \mathcal{B}_{0m}^{-}} \boldsymbol{t}_{0m} \boldsymbol{X}_{m} d\partial \mathcal{B}_{0m} ,$$

$$\widehat{\boldsymbol{M}}_{M} = \int_{H} \left[\frac{1}{A_{0m}} \int_{\mathcal{M}_{0m}} \theta^{3} \widehat{\boldsymbol{P}}_{m} dA_{0m} \right] d\theta^{3} = \frac{1}{A_{0m}} \int_{\partial \mathcal{B}_{0m}^{+} \cup \partial \mathcal{B}_{0m}^{-}} \theta^{3} \boldsymbol{t}_{0m} \boldsymbol{X}_{m} d\partial \mathcal{B}_{0m} ,$$
(20)

for heterogeneous RVE sections [4]. Therewith the coefficients of the generalized forces are

$$\widehat{\boldsymbol{N}}_{M}^{i} = \widehat{\boldsymbol{N}}_{M} \cdot \boldsymbol{G}_{M}^{i} ,$$

$$\widehat{\boldsymbol{M}}_{M}^{\alpha} = \widehat{\boldsymbol{M}}_{M} \cdot \boldsymbol{G}_{M}^{\alpha} .$$
(21)

5 CHARACTERIZATION OF THE MICRO LEVEL AND EXAMPLES

For the exemplary application of the introduced methods a periodic woven RVE is considered. With a size of $4 \times 4 \times 2$ mm³ the RVE is composed of 20-node hexahedral elements. The constitutive law of the fiber material is chosen to be isotropic, linear elastic, so the stress-strain relation can expressed by Hooke's Law

$$\boldsymbol{\sigma} = \frac{E}{1+\nu} \left[\frac{\nu}{1-2\nu} tr(\boldsymbol{\varepsilon}) \boldsymbol{I} + \boldsymbol{\varepsilon} \right] , \qquad (22)$$

where I is the identity matrix and $tr(\varepsilon)$ is the trace of the linearized strain tensor ε . E is the Young's modulus and ν is the Poisson ratio. After characterization of the material behavior also the interaction between the fibers has to be considered. Contact is a unilateral, nonlinear coupling condition where forces are transferred in the common contact zone.

Thereby stresses are acting between the contact partners, which can be classified in stresses t_n normal to the contact plane and tangential stresses t_t in the contact plane with the contact stresses

$$\boldsymbol{t}_c = \boldsymbol{t}_n + \boldsymbol{t}_t. \tag{23}$$

Further the relation between normal and tangential stress is introduced

$$|\boldsymbol{t}_n| = \mu |\boldsymbol{t}_t| , \qquad (24)$$

known as Coulomb friction with the friction coefficient μ chosen in this exemplary model to $\mu = 0.5$, realized by a Penalty method.

For the testing of the introduced methods calculations on the micro level were accomplished to evaluate the nonlinear textile behavior. Considering two typical deformations like bending (Fig. 3(a)) and membrane shearing (Fig. 3(b)) the deformed shape and the von Mises stress is plotted. Further the homogenized, macroscopic response over deformation for bending κ_{22} and shearing γ_{12} is shown.

6 CONCLUSIONS

This paper treats the basic principles to consider technical textiles in a multi scale scheme, connecting macroscopic shells with 3D-modeled fiber structures. It allows to



Figure 3: Macroscopic homogenized response for different RVE deformations

account for the coupling between structural heterogeneous shells and the underlying microstructural features that cause this behavior. Main focus was put on the extraction of the macroscopic phenomenological constitutive laws of the textile. A analysis of different 3D deformations was shown within the context of a shell response.

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A NEW DOUBLE CURVED ELEMENT FOR TECHNICAL TEXTILE ANALYSIS WITH BENDING RESISTANCE

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Key words: Textile Composites, Membrane Analysis, Total Lagrange, Nonlinear Continuum Mechanics, Thin Shell

Summary. A new double curved spatial element was developed to analyze textile structures. It has a bending resistance beside the in-plane stiffness. The element is based on a 8-node double curved membrane element¹. To find the equilibrium solution the Total Lagrange strategy is used with the dynamic relaxation method (DRM). Deformations are calculated with a continuum mechanical method.

1 INTRODUCTION

For the analysis of textile structures membrane elements with in-plane stiffness are widely used. To analyze the shape of wrinkles or any not tensioned stage we need bending resistance.

The new method of present paper is based on a double curved membrane element¹. The strategy of the analysis is the following:

- The numerical description is based on a double-curved finite element with a parametric surface coordinate system.
- Deformations are calculated between the initial (deformation free) and the actual geometry (Total Lagrange Method).
- Deformations are calculated by tensor analysis (Nonlinear Continuum Mechanics).
- The equilibrium shape is searched by dynamic relaxation.

The original method¹ used C0 finite elements. It was sufficient to describe the proper geometry of a surface for membrane analysis. For bendable shell analysis a C1 element is needed and the strategy of the analysis is presented here.

In our work we apply the thin shell theorem, namely the out of plane shear stresses are calculated from the bending moments².

2 THE DISCRETE MESH

9-node quadrilateral double curved, C1 continuous elements are chosen³. All nodes have freedom to move spatially and rotate except the internal node; it has only rotational freedom.

By taking into account large distortions of the geometry the side nodes and the internal nodes of the elements leave the middle position. For a linear transformation (and proper integration) between the parametric coordinate system of the element and the 3D global coordinate system we must control the movement of the internal nodes. By splitting the geometry of the surface and the geometry of the material it is possible to control the difficulty mentioned above. For this an advanced 8-node membrane element⁴ was used in the membrane analysis¹. That element is C0 continuous.

In practice the difference between the surface and the material geometry is small. The computational cost of using the C0 advanced 8-node element is acceptable. But if we want to use an advanced finite element technique for a C1 element, the computation becomes even more complicated and it requires an extreme computational cost. So in the technique presented here we neglect the difference between the surface and the material geometry.

3 CALCULATION OF THE DEFORMATIONS

3.1 Membrane deformations

Calculation of the membrane deformations are well described in ¹. The most important steps are shown here. The deformation is calculated from the change of the geometry between the free initial geometry (described by the \bar{r}^0 position vector) and the actual geometry (described by \bar{r}). The bases of the two stages are:

$$\overline{g}_{k}^{0} = \frac{\partial \overline{r}^{0}}{\partial x^{0k}},\tag{1}$$

$$\overline{g}_p = \frac{\partial \overline{r}}{\partial x^p},\tag{2}$$

where \overline{g}_k^0 and \overline{g}_p are the basis vectors, *k* and *p* can take the value ξ , η and 3 according to the surface coordinate system (3 is the normal direction)⁵. The expression of the strain based on large elongations is the following:

$$\mathcal{E}_{e} = \frac{ds}{ds^{0}} - 1 = \sqrt{C_{kl} e^{0k} e^{0l}} - 1,$$
(3)

$$\frac{1}{2}\sin\gamma_{I,II} = \frac{H_{kl}e_{I}^{0k}e_{II}^{0l}}{(1+\varepsilon_{I})(1+\varepsilon_{II})},$$
(4)

where ε_e and $\gamma_{I,II}$ are the strains in the e directions and the distortion between the e_I and e_{II} directions, e^{0k} are the contravariant components of the directions in the initial stage (the material law is known usually in the initial stage, so the best is to calculate the deformations to the initial stage), C_{kl} are the covariant components of the Green-deformation tensor (it

describes the transformation of the metric tensor of the two stages based on the initial stage), and H_{kl} are the covariant components of the Lagrangian-deformation tensor (it describes the difference between the scalar product of the differential lines of the two stage). The \bar{e}_l and \bar{e}_{ll} directions are ordinary directions, in practice they are parallel with the directions of the yarns of the technical textiles.



Figure 1: The basis system of the initial and the actual geometry.

The calculation of the Green-deformation tensors is the following:

$$g_{pq} = \overline{g}_k^0 \cdot \overline{\overline{F}}^T \cdot \overline{\overline{F}} \cdot \overline{g}_l^0 = \overline{g}_k^0 \cdot \overline{\overline{C}} \cdot \overline{g}_l^0$$
⁽⁵⁾

$$\overline{\overline{C}} = \overline{\overline{F}}^T \cdot \overline{\overline{F}} = \overline{g}^{0k} \otimes \overline{g}_p \cdot \overline{g}_p \otimes \overline{g}^{0k}, \qquad (6)$$

where $\overline{\overline{F}}$ is the deformation gradient and g_{pq} is the metric tensor of the actual stage. The expression of the Lagrange-deformation tensor is the following:

$$dr_{I} \cdot dr_{II} - dr_{I}^{0} \cdot dr_{II}^{0} = dr_{I}^{0} \cdot \overline{F}^{T} \cdot \overline{F} \cdot dr_{II}^{0} - dr_{I}^{0} \cdot dr_{II}^{0} =$$

$$dr_{I}^{0} \cdot \left(\overline{\overline{F}}^{T} \cdot \overline{\overline{F}} - \overline{\overline{I}}\right) \cdot dr_{II}^{0} = dr_{I}^{0} \cdot 2\overline{\overline{H}} \cdot dr_{II}^{0}$$

$$\overline{\overline{H}} = \frac{1}{2} \left(\overline{\overline{F}}^{T} \cdot \overline{\overline{F}} - \overline{\overline{I}}\right) = \frac{1}{2} \left(\overline{\overline{C}} - \overline{\overline{I}}\right),$$

$$(8)$$

where dr_I , dr_{II} , dr_I^0 and dr_{II}^0 are line elements in the surface. The deformation gradient expressed by the base vectors of the two stages can be obtained as:
$$\overline{\overline{F}} = \overline{g}_p \otimes \overline{g}^{0k} \,. \tag{9}$$

For small elongations the Lagrangian-deformation tensor gives a good approximation of the strain. But when elongations are large, the (3-4) formulas must be used.

3.2 Changing of curvatures

The bending moments can be calculated from the varying curvatures. The curvature of a surface can be obtained from the gradient of the normal vector of the surface²:

grad
$$\overline{n} = \frac{\partial \overline{n}}{\partial \xi^k} \otimes g^k$$
 and, (10)

$$\overline{n} = \overline{g}_{3} = \frac{\overline{g}_{\xi} \otimes \overline{g}_{\eta}}{\left| \overline{g}_{\xi} \otimes \overline{g}_{\eta} \right|}$$
(11)

where \overline{n} is the normal vector of the surface (equivalent with \overline{g}_3). The gradient of a surface is calculated by the simplified formula:

$$grad \,\overline{n} = - \begin{bmatrix} \overline{n} \frac{\partial \overline{g}_1}{\partial \underline{\xi}} & \overline{n} \frac{\partial \overline{g}_2}{\partial \underline{\xi}} & 0\\ \overline{n} \frac{\partial \overline{g}_1}{\partial \eta} & \overline{n} \frac{\partial \overline{g}_2}{\partial \eta} & 0\\ 0 & 0 & 1 \end{bmatrix}.$$
(12)

The curvature of the surface can be obtained by the element vector of any direction:

$$\omega_e = \operatorname{grad} \bar{n}_{kl} e^k e^l \,, \tag{13}$$

where ω_e is the curvature in the direction of the *e* vector.

In practice, the initial geometry is flat for textile membranes (i.e. the flat pattern is the initial geometry), so the variation of the curvature is in the gradient of the actual geometry. If the initial geometry is not flat, the difference between the initial and the actual curvatures must be taken into account:

$$\Delta \omega_e = \operatorname{grad} \overline{n}_{kl} e^k e^l - \operatorname{grad} \overline{n}_{kl}^{-0} e^{0k} e^{0l}.$$
⁽¹⁴⁾

4 CALCULATION OF THE INTERNAL FORCES

4.1 Membrane forces

The membrane forces can be calculated from the membrane strains. To study the equilibrium of the actual stage the Cauchy stress tensor is needed (all the geometry and directions are described in the actual stage). Traditionally during the measurements the material law is connected to the initial geometry, that is why the deformation tensors of the initial stage are used in this paper. In ordinary nonlinear problems (large displacements with small enlargement) the Second-Piola-Kirchoff (SPK) stress tensor is used: the stress directions are in the initial coordinate system and the geometry of the structure is described in the initial geometry. But the last statement is not exactly true: during the transformation not only the area of the elementary surface is changed but the stress vector, too:

$$dA = dA^0 \cdot \overline{\overline{F}}^T \cdot \frac{1}{\left|\overline{\overline{F}}\right|}$$
 and (15)

$$\overline{\overline{T}} = \overline{\overline{F}} \cdot \overline{\overline{S}} \cdot \overline{\overline{F}}^T \cdot \frac{1}{\left|\overline{\overline{F}}\right|},\tag{16}$$

where \overline{T} is the Cauchy stress tensor (the real stress tensor of the actual geometry), \overline{S} is the SPK stress tensor (the stress tensor of the initial geometry) and dA is the elementary area. In equation (15) there is a similar transformation of the area to equation (16), but to change the direction from the initial to the actual state there is a second transformation (multiplication by the gradient tensor \overline{F}), which transformation scales the size of the vector beside changing the direction. It is generally a negligible side-effect if there are no large elongations.

The material-laws give back not the SPK stress tensor but the Biot stress tensor. To get the Cauchy stress tensor from the Biot stress tensor we need a little bit more complication in the transformation process:

$$\overline{\overline{T}} = \overline{\overline{R}} \cdot \overline{\overline{T}_0} \cdot \overline{\overline{F}}^T \cdot \frac{1}{\left|\overline{\overline{F}}\right|},\tag{17}$$

where $\overline{\overline{T}_0}$ is the Biot stress tensor and $\overline{\overline{R}}$ is the rotation tensor. Compared to the usage of the SPK stress tensor the rotation tensor must be used. That can be obtained by the following:

$$\overline{\overline{R}} = \overline{\widetilde{g}}_{p} \otimes \overline{g}^{0k} , \qquad (18)$$

where $\overline{\tilde{g}}_{p}$ is the non stretched basis vector system, and can be obtained from the basis vectors of the initial stage:

$$\overline{\widetilde{g}}_k \cdot \overline{\widetilde{g}}_l = \overline{g}^0{}_k \cdot \overline{g}^0{}_l, \tag{19}$$

if the normal vector of the actual stage is used as $\overline{\widetilde{g}}_{_3}$ and $\overline{\widetilde{g}}_{_{\xi}}$ has the direction of $\overline{g}_{_{\xi}}$ and the length of $\overline{g}_{_{\xi}}^{_{0}5}$.

4.2 Bending moments

The moments of the structure can be calculated from the above presented equation (13):

$$\overline{\overline{M}}_0 = \overline{\overline{D}} \cdot \overline{\overline{\omega}} , \qquad (20)$$

where \overline{M} is the vector of the moment, \overline{D} is the matrix of the constitutive law and the $\overline{\omega}$ is the vector of the rotations and the torsion of the sections. The rotation vector is invariant so the constitutive law can be used in the initial coordinate system, and then the values of the moment vector can be transformed to the actual stage like the membrane stresses:

$$\overline{\overline{M}} = \overline{\overline{R}} \cdot \overline{\overline{M}_0} \cdot \overline{\overline{F}}^T \cdot \frac{1}{\left|\overline{\overline{F}}\right|} \,. \tag{21}$$

4.3 Out of plane shear forces

In the thin shell theorem the out-of-plane shear forces derived from the bending moments² are the following:

$$\overline{V} = \begin{bmatrix} \frac{\partial}{\partial \xi} & 0 & \frac{\partial}{\partial \eta} \\ 0 & \frac{\partial}{\partial \eta} & \frac{\partial}{\partial \xi} \end{bmatrix} \begin{bmatrix} M_{\xi} \\ M_{\eta} \\ M_{\xi\eta} \end{bmatrix}.$$
(22)

where \overline{V} is the vector consists of the outof-plane shear forces and the moments are presented in vector format instead of tensor format.

5 THE NODAL EQUILIBRIUM

To find the equilibrium of the structure we seek the nodal equilibrium. Having a nonlinear problem an iterative process is needed. The dynamic relaxation method (DRM) is our choice. The DRM uses fictive dynamic analysis⁶ with appropriate dumping to find the equilibrium configuration. In membrane analysis the nodal movement is followed. Due to the bending resistance the rotation of the tangent plane of the surface must be used, too. With C1 finite elements it is possible to carry out this process.

5.1 External loads

By ordinary finite element transformations it is possible to reduce the surface loads to the nodes:

$$\bar{f}_{ex} = \int_{A} \overline{\overline{N}}^{T} \cdot \overline{q} \, dA \,, \tag{24}$$

where \overline{f}_{ex} is the nodal force vector from the external loads, \overline{N} is the matrix with the shape functions and \overline{q} is the surface load vector.

5.2 Internal forces

$$\bar{f}_{in} = \int_{A}^{\overline{B}_{\sigma}^{T}} \cdot \overline{\sigma} \, dA + \int_{A}^{\overline{B}_{\tau}^{T}} \cdot \overline{\tau} \, dA + \int_{A}^{\overline{B}_{M}^{T}} \cdot \overline{M} \, dA \,, \tag{25}$$

where \overline{f}_{in} is the nodal force vector from the internal forces, \overline{B}_{σ} , \overline{B}_{τ} and \overline{B}_{M} are the transformation matrices of the internal forces, $\overline{\sigma}$ is the vector of the membrane forces, $\overline{\tau}$ is the vector of the out-of-surface shear forces and \overline{M} is the vector of the bending moments.

To get the \overline{B} matrices the operator matrices of the deformations are needed²:

$$\overline{\varepsilon}_{(\xi\eta)}^{\hat{X}\hat{Y}} = \overline{\overline{L}}_{\varepsilon} \cdot \hat{u}_{(\xi\eta)} = \begin{bmatrix} \frac{\partial}{\partial \hat{X}} & 0\\ 0 & \frac{\partial}{\partial \hat{Y}}\\ \frac{\partial}{\partial \hat{Y}} & \frac{\partial}{\partial \hat{X}} \end{bmatrix} \begin{bmatrix} u_{(\xi\eta)}^{\hat{X}}\\ u_{(\xi\eta)}^{\hat{Y}} \end{bmatrix},$$
(26)

where \hat{X} and \hat{Y} are the coordinates of the invariant surface coordinate system used for the integration¹ $\bar{\varepsilon}_{(\xi\eta)}^{\hat{X}\hat{Y}}$ gives back the membrane strains with the parametric coordinates of the invariable coordinate system of the surface, $\overline{L}_{\varepsilon}$ is the operator matrix, $\overline{u}_{(\xi\eta)}$ is the in-plain displacement vector according to the parametric coordinate system. (Details of the transformations can be found in ¹)

The expression of the shear deformation is the following:

$$\overline{\gamma} = \overline{\overline{L}}_{\gamma} \cdot \overline{u}_{(\xi\eta)} = \begin{bmatrix} \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \eta} \end{bmatrix} [u_3], \qquad (27)$$

where $\overline{\gamma}$ is the out of plain strain vector, \overline{L}_{γ} is the operator matrix, $\overline{u}_{(\xi\eta)}$ is the out of plain movement vector according to the parametric coordinate system. Strains perpendicular to the surface are neglected. The following formula shows the calculation of the curvatures:

$$\overline{\omega} = \overline{\overline{L}}_{\omega} \cdot \overline{\Theta}_{(\xi\eta)} = \begin{bmatrix} \frac{\partial}{\partial \xi} & 0\\ 0 & \frac{\partial}{\partial \eta} \\ \frac{\partial}{\partial \xi} & \frac{\partial}{\partial \eta} \end{bmatrix} \begin{bmatrix} \Theta_{\xi} \\ \Theta_{\eta} \end{bmatrix},$$
(28)

where $\overline{\omega}$ is the vector of the change of the curvature of the surface, \overline{L}_{ω} is the operator matrix, $\overline{\Theta}_{(\xi\eta)}$ is the rotation according to the parametric coordinate system.

The operators of the membrane strains and the curvature variation are identical. It means, the $\overline{\overline{B}}$ matrices are identical too. The $\overline{\overline{B}}$ matrix of ¹ can be used, the detailed development of the formula can be found in the original paper¹. Here just the final formula is expressed:

$$\overline{\overline{B}} = \frac{1}{|J|} \begin{bmatrix} \hat{X} \cdot \left(\overline{g}_{2}^{g} \cdot \hat{Y} \frac{\partial \overline{\overline{N}}}{\partial \xi} - \overline{g}_{1}^{g} \cdot \hat{Y} \frac{\partial \overline{\overline{N}}}{\partial \eta}\right) \\ \hat{Y} \cdot \left(-\overline{g}_{2}^{g} \cdot \hat{X} \frac{\partial \overline{\overline{N}}}{\partial \xi} + \overline{g}_{1}^{g} \cdot \hat{X} \frac{\partial \overline{\overline{N}}}{\partial \eta}\right) \\ \hat{Y} \cdot \left(\overline{g}_{2}^{g} \cdot \hat{Y} \frac{\partial \overline{\overline{N}}}{\partial \xi} - \overline{g}_{1}^{g} \cdot \hat{Y} \frac{\partial \overline{\overline{N}}}{\partial \eta}\right) + \hat{X} \cdot \left(-\overline{g}_{2}^{g} \cdot \hat{X} \frac{\partial \overline{\overline{N}}}{\partial \xi} + \overline{g}_{1}^{g} \cdot \hat{X} \frac{\partial \overline{\overline{N}}}{\partial \eta}\right) \end{bmatrix},$$
(29)

where |J| is the Jacobian determinant. Finally, $\overline{\overline{B}}_{\tau}$ is:

$$\overline{B}_{r} = \frac{1}{|J|} \begin{bmatrix} \hat{X} \cdot \left(\overline{g}_{2}^{g} \cdot \hat{Y} \frac{\partial \overline{N}}{\partial \xi} - \overline{g}_{1}^{g} \cdot \hat{Y} \frac{\partial \overline{N}}{\partial \eta}\right) \\ \hat{Y} \cdot \left(-\overline{g}_{2}^{g} \cdot \hat{X} \frac{\partial \overline{N}}{\partial \xi} + \overline{g}_{1}^{g} \cdot \hat{X} \frac{\partial \overline{N}}{\partial \eta}\right) \end{bmatrix}.$$
(30)

5.3 Dynamic Relaxation Method

The difference of the external and the internal nodal forces accelerate the fictitious mass of the nodes:

$$a_{i} = \frac{f_{ex,i} - f_{in,i}}{m_{i}},$$
(31)

where a_i is the acceleration of the i^{th} freedom, and m_i is the mass of the i^{th} freedom. The freedom can be a movement or a rotation. To handle movements and rotations, we assume a (fictive) masses and (fictive) moment of inertia at each node, respectively. Both viscous dumping and kinetic dumping can be used¹ to find the equilibrium position of the system.

6 CONCLUSIONS

A new method was developed for analysis of thin shell structures. It extends the membrane analysis technology¹ with bending stiffness. The calculation of the deformations by tensor analysis gives the chance to omit any approximation. Only the quality of the discrete mesh is a limitation of the accuracy of the analysis.

The new method gives an effective tool for analyzing thin shells and wrinkling of textiles and textile membranes.

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EVALUATION OF THE STRUCTURAL BEHAVIOR OF TEXTILE COVERS SUBJECTED TO VARIATIONS IN WEATHER CONDITIONS

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INTRODUCTION

A simplified model of an anticlastic membrane is that of two perpendicular ropes, which meet at a point. If the ropes are tensed in opposite directions, the meeting point becomes is fixed. As we increase the tension in the two ropes, more and more force will be required to move the meeting point of the two ropes. In other words, the system becomes more rigid when the tension of the ropes is increased and when applying an external force it will deform less.

The tension applied to a cable system or an anticlastic membrane for it to become more rigid is called pretension. A membrane or anticlastic net has an adequate structural behavior only if it is in a tensed state.[1]

Loss of pretension reduces the rigidity of the system, increasing its deformation due to external loads. If loss of pretension exceeds certain limits, the membrane will start to flutter or will be deformed, with the risk of accumulating water or snow, in both cases compromising the durability of the membrane. Therefore, it is of utter importance to know and be able to predict the process of loss of pretension in order to establish maintenance plans that allow optimal levels of initial tension in the anticlastic structures, avoid that loss of pretension reaches critical levels. Loss of pretension is due to the natural behavior of the material, but additionally, there are external factors that influence loss of pretension of the membranes, among these, we can mention the weather as a factor that affects tensional life of membranes.

In this work we will try to prove this hypothesis. In order to do this, the first stage will be the development of a trial bench that allows us to study the effect of superficial temperature, humidity and wind loads on the loss of pretension. The trial bench can reproduce, in a controlled and independent way, each one of the different variables of interest to this study. It allows us to study these variables in physical scale models and with accelerated cycle processes, which can in less time, simulate the behavior of the membranes in their normal life cycle, lowering the cost of the study. On the other hand, the result of these studies will let us validate a mathematical model that is developed at the same time.



Figure 1: Model

To carry out the trials, it is necessary for the trial bench to be automated, since it must perform in a repetitive manner each of the established cycles, and, likewise, it should include a data acquisition system, which enables it to keep track of how the traction forces on the membrane vary while it is subjected to load and unload cycles, under diverse temperature and humidity conditions.

For the design of the bench, working conditions, the geometry of the membrane, pretension force, temperature and humidity ranges, and wind forces were established in order to dimension the structure of the bench as well as the application and measuring systems.

1. Definition of working conditions

1.1-Weather Conditions:

Working conditions were selected based on the characteristics of the weather conditions defined by the building environmental group of the IDEC of the Faculty of Architecture of the UCV. [2].

In order to determine the superficial temperature, extreme factors of temperature, insolation and wind speed, that rarely occur simultaneously in Venezuelan territory, but which allowed us to establish the maximum extreme of superficial temperature. (Table 1)

Determination of superficial temperature:

Tsun- Tair= Tair + (α Es- hr (10°C))+hcr [4]

```
α= 0,2
```

hcr = 15 Watts/m² °C Thermal conductivity

 $hr= 5 Watts/m^2 \circ C$ Radioactive Exchange coefficient

 $Es = 1000 Watts/m^2$

$T=35^{\circ}C$

Temperature in maximum insolation conditions 45-50°C, little wind and maximum temperature.

WORKING RANGES					
	Minimum	Maximum			
Forces	40Kg/m ²	75Kg/m^2			
Wind speed		120 Km/h			
Temperature	30°C	50°C			
Humidity		80%			

Table 1: selected working conditions

1.2- Geometric/Structural Conditions:

A virtual model was built to determine the resulting forces, required pretension, and the geometry that will allow us to build the physical model.

For this we used the EASY program (by Technet), where nets for paraboloids were built, with the following characteristics: 0.90x0.90 mts and 1.80x1.80 mts with Sag/Span relations of 1:5, 1:15. FIG. 2. They were loaded with vertical loads of 40 Kg/m² both in pressure and in suction (based on ranges previously determined in the weather conditions for the areas that would be studied). FIG. 3.

In the model a 300X300 reticle was set in, where vertical loads and node deformations were calculated, which will allow us to place the burden in the trials where the wind loads are applied. TABLE 2. The model allows us to define the pattern design for the construction of the physical model of the same geometry of the virtual model.



Figure 2 : Geometry of the paraboloids of relations Arrow/light 1:5(Geom.1), 1:15 (Geom.2).



Figure 3. Graphic outcome of the program, which show resulting forces in the edge reinforcement and on the surface of the membrane on the right.

2. Design of the trial bench and data acquisition system:

The work bench is designed based on the previously established conditions. This allows it to hold a small membrane of approximately 2.8 m^2 with changeable Sag and space to house the various devices required to simulate the different trial conditions as well as the measuring equipment.

The supporting structure consists of a not easily deformable square base prism-shaped frame built with ECO(1) 100X100 mm tubular structures, welded at the edges The frame has adjustable legs that allow it to be leveled. (FIG. 4). Metallic bases are fixed on the studs of the vertical edges for the membrane to be connected. These bases are adjustable so as to produce paraboloids of Sag/Span relations 1:5, 1:10, 1:15. (FIG.5)

A second structure is added to this basic frame. This structure is shaped like a table, and contains the mechanism for the wind application system. When necessary it is fixed to the basic frame by means of four bolts.

GEOMETRY	LOAD	RESULTING EXTREMES KN	FORCE ON THE MATERIAL U/T KN	DEFORMATION MTS
1	PRETENSION	5,3	0,1/0,1	
1	40 KN	6,1	0,1/0,2	0,01
1	80 KN	6,2	0,1/0,2	0,01
2	PRETENSION	4,4	0,1/0,1	
2	40 KN	5,4	0,1/0,2	0,03
2	80 KN	7,1	0,1/0,3	0,03

TABLA 2: Summary of results



Figure 4 : supporting structure. Figure 5 : bases are adjustable so as to produce paraboloids of different Sag/Span relations .

2.2- Introduction of the tension and measurement

The connection of the membrane with the structural frame is carried out through an element that allows the introduction of the tension (tensor) FIG. 7 and by means of a load cell. The tensors are placed, one in the extreme of the paraboloid that is the highest in relation to the horizontal base plane (high point) and the other in the extreme perpendicular to this one that is placed in a low point of the paraboloid. The load cells are placed in the extremes opposite the tensors. In both cases the terminals allow two degrees of freedom to ensure a perfect alignment, also ensuring there will only be axial loads on the tensors and load cells. The system permits the introduction of traction loads of up to 2400 Kg.

In order to measure the traction loads on the tensile structure, (FIG.6) HBM (brand) Type S load cells capable of measuring up to 1360,77 Kg. (3000 lb.) are used. These load cells may exceed their maximum load up to 120%. These cells use stain gauges placed in a Wheatstone bridge shape, which detect the deformation suffered by the cell and report it in an analogical manner in an equivalent to the force applied in a relation of 3mV per feeding Volt, with a 3 mV/V sensitivity, the operating temperature range is from -30 a +70 °C. The cells are connected to a signal conditioning circuit, made up basically by an instrumental amplifier, with low-pass filter (fc = 50 Hz) and high quality tension reference, and to an analogical entrance for a USB-6009 data acquisition module, that allows the computer to pick up the load data. [6]

2.3- System for wind load application:

The main problem to reproduce the load applied by the wind is the homogeneous distribution of the load on the membrane surface. Several methods were studied; having chosen the one that was easiest to apply and at the same time provided the most homogeneous load distribution. In the chosen method, the wind load is applied through an air mattress rested against the membrane.

A table-like structure is connected to the main frame of the bench. Inside the structure there is a metallic frame to which a rigid surface has been fixed (a chipboard plate). This frame hangs from the main structure through a collapsible system (FIG. 8) made of articulated bars, which limit the horizontal movement of the frame, but at the same time allow vertical movement, thus ensuring that the frame will remain horizontal during the entire route. The movement of the plane is carried out with the aid of a linear actuator (Dynamat) with a 3KN thrust capacity at a speed of 10mm/sec, placed in the center of the frame and main structure. This engine applies the required force. The applied force is measured with a load cell placed between the linear actuator and the frame. A table-like structure is connected to the main frame of the bench. Inside the structure there is a metallic frame to which a rigid surface has been fixed (a chipboard plate). This frame hangs from the main structure through a collapsible system (FIG. 8) made of articulated bars, which limit the horizontal movement of the frame, but at the same time allow vertical movement, thus ensuring that the frame will remain horizontal during the entire route. The movement of the plane is carried out with the aid of a linear actuator (Dynamat) with a 3KN thrust capacity at a speed of 10mm/sec, placed in the center of the frame and main structure. This engine applies the required force. The applied force is measured with a load cell placed between the linear actuator and the frame.



Figure 6: Load Cell



Figure 7: Tensor

A mattress made of vinyl is hung on the surface of the movable frame. This mattress is partially filled with air, so that when it touches the surface of the membrane it adopts its form. As the distance between the frame and the membrane decreases, the pressure inside the mattress increases, transmitting the force that the engine applies to the mattress—through the movement of the rigid plane—to the membrane in the form of pressure. The pressure is normal at the surface and equal per area unit, which ensures that the force is applied homogeneously and in the same manner as in the computer model. [8]

2.4- Temperature application system:

Since the aim is to reproduce the superficial temperature produced by the amount of insolation and not the temperature of the environment, we discarded convection heating systems, in which air surrounding the membrane is heated, and chose instead a radiation heating system, which allows the surface of the membrane to be heated without heating the

environment. The chosen system consists of sixteen (16) infrared 250-Watts industrial lamps (FIG. 10), which have a total maximum power of 4000 Watts. The lamps are placed under.







The tensile structure to cover an area of 2.4 m². With infrared radiation, the radiated heat can be directed very accurately, avoiding loss of energy, which renders this method very effective. Due to the tridimensional geometry of the membrane, an adjustable base was used to keep the distance and the perpendicularity between the lamp and the membrane. The regulation of the lamps allows us to take the superficial temperature to 60°C. [6]

A control system was designed to allow uniform heating of the surface. The lamps are separated into four independent circuits, each one controlled by a sensor (digital dual temperature/humidity sensors), placed under the membrane in pockets to avoid direct exposure of the lamps, and in the center of the action area of the corresponding circuit, producing four independent control bows. The system allows the control of the amount of power given to the lamp, managing to make it radiate the necessary amount of energy at all times so that the temperature in the surface remains stable. The control system can compensate the effect of the Lamp/membrane distance difference and can even allow the compensation of the cooling effect due to air currents. Although the system remains closed during the trials, all the faces [sides] of the bench were open during the adjustments to the control system in order to make the access easier. The lamps facing the air current was turned on more frequently, than the ones that were further away, to compensate the cooling, thus ensuring uniform temperature throughout the entire surface. The heating period of the membrane to 50 °C is 3 minutes. To accelerate cooling, an extractor is used. Air extraction takes place at a rate of 11.4 m³/min, air injection 8.6 m^3 /min (power: 40 Watts), which allows the total volume (2.6 m^3) to be changed in 54 seconds. The extractor is activated through triac controlled from the computer. The environment temperature is recorded with a digital sensor connected to the control system and allows environment temperature to be recorded independently from the membrane temperature.

2.5 – Humidity Application System:

Humidity is varied by introducing water in the form of fog by means of a fogger inside the same isolated system used for the temperature trials. For this, a linear engine or *plunger* is used. It works (opening or closing) a multifunction hose-tip fogger (FIG. 11). Humidity is

measured inside the box through dual integrated sensors, which send the information in a digital manner to the computer that adjusts the fogging time and temperature necessary to reach the level of humidity (80%). To shut down the cycle, the extractors and the lamps are activated to take out the humid air and take humidity to the environment level.

2.6 – Automatic Control System:

The trial bench should work for long periods under little human supervision. In case the process stops abruptly, the supervision system must automatically store data of interest for the process, and it must be ensured that the system will not go haywire in case of power failure or loss of communication." [6]

The control of the bench is carried out with a computer equipped with a data acquisition card from National Instruments model USB 6009. The control system is complemented with three PIC18 micro controllers, which receive the data from the sensors (dual digital sensors), the orders from the computer, and activate the various devices, which is done through triac that supply the lamps, extractors, etc. with power.



Figure 10: Infrared Lamps.



Figure 11: Fogger with Plunger

There are two power modules: the high power module, responsible for the heating system and the power module used for handling the devices of the process, such as actuators or diffusers. The system allows the acquisition of temperature, humidity, and tension data. In figure 12 you may see a block diagram of how the various sections that compose the modules interconnect.

3. Trials

3.1 – Test Samples

The membrane is made according to the pattern design obtained from the informatics model (EASY program). Its geometry is a symmetric hyperbolic paraboloid that measures 1800mm on the side (for the first trial group with a Span/Sag ratio of 1:5, geometry 1).

The pattern cutting is carried out in such a manner that when the weave and the warp are ensambled, they are placed in the same direction in the four patterns, to ensure uniformity in the behavior and so that all the Test samples are comparable. The membrane is reinforced at the edges with a 6 mm 6x19iwrc, steel cable [edge reinforcement] with stainless steel terminals, threaded at the tips. All of this is placed in an edge pocket. On the vertexes, the membrane ends in identical corner plates that trap the extreme of the membrane and receive the steel cables of the edge reinforcement. A pocket is added to the membrane with a plastic bar to keep it from slipping. The corner plates, as well as the edge reinforcement, are removable, and are used in all the membranes that will be tested. The corner plates are built with two pieces made in a 1 mm sheet of stainless steel, folded at the edges. The tubes that permit the steel cable to be passed are welded to one of the pieces. (FIG. 13)



Figure 12: Structure of the hardware

3.2 – Trials of the equipment

Trials with the systems were carried out to verify that it behaved according to the specifications of the design and the virtual simulations that were carried out during the design of the control circuits and that were robust enough to withstand long working periods.

Rehearsal routines were programmed for the temperature system for it to carry out several cycles, using the temperature of the environment as minimum and taking the membrane up to 55 °C. In graph 1 the system's behavior is shown. The black line reflects the temperature of the environment on the membrane, measured by each one of the sensors. Similar behavior is seen in the various sensors. The differences are in the slope to reach the maximum and minimum values as rapidly as possible.

Regarding humidity, rehearsals were carried out to perform cycles of 30% to 55% of humidity. The system's behavior is shown on graph 2, where the lack line represents humidity inside the rehearsal bench and colored lines the humidity on the membrane, recorded through the various sensors. The variation is due to the differences in location of the sensors in relation to the fogger and the extractor, mainly affecting initial measurements. The system takes humidity to the maximum established value, regulating the average humidity inside the bench.



Figure 13: Test tube

Graph 1. Behavior of the temperature

As for the load measurement of the loads, these behave in the expected way, increasing when the temperature decreases (contraction of the material) and lowering when the membrane is heated (expansion of the material); and, on the contrary, when humidity decreases, the load decreases and vice versa.



Graph 2. Behavior of humidity

Graph 3. Loss of the Pattern's pretension

3.3 – Pattern

A new membrane is placed on the trial bench, it is taken to a pretension of 440 Kg, pretension is allowed to drop for 15 minutes and is pretensed again to the previous load. The membrane is left on the bench for 36 days with no further manipulation. The tension on the load cells, the environment temperature, and humidity were measured. With this pattern it is possible to establish the process of loss of pretension due to the material's characteristics.

Graph 3 reflects a rapid fall in pretension; pretension rises when tension is applied a second time and then starts to decrease, until it becomes stable at 80 hours. This rapid fall occurs because of the fiber's readjustment inside the membrane (creep), loss of pretension will continue very slowly, only affected by changes in the environment temperature between day and night in the structure laboratory.

CONCLUSIONS

- The trial bench allows rehearsing Hyperbolic Paraboloids of a 2.8 m² area with Sag/Span ratio of 1:5, 1:10, 1:15. It allows the application and measurement of tensions of up to 1360.77 Kg with an accuracy of +-1 Kg.

- It allows heating the membrane by means of radiation in a uniform manner from 20°C to 60°C and returning to environment temperature in 30-minute cycles; for the maximum working temperature, the cycle can be performed in 25 minutes or 57 cycles per day.

- Humidity in the bench can be kept in a range of 20% to 80%, and it is possible to carry out entire cycles in 12.5 minutes.

- The bench allows us to take the membrane to the conditions and in the ranges established in the working conditions.

- The rehearsed membrane establishes the pattern of loss of pretension due to the readjustment of the fibers in the material. Its incline allows us to extrapolate this loss in a bigger time frame, and will allow us to compare the loss when other conditions are applied.

- At the time this paper was written control system of the mechanism for simulation of the wind force have not ended and temperature and humidity rehearsals are being carried out.

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An orthotropic membrane model replaced with line-members and the large deformation analysis

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Key words: Finite Displacement, Membrane Structures, Compression-Free Model.

Summary. The paper proposes a method for analyzing large deformation of membrane structures by replacing the structures with elements composed line-members. The replacement has the advantage that the structural model is completely compression-free, so that the analysis always has a unique equilibrium solution without falling into multi equilibrium problems and the method is stable and easy even for how large deformation.

1 INTRODUCTION

Many methods to analyze membrane structures are popularly using elements in uniform strain with a bilinear relation between the stress and the strain, and the analyses are naturally geometric nonlinear. However, the bilinear stress-strain relation frequently brings weak compressive force in the structures, so that the analyses are apt to be a problem of solving multi equilibrium. For example, when both of tense area and slack one intermingle in the structure and the areas gradually change, the multi-equilibrium problem occurs and the analyses need some troublesome handling to get a solution. The appropriate solution probably has the minimum energy because membrane is generally flexible.

The proposed method uses replacing the structure with flexible line-members, so that the method is to guarantee a unique equilibrium solution with the minimum energy. When a triangular element composed by the line-members is in tense condition, the mechanical quality almost equals to the uniform strain element. On the other hand, when the line-member element is in slack condition, the element end forces always keep small tensile values and the element can deform until the three apexes concentrate into a point.

The study¹ presented in the last conference used elastic catenary cables as line-members, but restricted a triangle shape of the element. The modified method in the paper has no such restriction and can be easily applied to orthotropic material. The main modifications are to use different compatibilities between the main frame and the sub-frame in the element and to use two iterative methods of the Newton-Raphson Method and the Dynamic Relaxation Method. Solutions obtained by the method surely satisfy the strict equilibrium condition, so that the solutions do not have such errors in commercial software of FEM at all that was pointed out in the last conference².



Figure 1: Membrane element and the axial member frame substituted for the uniform strain element



Figure 3: Frame types mechanically equal to the element

2 ANALYTICAL THEORY

the sides in the element

The method is to solve purely the equilibrium equation expressed by the nodal force and the element end forces at the node. Therefore, the equilibrium equation does not include the nodal displacement at all. The displacement in the method is used for renewing the nodal position, and the nodal displacement is obtained by solving the tangent stiffness equation derived by differentiating the equilibrium equation. The nodal positions can determine the deformations of the element on geometric strictness without using the displacement, and the deformations determine the element end forces that balance with the axial forces of the linemembers. Then, the equilibrium equation that consists of the element end forces are used for obtaining the nodal displacement to renew the nodal positions. The concept and the flow are based on the theory^{3, 4}.

2.1 Equilibrium equation of membrane structure and the tangent stiffness equation

When a triangular element in the total structure shown in Figure 1 is positioned in the universal coordinate as shown in Figure 2, the nodal forces of $\{U_1 \ U_2 \ U_3\}^T$ balance with the element end forces,

$$\begin{cases} \mathbf{U}_1 \\ \mathbf{U}_2 \\ \mathbf{U}_3 \end{cases} - \begin{bmatrix} \mathbf{0} & \mathbf{\alpha}_2 & -\mathbf{\alpha}_3 \\ -\mathbf{\alpha}_1 & \mathbf{0} & \mathbf{\alpha}_3 \\ \mathbf{\alpha}_1 & -\mathbf{\alpha}_2 & \mathbf{0} \end{cases} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \mathbf{0}, \qquad \begin{cases} \mathbf{U}_1 \\ \mathbf{U}_2 \\ \mathbf{U}_3 \end{bmatrix} - \mathbf{\alpha}_e \mathbf{P}_e = \mathbf{0}, \qquad (1a, b)$$

where α_1 , α_2 and α_3 are the direction cosine vectors of the three sides.

When the membrane element is replaced with line-member frame, the frame is divided into the main frame and the sub-frame, as shown in Figure 1. The main frame makes the element shape, and the sub-frame consists of the line-member connected the three apexes and the subnode. The element end force \mathbf{P}_e in Equation (1b) is the sum of the end forces \mathbf{P}_m and \mathbf{P}_s in the two frames,

$$\mathbf{P}_e = \mathbf{P}_m + \mathbf{P}_s \,. \tag{2}$$

The end forces in each frame are balanced with the axial forces of the line-members. The axial forces are obtained from the deformations of the line-members, and the axial deformations are derived from the element deformation. The element deformation is expressed by the nodal positions with geometric strictness, as follows,

$$\Delta \mathbf{l} = \begin{cases} \Delta l_1 \\ \Delta l_2 \\ \Delta l_3 \end{cases} = \begin{cases} \sqrt{(\mathbf{u}_3 - \mathbf{u}_2) \bullet (\mathbf{u}_3 - \mathbf{u}_2)} - l_{10} \\ \sqrt{(\mathbf{u}_1 - \mathbf{u}_3) \bullet (\mathbf{u}_1 - \mathbf{u}_3)} - l_{20} \\ \sqrt{(\mathbf{u}_2 - \mathbf{u}_1) \bullet (\mathbf{u}_2 - \mathbf{u}_1)} - l_{30} \end{cases},$$
(3)

where l_{n0} is each length of the three sides in the non-stress element, and the dot is the scalar product.

Since Equation (3) is the elongation of the side length, the axial forces of the main members are directly obtained from the elongations. Therefore, the main frame uses the strict compatibility of Equation (3). On the other hand, the axial forces in the sub-frame are derived from the imposed displacements at the apexes, Δx_2 , Δx_3 and Δy_3 , that are transformed from the elongation of Equation (3) to with the linear compatibility, as shown in Figure 1.

When the sum of the second term in Equation (1) of all elements equals to the force vector U working at all nodes, the nodal positions are the solution. When the present positions of the nodes are not correct, the unbalanced force vector ΔU exists, as follows,

$$\mathbf{U} - \sum_{e} \boldsymbol{\alpha}_{e} \mathbf{P}_{e} = \Delta \mathbf{U} \,. \tag{4}$$

The unbalanced forces decrease by renewing the nodal positions with the tangent stiffness equation. Differentiating Equation (1) gives the tangent stiffness equation,

$$\begin{cases} \delta \mathbf{U}_{1} \\ \delta \mathbf{U}_{2} \\ \delta \mathbf{U}_{3} \end{cases} = \begin{bmatrix} \mathbf{0} & \delta \mathbf{\alpha}_{2} & -\delta \mathbf{\alpha}_{3} \\ -\delta \mathbf{\alpha}_{1} & \mathbf{0} & \delta \mathbf{\alpha}_{3} \\ \delta \mathbf{\alpha}_{1} & -\delta \mathbf{\alpha}_{2} & \mathbf{0} \end{bmatrix} \begin{cases} P_{1} \\ P_{2} \\ P_{3} \end{cases} + \mathbf{\alpha}_{e} \delta \mathbf{P}_{e} \\ P_{3} \end{cases}$$
$$= \begin{cases} \begin{bmatrix} \mathbf{k}_{G2} + \mathbf{k}_{G3} & -\mathbf{k}_{G3} & -\mathbf{k}_{G2} \\ \mathbf{k}_{G3} + \mathbf{k}_{G1} & -\mathbf{k}_{G1} \\ \mathbf{sym.} & \mathbf{k}_{G1} + \mathbf{k}_{G2} \end{bmatrix} + \mathbf{\alpha}_{e} \mathbf{k}_{e} \mathbf{\alpha}_{e}^{\mathrm{T}} \end{cases} \begin{cases} \delta \mathbf{u}_{1} \\ \delta \mathbf{u}_{2} \\ \delta \mathbf{u}_{3} \end{cases}. \tag{5}$$

where, \mathbf{k}_{Gn} is the stiffness produced from infinitesimal change of the direction of the element end force P_n , and it is,

$$\mathbf{k}_{\mathrm{G}n} = P_n(\mathbf{e} - \boldsymbol{\alpha}_n \boldsymbol{\alpha}_n^{\mathrm{T}}) / l_n, \qquad (6)$$

where **e** is the unit matrix of 3x3. \mathbf{k}_e in Equation (5) is the sum of the stiffness of the two frames,

$$\delta \mathbf{P}_e = (\mathbf{k}_m + \mathbf{k}_s) \delta \mathbf{l} = \mathbf{k}_e \delta \mathbf{l} \,. \tag{7}$$

 $\{\delta \mathbf{u}_1 \ \delta \mathbf{u}_2 \ \delta \mathbf{u}_3\}^{\mathrm{T}}$ in Equation (5) is the infinitesimal change of the nodal position, that is, the infinitesimal displacement of the nodes. The tangent stiffness equation of the total structure is given by summing Equation (5) of all elements. When $\Delta \mathbf{U}$ in Equation (4) is used for $\delta \mathbf{U}$ in Equation (5), solving the tangent stiffness equation gives the nodal displacements to renew the nodal positions.

2.2 Theory of replacing orthotropic element with line-members

The method replaces a triangular element in uniform strain with which of the frames composed by line-members in Figure 3. The frame structure and the member stiffness are determined by equalizing to the stiffness matrix of the triangular element.

An orthotropic element has the following stiffness equation expressed by the relation between the element end forces and the elongations of the element sides.

$$\begin{cases} P_1 \\ P_2 \\ P_3 \end{cases} = \mathbf{a}^{\mathrm{T}} \boldsymbol{\omega}^{\mathrm{T}} \mathbf{E} \, \boldsymbol{\omega} \, \mathbf{a} \, \Delta \mathbf{l} = \begin{bmatrix} K_{11} & K_{12} & K_{31} \\ K_{22} & K_{23} \\ \text{sym.} & K_{33} \end{bmatrix} \begin{bmatrix} \Delta l_1 \\ \Delta l_2 \\ \Delta l_3 \end{bmatrix},$$
(8)

where,

$$\mathbf{E} = \begin{bmatrix} E_{11} & E_{12} & 0 \\ E_{22} & 0 \\ \text{sym.} & E_{33} \end{bmatrix}, \quad \mathbf{a} = \frac{1}{2A} \begin{bmatrix} 0 & 0 & a_{30} \\ c_{30}l_{10} / a_{30} & d_{30}l_{20} / a_{30} & -c_{30}d_{30} / a_{30} \\ -l_{10} & l_{20} & d_{30} - c_{30} \end{bmatrix}, \quad (9),(10)$$

and $\boldsymbol{\omega}$ is the transforming matrix to change the first axis from the orthotropic axis to the

direction of the third side in the element. a_{30} is the length of the perpendicular from the third apex, and c_{30} and d_{30} are the two lengths given by dividing the third side with the perpendicular.

The stiffness matrix of the frame in Figure 3 (1) is equalized into Equation (8). From the nondiagonal elements in Equation (8), the position of the sub-node, that connects the three sub-members, is determined by,

$$h_1 = K_{23} / K_a, \qquad h_2 = K_{31} / K_a, \qquad h_3 = K_{12} / K_a, \qquad (11)$$

where h_1 , h_2 and h_3 are the distances between the position and the three sides, and

$$K_a = K_{23} / a_{10} + K_{31} / a_{20} + K_{12} / a_{30} .$$
⁽¹²⁾

When h_n is plus, the sub-node is positioned on the element side against the side *n*. The type of the frame in Figure 3 complies with plus or minus of each of h_1 , h_2 and h_3 .

The three sub-members have the same stiffness as each other and the stiffness is,

$$k_{s} = 2\left(\frac{K_{23}K_{31}}{K_{12}} + \frac{K_{12}K_{23}}{K_{31}} + \frac{K_{31}K_{12}}{K_{23}} + K_{23}\cos\theta_{10} + K_{31}\cos\theta_{20} + K_{12}\cos\theta_{30}\right)$$
(13)

where θ_{n0} is the interior angle.

The stiffness of the main member is,

$$k_1 = K_{11} - \frac{K_{31}K_{12}}{K_{23}}, \quad k_2 = K_{22} - \frac{K_{12}K_{23}}{K_{31}}, \quad k_3 = K_{33} - \frac{K_{23}K_{31}}{K_{12}}$$
 (14)

The stiffness of the sub-member occasionally takes a minus value, and one of main members occasionally becomes minus.

2.3 Hyperbolic axial line-member

When hyperbolic functions are applied to axial forces of all line-members in a frame, the element replaced with the frame can surely keep compression-free. The hyperbolic function keeps the axial force in either phase of tension or compression, and the function can easily handle the case that a line-member has minus stiffness. When an element is in tense condition, the frame does not consist of only tensile members, but needs compressive members with plus stiffness or minus stiffness on certain occasions. For example, in the type 2 or 3, one or two of the sub-members must be compressive in tense condition of the element. Each phase of the axial forces in tense condition of the element must not change in slack condition, and the hyperbolic function conforms to this. Further, the function can easily express which of the four types of combinations of plus or minus stiffness and tension or compression.

The variable in the hyperbolic function is the deformation Δl that is change of the distance between both ends of the line-member, as shown in Figure 4. The axial force N is,

$$\overline{N} = \frac{1}{2}k\Delta\overline{l} \pm \sqrt{N_h^2 + \frac{k^2\Delta\overline{l}^2}{4}},$$
(15)







Figure 6: Position of the sub-node to the angle between the orthotropic axis and the third side in the equilateral triangle element



Figure 5: Hyperbolic tension with plus stiffness



Figure 7: Change of the stiffness, k_1 and k_s to the direction of the orthotropic axis

where the plus in the double sign uses for tensile member and the minus for compressive member, and

$$\Delta \overline{l} = \Delta l - \Delta l_d, \qquad \overline{N} = N - N_d, \qquad (16), (17)$$

$$\Delta l_d = \frac{N_0^2}{k(k\Delta l_c + N_0)}, \quad N_d = k\Delta l_d, \quad N_h = k\sqrt{\Delta l_c\Delta l_d}$$
(18), (19), (20)

The three parameters of k, Δl_c and N_0 determine the hyperbolic function, as shown in Figure 5 and the equations. k is the elongation stiffness of a line-member given by Equation (13) or (14). Δl_c is the limit deformation of the line-member that the three apexes concentrate into a point. N_0 is the axial force in zero elongation, and the axial forces are balanced with the element end forces that keep the same shape under gravity as the non-stress element.

Differentiating Equation (15) by Δl gives the tangent stiffness of the line-member, and the stiffness is used for Equation (7).



Figure 8: The element used for the comparison and the imposed displacement



Figure 9: Comparison between the three kinds of the elements

3 EXAMPLE OF REPLACEING AN ORTHOTROPIC ELEMENT

Figure 6 and 7 show an example of replacing an orthotropic element in uniform strain with a frame of line-members. The element is an equilateral triangle with the side length of 1m, and it has the following stiffness in Equation (9),

$$E_{11} = 1031.729$$
, $E_{12} = 319.145$, $E_{22} = 935.809$, $E_{33} = 303.876$

where the unit is kN/m.

When the directions of orthotropic axes change in the element, the position of the sub-node changes as shown by the red curved line in Figure 6. The blue lines show the sub-members that the orthotropic axis of the larger stiffness is at an angle of 110 degrees to the third side in the element. When the element is isotropic, the sub-node is at the gravity center, so that the frame forms only the type 1 in Figure 3. On the other hand, the frame equalized to the orthotropic element becomes any of the three types to the direction of the orthotropic axis.

The stiffness k_1 of the main member is always plus regardless of the orthotropic direction, as shown in Figure 7. Since the element is the equilateral triangle, another stiffness of the main member is always plus also. However, the sub-members occasionally have minus stiffness to the direction of the orthotropic axis. Even if the frame considerably changes like this, the equations in Section 2.2 and 2.3 can systematically form the frame and the frame keeps compression-free.

4 MECHANICAL COMPARISON BEWTEEN SEVERAL KINDS OF ELEMENT

The replacement with frames in the paper can give also the line-members bilinear characteristic that decreases the member stiffness in the element in slack condition from that in tense condition, without using hyperbolic function. The frame has similar behavior to the element with the bilinear stress-strain relation that is widely used for membrane structures.

The mechanical quantity of the frame does not strictly equal to that of the element, because the equivalent frame must be exactly formed by Equation (8) that changes on the bilinear stress-strain relation. However, the difference mainly results from the sub-members, and the difference may be small because of very small axial forces in slack condition.

Figure 9 shows comparison between three elements of the frame with bilinear characteristic named bilinear element in the figure, the frame composed by hyperbolic axial members named hyperbolic element and the elastic element in uniform strain. The element used for the comparison is shown in Figure 8, and it is the same as that used in Figure 6. The frame is also the same as that in Figure 6 that the orthotropic axis with the larger stiffness is at the angle of 110 degrees with the third side in the element. The end force of V_3 in each of the three elements is obtained from giving the imposed displacement at the second apex and the third one equally $\Delta u_2 = \Delta v_3$, and the result is Figure 9. In the bilinear element, the ratio of the stiffness in slack condition and that in tense one is 1/100.

When the imposed displacement is positive that the elements are tense, the force of the hyperbolic element becomes gradually equal to the others together with increasing the displacement. When the displacement is 3mm, the three forces almost equally become 2243N. On the other hand, when the imposed displacement is minus that the elements are slack, the three elements are distinct from each other. The hyperbolic element keeps the force tension, and the force in the bilinear element is compression of 1/100 of that in the uniform strain element. Though the difference between both of the frame elements is very small, the different phase between tension and compression affects computing large deformation of a membrane structure.

5 COMPUTATIONAL EXAMPLE OF MEMBRANE STRUCTURE

A computational example is to analyze behavior during lifting a suspended membrane that both of slack area and tense one intermingled appear and the areas change with the deformation by gradual lift.

The two sides of the rectangular membrane are fixed and the primary shape is shown in Figure 10. The weight per unit area is 9.8N/m² and the stiffness in Equation (9) with the unit of kN/m is,

$$E_{11} = 1050, \quad E_{12} = 420, \quad E_{22} = 1050, \quad E_{33} = 315,$$

The membrane model is formed from each of the hyperbolic element, the bilinear element with the stiffness ratio of 1/100 and the element of the frame composed by elastic catenary cables presented in the last conference, that is named cable element as shown in Figure 11 and 12. The model that uses the hyperbolic element or the bilinear element consists of 800 elements of only the type 2 in Figure 3. The model with the cable elements consists of 410 elements of the type 1 and 2. In the latter case, the elements of the type 2 are restrictedly used for elements with fixed sides, because deformation of the elements must be small, and this is a fault in the previous method. The previous method applied the strict compatibility to the submembers as well as the main members. However, when the compression-free element is in a extremely slack condition, the sub-members in the type 2 or 3 become so unstable that an



Figure 12: Deformation of the suspended membrane by lifting

equilibrium after deformation is unsettled. The sub-node in the types under such slack condition that the three apexes approach each other is quite movable by very small changes of axial forces of tension and compression that are also small values. Therefore, the modified method in the paper applies the linear compatibility to only the sub-members. The use of the linear compatibility may not so influence the analysis because the axial forces are very small in slack condition and the element deformation is small in tense condition.

Figure 11 shows the relation between the vertical position of the nodes imposed the displacement every 1m and the sum of two forces to lift the two nodes. The force in the model composed by the bilinear elements is largest in that of the three elements and the difference from the other two of the compression-free models is remarkable. Further, we stopped the computation with the bilinear element when the vertical position of the nodes reaches -1m because the computation needed more than 6000 times of the iteration. The cause of requiring

so many times of iteration is that slack areas gradually changed with the lift and that the analysis became a multi equilibrium problem, as clearly comprehensible from comparing Figure 12 (2) with (1). The axial forces in the bilinear elements in slack condition are mainly compression though very small. The computation first takes the Dynamic Relaxation Method and then the Newton Raphson Method, named DRM and NRM from now on respectively. However, the obtained equilibrium solution that satisfies the allowable unbalanced force of 10^{-4} N does not clearly indicate the solution with the minimum energy. When a membrane structure is flexible and the deformation includes both of slack area and tense one, the other two of the compression-free elements are adequate to the analysis.

Figure 12 (3) and (4) are obtained from the two of the compression-free elements. Since the model of the hyperbolic element uses more elements than that of the cable element, the form of (3) is more correct as the two edges connecting the lifted nodes and the corners in the membrane appear in Figure 12 (3).

The case of the cable element can obtain the solution by using only NRM and the iteration of less than 50 times in the displacement of 1m reaches the solution on the allowable unbalanced force of 10^{-4} N. On the other hand, the computation by the hyperbolic element takes also DRM together with NRM as well as the bilinear element. The total times of the iteration is less than 500 times. The hyperbolic element is compression-free and an equilibrium solution is always unique, but NRM cannot obtain the solution. The causes are the use of different compatibilities in an element and the application of the hyperbolic function to the axial force in the line-member. The latter cause is comprehensible from a value of an allowable unbalanced force in DRM to change from DRM to NRM. The example takes the value of 0.1N in the bilinear element and 10N in the hyperbolic element. When the unbalanced forces in all nodes become less than the value, the computation changes from DRM to NRM and then the computation continues until the unbalanced forces become less than 10^{-4} N. The threshold value that can use NRM depends on the function of the axial force. particularly the maximum curvature in the function. The maximum curvature in the bilinear element is infinite as the stiffness is not continuous, and the next is that of the hyperbolic element, and then that of the elastic catenary element is smallest.

6 CONCLUSIONS

- Membrane structures can be replaced with frames composed line-members that the axial forces are expressed by hyperbolic functions, and the structural models become completely compression-free.
- The proposed method with frames solves equilibrium equations at nodes on the geometrically nonlinear condition and finite deformation of element. The solution satisfies the equilibrium condition under the allowable unbalanced force.
- The structure of the frame with the stiffness equalized to a uniform strain element is classified into three types, and the type changes by a direction of an orthotropic axis in a shape of an orthotropic element.
- A line-member keeps a phase of the axial force by applying the hyperbolic function that is classified into 4 combinations of positive stiffness or minus one and tension or

compression.

The compression-free model of a membrane structure replaced with the frames give the equilibrium solution with the minimum energy by applying the Newton Raphson method together with the dynamic relaxation method. The proposed method is adequate for analyzing large deformation of flexible membrane structures.

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FORM-FINDING OF EXTENSIVE TENSEGRITY USING TRUSS ELEMENTS AND AXIAL FORCE LINES

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Summary. Tensegrity structure, which consists of cables and struts, are expected to be used as systems for cosmological, foldable and/or inflatable structures. The equilibrium shape of the tensegrity can be determined by iteration of solving the tangent stiffness equation. Here, it is rational to use the truss elements for struts and the axial force line elements for cables. In this study, a way to find the shapes of "extensive tensegrity", which counts their self-weight and permits support conditions of statically indeterminate. As results of numerical examples, even the case where many solutions exist under the same loading conditions like the tower tensegrity, expected one equilibrium solution can be obtained, and its equilibrium path can be drawn.

1 INTRODUCTION

Tensegrity structures have very unique morphology that is formed by continuous tension and discontinuous compression, and so many researchers have been tried to determine their shapes. Force Density Method [1] (FEM) is one of the form-finding method for tensegrity structures used most frequently. The method gives equilibrium solutions by a linear stiffness equation without any iteration and is useful, for example, for form-finding of cable net structures under constant external forces and stable support conditions. However, we have to choose suitable force density ratio between every element to find smooth and proportionate shape with unified element size. Furthermore, in order to get the spatial shapes of the "pure" tensegrities, which is in state of self-equilibrium without self-weight and external forces, FDM needs to determine the feasible sets of force density by non-linear analysis before solving the linear stiffness equation. Since Vassart and Motro[2], some procedures to find the feasible sets of pure tensegrities have been proposed[3]-[5].

On the other hand, the authors have developed the measure potential which produces the elements with virtual stiffness, and have applied to the form-finding of the cable net structures, membrane pneumatic structures and tensegrity structures [6]-[8]. The measure potential can be defined freely as a function of "element area" or "element length", so if we define the potential of a triangular element as is proportionate to its area, the element behaves as soap film and the form-finding of an isotonic surface realizes. Moreover, if we define the potential of a line element as is proportionate to (n+1)-th power of its length, its axial force proportionate to *n*-th power of its length and we call it "*n*-th axial force line element". Especially, when n=1, the stiffness equation becomes linear and the process of computation becomes quite equal to FDM. (In that meaning, our idea for this potential may be close to Miki and Kawaguchi's one.) However, *n* that magnitude is bigger than 2, gives the solutions more regulated element length and brings the performance of the form-finding better. Our recent paper[8] has tried to apply the measure potential to the pure tensegrities, and here we used the axial force line elements with n=2 for cables and the rigid bars for the struts. NR method by iteration of solving the tangent stiffness equation converges surely and perfect equilibrium solutions can be obtained.



Figure 1 Snelson's needle tower

Also in this study, elements for struts are modified to truss element with real material stiffness in other to get better convergence. Moreover, this modification made it easy to apply the form-finding analysis to extensive tensegrity structures, which allow external forces and connection between struts.

In this study, some numerical examples of form-finding for tower tensegrities just like Snelson's needle tower (Figure 1) are shown. This type of tensegrity has self-weight and requires stable support conditions, therefore spatial forms may be obtained even if FDM uses any value for force densities. However, a tensegrity has many equilibrium shapes corresponding to one condition of connectivity and loading, and then it is difficult to obtain an expected solution such as Figure 1 that the modules with uniform geometry are lined up in vertical direction in good order. This study shows that the combination of the loading control and the displacement control is effective to find the equilibrium of self-reliance with its selfweight. Letting top nodes of control points displace up compulsorily and searching where the control points have no reaction forces gives us a solution with tower geometry.

Furthermore, in this study, another numerical example is shown. As mentioned above, tower tensegrity have so many solutions for a condition, so the searching equilibrium paths attracts us and that gives us a lot of information to make clear the character of tensegrity. As a result of computation, five main paths, in which the shape deforms keeping symmetry, have found and they are independent each other.

2 FORM-FINDING BY THE TANGENT STIFFNESS METHOD

2.1 Tangent stiffness equation

Let the vector of the element edge forces independent of each other be indicated by S, and let the matrix of equilibrium which relates S to the general coordinate system by J. Then the nodal forces U expressed in the general coordinate follow the equation:

$$\mathbf{U} = \mathbf{J}\mathbf{S} \tag{1}$$

The tangent stiffness equation is expressed as the deferential calculus of Eq. (1),

$$\delta \mathbf{U} = \mathbf{J} \delta \mathbf{S} + \delta \mathbf{J} \mathbf{S} = (\mathbf{K}_0 + \mathbf{K}_G) \delta \mathbf{u}$$
⁽²⁾

In which, K_0 is the element stiffness which provide the element behaviour in element (local) coordinate, and K_G is the tangent geometrical stiffness. δu is nodal displacement vector in general coordinate.

2.2 Element potential function

In order to regulate the element behavior in element (local) coordinate, we define the element measure potential, which is expressed as the function of measurement such as element length or element area. Defining element measure potential is equal to assuming the "virtual" elemental stiffness. Moreover, it has no relationship with material's stiffness.

Let element measure potential is P, and let the vector of elements' measurements whose component is independent of each other is s

$$\mathbf{S} = \frac{\partial P}{\partial \mathbf{s}} \tag{3}$$

Then we can get the element edge force S.

2.3 Axial force line element

The line element is connected with nodal point 1 and nodal point 2. Supposing that the element measure potential is proportional to the power of length of line element, the element measure potential can be expressed as:

$$P = Cl^{n+1} \tag{4}$$

The axial line element force can be obtained by differential calculus of element measure potential:

$$N = nCl^n \tag{5}$$

where, C is the coefficient to be able to set freely.

Let α are the components of cosine vector of an axial force line element, which connects node 1 and node 2, and we can rewrite the Eq. (1) as:

$$\begin{bmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \end{bmatrix} = \begin{bmatrix} -\boldsymbol{\alpha} \\ \boldsymbol{\alpha} \end{bmatrix} N \tag{6}$$

(7)

Substituting the Eq. (6) to the above Eq. (2), and make it matrix.

$$\delta \begin{bmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \end{bmatrix} = \mathbf{K}_{\mathbf{T}}^{\mathbf{L}} \delta \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \end{bmatrix}$$
(7)

$$\mathbf{K}_{\mathbf{T}}^{\mathbf{L}} = nCl^{n-2} \begin{bmatrix} \mathbf{e} + (n-2)\boldsymbol{\alpha}\boldsymbol{\alpha}^{\mathrm{T}} & -\mathbf{e} - (n-2)\boldsymbol{\alpha}\boldsymbol{\alpha}^{\mathrm{T}} \\ -\mathbf{e} - (n-2)\boldsymbol{\alpha}\boldsymbol{\alpha}^{\mathrm{T}} & \mathbf{e} + (n-2)\boldsymbol{\alpha}\boldsymbol{\alpha}^{\mathrm{T}} \end{bmatrix}$$
(8)

For the Eq. (5), in the case of n=2, the element forces become constant, and for the Eq. (7), the tangent geometrical stiffness of line element becomes the same form as truss element's. Therefore, the axial forces can be designated as a constant value.

In addition, in the case of n=2, axial force is proportional to the length of line element, and Eq. (7) is linear. However, in the case of n>2, iterative steps are required because of nonlinearity. The magnitude of *n* become larger, the length of all line elements on the solution surface tend to be more uniform [6],[7].

2.4 Truss element with real stiffness for struts

In the Ref.[8], rigid bares are used for struts, but it seems that they causes the convergence worse. However, it became evident that the rigid-bars bring the aggravation of convergence, because of the non-linearity of the degeneration matrix.

Therefore, the ordinary truss members are applied to struts in this study, and the convergence property was improved dramatically instead of sacrificing just one degree of freedom for an element. Namely, when a huge value was applied to Young's modules, the member behaves like a rigid-bar. Referentially, the element force equation and the tangent stiffness equation of a truss member are shown in Eq.(9) and (10), respectively.

$$N = \frac{EA}{l_0} \Delta l \tag{9}$$

$$\delta\begin{bmatrix}\mathbf{U}_1\\\mathbf{U}_2\end{bmatrix} = \left\{\frac{EA}{l_0}\begin{bmatrix}\mathbf{a}\mathbf{a}^{\mathsf{T}} & -\mathbf{a}\mathbf{a}^{\mathsf{T}}\\-\mathbf{a}\mathbf{a}^{\mathsf{T}} & \mathbf{a}\mathbf{a}^{\mathsf{T}}\end{bmatrix} + \frac{N}{l}\begin{bmatrix}\mathbf{e}-\mathbf{a}\mathbf{a}^{\mathsf{T}} & -\mathbf{e}+\mathbf{a}\mathbf{a}^{\mathsf{T}}\\-\mathbf{e}+\mathbf{a}\mathbf{a}^{\mathsf{T}} & \mathbf{e}-\mathbf{a}\mathbf{a}^{\mathsf{T}}\end{bmatrix}\right\}\delta\begin{bmatrix}\mathbf{u}_1\\\mathbf{u}_2\end{bmatrix}$$
(10)

where, *EA* is elongation rigidity, and l_0 is non stressed length of the member.

3 COMPUTATIONAL EXAMPLES

3.1 Comparison of element application for struts

The example compares two models of element application for struts, one is to use rigid bodies and the other is to use truss. Both models have no external force, so the solutions should be in self-equilibrium state. Figure 2 is the primary shape consists of 64 axial force line elements and 9 rigid bodies with four-nodes. Here, the coefficient and the power in Eq.(5) are designated as c=2 and n=2, respectively. As a result, the equilibrium shown in Figure 3 can be obtained. Figure 4 is the primary shape where the tetrahedral truss-units are placed instead of the rigid bodies of previous model. The equilibrium shown in Figure 5 can be obtained. Comparing these two models, all the nodes are located at almost same position and equilibrium shapes are evaluated as equal.

Figure 6 is comparison of the convergent process of maximum unbalanced force. When the rigid body is applied to struts, the convergence process is gradual as the unbalanced force reduces to half in an incremental step. On the other hand, the truss units make the convergence process accelerate, and the uniqueness of the tangent stiffness method that the unbalanced force converges suddenly and rapidly can be recognized.



Figure 6 Comparison of convergent process

3.2 Tower shape with gravity

The model with five units layered has the same connectivity as the needle tower by Snelson, as shown in Figure 1. Figure 7 is the primary shape in which (a) is top view and (b) is side view. Where, non-stressed length of the struts is 1m respectively. The initial idea was to obtain the objective equilibrium solution with self-weight via a solution of self-equilibrium without any nodal force (Figure 8 (a)). Figure 8 (b) is a solutions with 0.1kN of self-weight for each node under the condition of n=5, C=1.5 in eq.(5), but in almost cases of n and C, similar solution will be obtained. They are different from the expected shape of tower.

The second idea is to once displace the top nodes of the tower compulsorily up to a height corresponding to the expected shape. Then displacing them gradually down as to the reaction forces of the control points become zero, we can get the solution of self-reliance with its self-weight. Figure 9 (a) is the shape with the displacement of 5m which is 5 times for the length of a strut. Here, eq. (5) is set by n=5, C=1.5. After that, 0.2m of compulsory displacement is added to the equilibrium of Figure 9 (a) step by step, and the reaction forces have changed from negative to positive at 5th step. Then releasing the restriction of the control points gives the perfect equilibrium shape of self-reliance with its self-weight as shown in Figure 9 (b). However, depending on the values of n and C, the expected solution may not be obtained. Figure 10 shows an example in case of n=5, C=1.5, but this is also a perfect equilibrium solution.



3.3 Path finding of tower tensegriy

It was suggested in Chapter 3.2 that tower shape tensegrity have a variety of equilibrium corresponding to one connectivity. Therefore, it is attractive for us to search the equilibrium path under the self-weight of struts. The model is a tower with two units layered, in order to observe the transition of the equilibrium shape with partial buckling. Figure.11 is the primary shape in which (a) is top view and (b) is side view. Five nodes of lower unit is fixed. The coefficient and the power in Eq.(5) are designated as c=2 and n=2, respectively. Moreover, nodal self-weight is 0.1[kN] and elongation rigidity is $2.0 \times 10^{9}[kN]$, and five node The analysis proceeds as follows;

- 1) Initial load is triggerd on five nodes of upper unit that are control points. Obtained solution be the primary solution of path finding. Depending on the magnitude of initial load deffernt pathes can be found. (Only for the path in Figure 16, the primary solution is obtained by 'snap through' during searching the path expressed by Figure 15.)
- 2) Path is drawn by incremental analysis with combination of the load control and the displacment control. The control is swiched by the current tangent of the path.

As the results of computation, five paths independent each other (Figure 12-16) have found and the all the solutions on these paths have symmetric shape. On Figure 17, all the paths found by the analysis are gathered on a coordinate, the point where the path crosses the horizontal axis is the solution of self-reliance with its self-weight. Therefore, thirteen of selfreliance solutions have been found by the analysis.

However, all the paths and all the solutions may not provided by this analysis. They are "some of all". Eigenvalue analysis of tangent stiffness matrix may be necessary to discover this equilibrium system completely.



Figure 11 Primary shape and connectivity



Figure 12 Path from origin of the solution by 2.4[kN] of initial load



Figure 13 Path from origin of the solution by 0.0[kN] of initial load


Figure 14 Path from origin of the solution by 1.45[kN] of initial load



Figure 15 Path from origin of the solution by 2.7[kN] of initial load



Figure 16 Path found by snap through from another path in Figure 15





4 CONCLUSIONS

Tensegrity structure has so many morphologies corresponding to one formation of connectivity; therefore, it may be difficult to find an expected shape even if using FDM. On the proposed procedure, the form-finding process is to get the equilibrium solutions of the virtual structure, which consists of the combination of axial force line elements and truss members. Using this virtual structure, the displacement and the length of the struts can be designated freely and it becomes easier to control the equilibrium shapes.

Furthermore, the path finding analysis gives us so many self-reliance solutions of tower tensegrity, and obtained paths bring us interesting information about the complicate equilibrium behavior.

Consequently, the proposed procedure is expected to be a reasonable form-finding process for various types of extensive tensegrity structures including cosmological, foldable and /or inflatable structures.

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ANALYSIS OF THERMAL EVOLUTION IN TEXTILE FABRICS USING ADVANCED MICROSTRUCTURE SIMULATION TECHNIQUES

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Abstract. Nowadays, membrane structures represent a modern construction element to be used as roof material in modern buildings or as design element in combination with traditional architecture. Membranes are mostly used in an outdoor environment. Therefore they are exposed to wind, radiation (solar and infrared), rain and snow. Specific membranes are three-dimensional fabrics which can be used as energy absorber or as insulation of membrane roofs. The applicability as energy absorber becomes important if the three-dimensional fabrics are designed as a porous flow channel streamed by air and convectively heated up. The transferred energy may be stored in a latent heat storage system.

Due to their porous structure, textile fabrics have a large heat-exchanging surface. If they are handled as homogenized porous structures, the heat transfer processes can not be described in a correct way. Therefore a microstructure model locally resolving all filaments of the three-dimensional fabrics has been formulated. By using an advanced meshing tool, a simulation technique has been developed taking into account the local heat conduction properties of the different materials.

To analyse the heat transfer processes inside the three-dimensional fabrics, numerical simulations have been performed using the phase-field solver (Pace3D) of the Karlsruhe Institute of Technology and the commercial CFD-Solver StarCCM+. For a better understanding of the thermal behaviour of the fabrics, different thermal loads including thermal conduction in the microstructure (filaments) and convection by the surrounding air have

been computed. The results show that the advanced simulation techniques allow to analyse the rate of conductive and convective heat transfer in three-dimensional fabrics. The results of the applied computational methods are compared.

1 INTRODUCTION

As a part of the polar bear project, a textile pilot building is constructed. The textile structure is used to generate heat and copy the fur of the polar bear [3]. The use of textile structures as a thermal collector offers many advantages. Textile structures are flexible and can be used as a self-supporting roof or on rooftops. Here, in addition to generating heat, the insulation is important. In the future textile structures will be used on roofs with an area ranging from several up to 10000 m^2 .



Figure 1: Design and principle of a thermal collector with membrane structures.

The operating principle is shown in Figure 1. To have a well insulating textile fabric that can be used as a thermal heat collector, it must have the following characteristics:

- 1. Solar radiation should be transmitted.
- 2. Heat radiation from the roof and the heated structure should be reflected.
- 3. The textile fabric should have a low thermal conductivity.
- 4. Flowing through air should absorb heat from the textile fabric.

The solution is a construction of three different textile structures. Figure 2 shows the structure. Layer 2 and 3 transmit solar radiation and reflect heat radiation. Layer 1



Figure 2: Thermal collector composed of 3 layers of textile spacer fabrics.

absorbs solar radiation and air flows through. All three layers act as insulation. In the following property 3 and 4 are considered in more detail.

To calculate the physical processes in the textile, a detailled model resolving the structure is needed. The complete structure is required for the calculation of heat conduction, the surface for convective heat transfer. The textile membranes are used industrially on a large scale. It is not possible to calculate large areas of textile structures with resolution of the finest fibers with today's computing capacity. Therefore, it is necessary to reproduce small units of the textile structure with all the fibres in a representative volume. Using these models the physical processes are calculated and we obtain, for example, a temperature profile or a pressure profile within the structure. Thus it is possible to better understand the processes in the textile structure and to calculate effective material parameters and dimensional parameters for the modeled structure unit. Effective material parameters and dimensionless parameters allow a calculation of large areas, because instead of an exact resolution of the structure, effective material parameters are used.

The following sections explain the pre-processing, calculation, evaluation and determination of effective material parameters for the textile structure in Figure 2.

2 PREPROCESSING

The model generation is described for layer 1 in Figure 3. In a first step, the structure is analysed. The structure consists of three different textile fiber elements, which recur periodically within the tissue. The geometric dimensions of the structure and the individual structural elements are required for the reconstruction.

In a second step the individual structural elements are replicated. Therefore the program Pace3D was used. Pace3D stands for Parallel Algorithm in Crystal evolution in 3D. The software package was developed for materials science applications in order to



Figure 3: Textile spacer fabrics at high resolution.

computationally design new materials with tailored properties and to get insight in microstructure formation processes. The mathematical and physical basis is a phase-field model [2]. Within the simulation program, a volume is defined which is discretized with hexahedrons. Each hexahedron is assigned a phase, the structure gets a solid phase and the surrounding air a gaseous phase. To model the structure area, the preprocessing of



Figure 4: Design elements to construct layer 1.

the software provides interfaces to define functional expressions to describe the three-

dimensional geometry. In the case of the deck fibre, it is a sine function

$$F1(z) = amp \cdot \sin\left(z \cdot \frac{2\pi}{\lambda}\right)$$
 (1)

The parameters were taken from the geometric dimensions. The fiber was then multiplied to get the deck structure.

When creating the second structure type, the intermediate fibers, the procedure was comparable. They were also modeled with a sine function. The duplication of the fiber is based on the sine function in equation (1). The twisted intermediate fibers form the third type of fiber. They have the special feature keeping overlying deck fibres seperate and connecting diagonally above the other deck fiber. Therefore, two functions overlap, a linear shift in x-direction and a sine-function in z-direction. Figure 4 shows the three structural elements.

In a third step, the structural elements are combined and superimposed. The result is the overall structure in Figure 5a. Layer 2 and 3 were created similarly. They are shown



Figure 5: Modeled layers of the thermal collector.

in Figure 5b and 5c.

By formulating suitable mathematical expressions, a twisted deck fiber in Figure 6 can be generated. It is composed of four single fibers twisted together and shaped in the form of a sine curve. In the further refinement of the structure it has to be estimated, whether this improves the accuracy of the calculation or just increases the computational cost and complexity.

3 CALCULATED LOADCASES

Based on the structural model in Figure 5a different calculations were performed. In addition to Pace3D, StarCCM+ was used for calculation. StarCCM+ is a commercial Computational Fluid Dynamics (CFD) code. There were three aims:

• Analysis of physical processes in the structures



Figure 6: Twisted deck structure fiber.

- Comparing the results of Pace3D and StarCCM+
- Calculation of effective material parameters

For this purpose two load cases LC1 and LC2 were defined. The textile fabric is made of polyamide surrounded by air. The used material parameters are given in Table 1 and are taken from the VDI-Wärmeatlas [1].

Material	Polyamide	Air	
	$T = 323.15 \ K$	$T = 333.15 \ K$	
Density $\rho \left[\frac{kg}{m^3}\right]$	$1150.0_{(T=293.15\ K)}$	1.046	
Heat capacity $c_p \left[\frac{J}{kg \cdot K}\right]$	1680.0	1008.2	
Heat conductivity $\lambda \left[\frac{W}{m \cdot K}\right]$	$330.0 \cdot 10^{-3}$	$28.8\cdot10^{-3}$	
Dynamic viscosity $\nu \left[\frac{m^2}{s}\right]$	-	$192.2 \cdot 10^{-7}$	

Table 1: Material values for LC1 und LC2.

3.1 Load case LC1

We calculated the steady-state heat conduction through the textile structure. For the calculation, the smallest periodic unit (unit cell) of the textile structure was considered and model-appropriate temperature conditions imposed. Figure 7a shows the selected boundary conditions. For the calculation with Pace3D, also a base plate was added (Figure 7c). The base plate is made of polyamide and has the height $\Delta y = 1 \text{ mm}$. Using this base plate it is possible to calculate the heat flow through the structure. For the calculation with StarCCM+ the construction of a base plate is not necessary (Figure 7b).

Due to the non-stationary solution scheme of Pace3D, the simulation has to run for a long physical time until a steady-state temperature profile has formed over the structure.



Figure 7: Boundary conditions and simulation environment (LC1).

This is reached after about 100 s. In addition, the initial conditions were chosen so, that they are close to a steady-state temperature profile. For the textile structure, we chose $T_{t=0,structure} = 333.15 K$ and for the base plate $T_{t=0,plate} = 373.15 K$. StarCCM+ calculates at steady-state conditions and implicitly, therefore the required computing time is lower.

3.2 Load case LC2

For load case LC 2, four unit cells like in Figure 7b were joined together. This structural unit is flown through by air in a channel and heated from below with a temperature $T_{yz,x=0} = 373.15 \ K$. The structure and air are heated. The convective and conductive heat transfer and pressure loss are considered. Figure 8 shows the geometry and boundary conditions. The channel has a length of $L_{channel} = 210 \ mm$. The structure is passed through in the direction of the x-axis with a speed of $u = 1.0 \ ms m^2$. The temperature of the fluid at the inlet is $T_{inlet} = 293.15 \ K$. To estimate the Reynolds number, a channel cross-section between two intermediate fibers of $d = 1.75 \ mm$ and an increased flow velocity of $u_{max} = 2.0 \ ms m^2$ is assumed. It results in a Reynolds number of

$$Re = \frac{u_{max} \cdot d}{\nu} = 182.1 . \tag{2}$$

The flow has established steady-state and remains at laminar conditions.

4 RESULTS

The LC1 for the layer 1 was calculated with Pace3D and StarCCM+. The results illustrate the temperature layers in Figure 9a for stationary state. Figure 9b shows the



Figure 8: Geometry and boundary conditions (LC2).



Figure 9: Temperature distribution and profile.

temperature distribution over the height of the structure. The diagram was obtained by averaging the temperature over the xz-plane.

The heat flow through the structure is calculated with StarCCM+ as $\dot{Q} = 60.2 \cdot 10^{-3} W$. For Pace3D, the heat flow can be calculated using the temperature difference over the base plate. The temperature difference is $\Delta T = 0.9 K$. With the corresponding geometric values, we get $\dot{Q} = 56.2 \cdot 10^{-3} W$ using

$$\dot{Q} = \frac{\lambda}{\Delta y} \cdot \Delta T \cdot A \ . \tag{3}$$

LC2 was calculated for layer 1 with StarCCM+. Figure 10 shows the stationary temperature profile for the xy-plane. By the hot plate, heat is applied to the structure and the

Variable	StarCCM+	Pace3D
Area $A = \Delta x \cdot \Delta z \ [m^2]$	$185.2 \cdot 10^{-6}$	$189.3\cdot10^{-6}$
Height $\Delta y_{structure} \ [m]$	$9.9\cdot10^{-3}$	$10.2\cdot 10^{-3}$
Temperature difference $\Delta T_{structure}$ [K]	80	79.4
Heat flow \dot{Q} [W]	$60.2 \cdot 10^{-3}$	$56.2\cdot10^{-3}$

Table 2: Geometric values and results of LC1.



Figure 10: Temperature distribution in the xy-layer (LC2).

air. The air is heated from $T_{inlet} = 293.15 \ K$ to $T_{outlet} = 300.29 \ K$. The supplied heat energy is $\dot{Q} = 1.762 \ W$. The averaged temperature of the structure is $\bar{T}_{structure} = 319.49 \ K$ and of the air between the structure $\bar{T}_{fluid} = 302.43 \ K$.



Figure 11: Pressure distribution and velocity vectors for the xz-layer (LC2).

During flow through the structure, a stagnation point is formed in front of the fibers. Between the fibers, the flow is accelerated due to the cross-section. Figure 11a depicts

the pressure distribution over the structure. The pressure loss during flow through the structure is $\Delta p = 3.7 \ Pa$. Figure 11b displays the velocity vectors in the *xz*-plane for a part of the structure. The velocity vectors reveal the flow around the individual fibers. To reflect these phenomena within the structure, it is necessary to accurately reproduce the structure. The maximum speed discovered in the computational domain is $u_{max} = 2.28 \ \frac{m}{s}$. With this value, our estimation used to calculate the Reynolds number turns out to be correct.

5 DISCUSSION

The results of the temperature versus height y in Figure 9b and obtained by StarCCM+ and Pace3D are almost identical. Position a) in the diagram shows the temperature curve above the base plate. Position b) and c) correspond to the lower and upper deck structure. Here the proportion of polyamide is higher than in the area of the intermediate fibers. The thermal conductivity is greater in these areas, because they have a lower temperature gradient. The proportion of polyamide and the course of the fibers have an influence on the thermal conductivity. For technical applications, the effective thermal conductivity of the entire structure is important. This is given by

$$\lambda_{eff} = \frac{\dot{Q} \cdot \Delta y_{structure}}{\Delta T_{structure} \cdot A} . \tag{4}$$

With the values from Table 2 for the layer 1, the effective thermal conductivities of the two simulation applications are $\lambda_{eff,StarCCM+} = 40.2 \cdot 10^{-3} \frac{W}{K \cdot m}$ and $\lambda_{eff,Pace3D} = 38.1 \cdot 10^{-3} \frac{W}{K \cdot m}$. The deviation of the calculated values is 5.2% with respect to $\lambda_{eff,StarCCM+}$. This small difference and the similar temperature distribution in Figure 9b serve as a validation of the results. LC1 was also used for the layers 2 and 3 with StarCCM+, and

Layer	$\lambda_{eff} \left[\frac{W}{m \cdot K} \right]$	$c_p \left[\frac{J}{kg \cdot K}\right]$	$\rho \left[\frac{kg}{m^3}\right]$
Layer 1	$40.2\cdot10^{-3}$	1048.5	70.0
Layer 2	$33.4\cdot10^{-3}$	1026.5	32.4
Layer 3	$32.8\cdot10^{-3}$	1018.7	19.1
Complete structure	$34.4 \cdot 10^{-3}$	1027.8	34.6

Table 3: Calculated values for the three layers with StarCCM+ (LC1).

the effective thermal conductivity was calculated. Table 3 contains the resulting values. With the effective thermal conductivity of the three layers, it is possible to determine an effective thermal conductivity for the whole structure of $\lambda_{eff,tot} = 34.4 \cdot 10^{-3} \frac{W}{m \cdot K}$, which is 19.4% larger than the thermal conductivity of pure air. The whole structure has good thermal insulating properties similar to air. This value can be used for technical design

of large textile surfaces. In conclusion, the thermal insulation properties can be correctly reproduced without resolution of the exact structures.

Another important feature to evaluate and interpret is the heat transfer from the structure to the fluid. The heat transfer coefficient α specifies the transferred heat power per surface and temperature difference. For textile membranes, there is no valid Nusselt number correlation. With the values of the LC2, the heat transfer coefficient for layer 1 can be estimated to

$$\bar{\alpha} = \frac{\dot{Q}}{A_{structure} \cdot (T_{structure} - T_{fluid})} , \qquad (5)$$

where $A_{structure} = 2.26 \cdot 10^{-3} m^2$ is the interface area between fluid and structure. This results in an averaged value of $\bar{\alpha} = 139.1 \frac{W}{m^2 \cdot K}$. For comparison, the heat transfer coefficient for an overflown, heated plate was calculated to $\bar{\alpha} = 22.0 \frac{W}{m^2 \cdot K}$ by the correspondig Nusselt number correlation of the VDI-Wärmeatlas [1]. The heat transfer in the flow is increased by the structure. The heat is taken away from the structure by the air flowing through, and can be stored for example in a latent heat storage.



Figure 12: Temperature distribution in the textile fiber.

As seen in Figure 10, the heat only penetrates into the flow region at the bottom of the channel. Figure 12 emphasizes this, the fiber is heated only in the lower third. The corresponding Biot number is the heat transfer between fluid and structure in relation to heat conduction in the structure. It can be estimated with $\bar{\alpha}$ as

$$Bi = \frac{\bar{\alpha} \cdot L_{structure}}{\lambda_{structure}} .$$
(6)

The characteristic length of the structure $L_{structure}$ is given by the structure height $\Delta y = 9.85 \cdot 10^{-3} \ m$ and $\lambda_{structure} = 330.0 \cdot 10^{-3} \ \frac{W}{m \cdot K}$ refers to $\lambda_{Polyamide}$. This results in the



Figure 13: Cross-sections of the structure.

dimensionless number Bi = 4.2. A Biot number greater one means that the heat transfer in the flow is greater than the heat conduction in the structure. Therefore the structure is heated marginal. The heat is completely removed at the bottom of the structure by the fluid. In technical applications, the structure is heated by radiation leading to a more uniform heat distribution and to higher heat transfer rates.

The flow through the structure yields not only an improved heat transfer, but also results in increased pressure loss important for applications to fans. To make advantage of this property, the structure was designed, so that, in the direction of flow, channels are formed (Figure 13a). The flow resistance is reduced. The view in Figure 13b shows an almost closed fiber structure for which the resistance for a flow in z-direction increases.

6 CONCLUSION

The microstructure simulation software Pace3D is capable to recreate small units of textile structures. All fibers and surfaces relevant to reflect physical processes have been modelled. With defined load cases, these processes were examined in the textile structures. The stationary heat conduction in three textile structures was simulated with StarCCM+ and Pace3D and an effective thermal conductivity was calculated. It was shown that the textile structures have good thermal insulation properties.

In a second load case, the flow through a textile structure was simulated with Star-CCM+. The convective and conductive heat transfer and pressure loss were calculated. The textile structure develops a higher heat transfer coefficients α . The Biot number and analysis of the temperature distribution in the structure indicate that the heat conduction in the structure is an insufficient heat transfer mechanism. This is because of the low thermal conductivity of the polyamide and the large surface of the structure.

A comparative analysis of stationary heat conduction with Pace3D and StarCCM+ showed almost identical results and served as validation of the two different approaches. Based on the results, predictions can be made about the influence of the particular structure of textile fabrics on heat transfer properties. Effective material parameters can be calculated for the structures starting from this detailed analysis.

For the complete design of thermal collectors, made of textile membranes, a forthcoming research will adress the following issues:

- the definition of additional load cases
 - to study the radiation behaviour of textile structures.
 - to create a pressure correlation for calculating the pressure drop at different operating points and flow lengths.
- validation by experimental data.
- structural optimization considering the required heat and flow characteristics.

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MECHANICS OF LOCAL BUCKLING IN WRAPPING FOLD MEMBRANE

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Key words: Buckling, Wrapping Fold, Solar Sail, Membrane Structure

Abstract. Mechanics of a local buckling, which is induced by wrapping fold of a creased membrane, is discussed experimentally, theoretically, and numerically in this paper to examine the condition for the local buckling. The theoretical analysis is performed by introducing one-dimensional wrapping fold model, and the dominant parameters of the condition for the local buckling are obtained, which are expressed by the tensile force, the membrane thickness, and the radius of the center hub. The experimental results indicate that the interval of the local buckling is proportional to the diameter of the center hub, and the results are qualitatively agreement with the FEM results.

1 INTRODUCTION

There is currently much interested in the use of large deployable space membranes for several lightweight space structures; solar sails[1], large aperture antennas, sunshields, solar power satellites, and et al. As the space membranes are folded and packed in a rocket and deployed in the space, the fold properties affect the deployment dynamics, and hence, the fold is one of the significant technical issues to realize the space membranes. For example, the membrane is desired to be folded simply with compact storage. Also, in the folding process, the damages to the membrane have to be avoided.

One of the folds for the space membrane is wrapping fold[2]. In the course of the wrapping fold of a creased membrane, local buckling is observed repeatedly as shown in Fig.1. This local buckling is induced by the compressive stress in the inner membrane. When the local buckling occurs, the layer thickness of the wrapped membrane is locally increased, and hence, the package volume is increased. Also, when the compressive stress is concentrated by the local buckling, the membrane would be damaged. Thus, it is significant to investigate the mechanics of the local buckling.

In this paper, the mechanics of the local buckling in a wrapping fold membrane is examined experimentally, theoretically, and numerically to determine the condition for the local buckling and the interval of the local buckling. We focus on the wrapping fold around a cylindrical center hub, which is used in IKAROS[1]. At first, the wrapping fold experiments for a creased membrane are performed to indicate the local buckling behavior in terms of the tensile force for wrapping, the diameter of the center hub, and the membrane thickness, where the local buckling is treated with the interval of that. Next, to determine the dominant parameters of the condition for the local buckling, theoretical analysis is performed by introducing a one-dimensional model. Then, the experimental and theoretical results are evaluated by FEM analyses. Finally, the condition for the local buckling and the interval are discussed.



Figure 1: Overview of local buckling

2 WRAPPING FOLD EXPERIMENTS

Figure 2 indicates the cross-section of the wrapping fold membrane. We assume that the cross-section of the wrapping fold is the repeating structure, and hence, the selected area in Fig.2 is modeled for the membrane specimen of the wrapping fold experiments. To evaluate the crease quantitatively, we introduce a layer pitch h, which indicates the thickness per a membrane of the folded thickness.

Figure 3 indicates the experimental setup. As shown in the front view in Fig.3a, the upper end of the membrane specimen is attached to the center hub by Kapton tape. To apply the tensile force to the membrane, the lower end of the membrane specimen is sandwiched by steel brackets, and weights are applied to the steel brackets as shown in Fig.3a,b. On the both sides of the membrane in Fig.3a, the creases are generated, which are cl and cr. These creases cl and cr correspond to the creases indicated in the cross-sectional view, Fig.3c.



Figure 3: Experimental setup

3 ONE-DIMENSIONAL THEORETICAL ANALYSIS

A one-dimensional model for a wrapping fold membrane is introduced, and the theoretical analysis for the model is formulated to examine the mechanics of the local buckling.

3.1 One-dimensional wrapping model

Figure 4a indicates a one-dimensional analytical model of a wrapping fold membrane. In the figure, r and T represent the radius of the cylindrical center hub and the tensile force, respectively. In the model, we treat the wrapped membrane as a one-dimensional model to simplify the mechanics of the wrapping fold. To this end, the cross-section has to be assumed. Fig.5a indicates the cross-section of the wrapping fold membrane of the membrane specimen for the experiments, where t, s, and l_0 represent the membrane thickness, the body fixed system of the membrane, and the overall length of the membrane along the body fixed system. As shown in the figure, the layer pitch around the crease is thicker than that of the other area, we introduce a cylindrical cross-sectional model as shown in Fig.5b, where the radius and the thickness are a and t'. In that case, we have to consider the equivalent in-plane stiffness as,

$$Etl_0 = Et'2\pi a \tag{1}$$

where, E is the Young's modulus.

For the one-dimensional model shown in Fig.4a, we assume that the local buckling occurs in $\cdot b_{n-2}$ and b_{n-1} . As the layer pitch becomes small in the position of the local buckling as shown in Fig.1, the bending moment is small at b_{n-1} , and thus, the bending

moment becomes maximum somewhere between b_{n-1} and A. We define that position as b_n , where the local buckling is about to occur, and l_b corresponds to the interval of the local buckling.

To examine the condition for the local buckling at b_n , we focus on the membrane element between b_{n-1} and b_n as shown in Fig.4b. We assume that the membrane element contacts with the center hub from $\theta = 0$ to θ_c , and the contact area is l_c , where $\theta = 0$ at b_n , and the membrane dose not contact with the center hub in the area δ . The equilibriums of the force in the membrane element in the z-direction and in the x-direction are,

$$T + R_{n-1}\sin\theta - T\cos\theta - \int_0^{\theta_c} q\sin\theta r d\theta = 0$$
⁽²⁾

$$R_n + R_{n-1}\cos\theta + T\sin\theta - \int_0^{\theta_c} q\cos\theta r d\theta = 0$$
(3)

Also, the equilibrium of the moment in the membrane element is,

$$(R_{n-1}\cos\theta + T\sin\theta)r\sin\theta - \int_0^{\theta_c} q\cos\theta r\sin\theta rd\theta - M_n + M_{n-1} = 0$$
(4)

where, q, R_n , R_{n-1} , M_n , and M_{n-1} represent the distributed contact force applied by the center hub, the reaction force at b_n , the reaction force at b_{n-1} , the moment at b_n , and the moment at b_{n-1} , respectively. The distributed contact force q is derived by the equation for a wrapped belt[3] as,

$$q = T/r \tag{5}$$

where, the one-dimensional wrapping by the tensile force T around the cylindrical center hub of r radius are assumed. As the layer pitch is small at b_{n-1} , we assume the layer pitch at b_{n-1} becomes 2t'. In that case, the moment at b_{n-1} is derived as,

$$M_{n-1} = \frac{EI_{n-1}}{r} \cong \frac{E\pi at'^3}{12r} \tag{6}$$

where I_{n-1} is the moment of inertia of area at b_{n-1} . By solving the simultaneous equations Eq.(2)-(4), and using Eq.(5) and (6), the moment at b_n is derived as,

$$M_n = \frac{E\pi a t^{\prime 3}}{12r} + Tr\{1 - \cos(\frac{l_b}{r})\cos(\frac{l_c}{r}) - \frac{1}{2}\sin^2(\frac{l_c}{r})\}$$
(7)

As the contact area between the membrane and the center hub cannot be derived by the one-dimensional model, the effect of δ in Fig.4b on the moment Eq.(7) is examined. We define F as,

$$F = 1 - \cos\left(\frac{l_b}{r}\right)\cos\left(\frac{l_c}{r}\right) - \frac{1}{2}\sin^2\left(\frac{l_c}{r}\right) \tag{8}$$

and, Eq.(8) is expressed with δ as,

$$F = \frac{1 - \cos(\frac{l_b}{r})\cos(\frac{l_b - \delta}{r}) - \frac{1}{2}\sin^2(\frac{l_b - \delta}{r})}{= 1 - \cos^2(\frac{l_b}{r})\cos(\frac{\delta}{r}) - \sin(\frac{l_b}{r})\cos(\frac{l_b}{r})\sin(\frac{\delta}{r})}{\frac{1}{2}\left\{\sin^2(\frac{l_b}{r})\cos^2(\frac{\delta}{r}) - 2\sin(\frac{l_b}{r})\cos(\frac{\delta}{r})\cos(\frac{l_b}{r})\sin(\frac{\delta}{r}) + \cos^2(\frac{l_b}{r})\sin^2(\frac{\delta}{r})\right\}}$$

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When we assume δ is sufficiently smaller than r, the following equations are derived as,

$$\sin(\frac{\delta}{r}) \cong 0, \cos(\frac{\delta}{r}) \cong 1 \tag{10}$$

Using Eq.(10) and (10), F is expressed as,

$$F \cong 1 - \cos^2(\frac{l_b}{r}) - \frac{1}{2}\sin^2(\frac{l_b}{r}) \tag{11}$$

Thus, in this paper, we treat l_c as l_b .

Next, we discuss the condition for the local buckling. The stress in the inner membrane at b_n is expressed as,

$$\sigma_{bn} = -\frac{M_n}{I_n}a + \frac{T}{2\pi a t'} \tag{12}$$

where I_n is the moment of inertia of area at b_n , and expressed as,

$$I_n = \frac{\pi \{ a^4 - (a - t')^4 \}}{4} \tag{13}$$

The 2nd term on the right hand side of Eq.(12) is the tensile stress by the tensile force, T. As the local buckling occurs when σ_{bn} is smaller than the buckling stress σ_{cr} , the condition for the local buckling is,

$$-\frac{M_n}{I_n}h + \frac{T}{2\pi a t'} < -\sigma_{cr} \tag{14}$$

As the cross-section has the curvature 1/a, the compressive buckling stress of the cylinder is applied to the buckling stress σ_{cr} as,

$$\sigma_{cr} = \frac{1}{\sqrt{3(1-\nu^2)}} \frac{Et'}{a}$$
(15)

where, ν is Poisson's ratio.



Figure 4: One-dimensional analytical model of wrapping fold membrane



Figure 5: Cross-sectional model

3.2 Dominant parameters for local buckling

The dominant parameters for the local buckling are examined using Eq.(14). By using Eq.(7) and (8), Eq.(14) is expressed as,

$$-\frac{a}{I_n}(M_{n-1} + TrF) + \frac{T}{2\pi at'} < -\sigma_{cr}$$

$$\tag{16}$$

As the layer pitch is small in the position of the local buckling, which is indicated in Fig.1, the bending moment M_{n-1} is sufficiently smaller than the 2nd term in the parentheses on the left hand side of Eq.(16), TrF. The value of a and I_n is determined by T, r, and t in the wrapping fold process. When the radius of the center hub is sufficiently larger than the interval of the local buckling, l_b is sufficiently smaller than r, and hence, F in the Eq.(16) becomes roughly constant. Based on the above discussion, the condition for the local buckling is mainly affected by T, r, and t' in Eq.(16). Considering the form of Eq.(16) and (1), the dominant parameters of the condition for the local buckling are T/t and Tr. Using these dominant parameters, Eq.(16) is expressed as,

$$(Tr) > \frac{1}{F} \{ \frac{I_n}{a} (\frac{1}{l_0} \frac{T}{t} + \sigma_{cr}) - M_{n-1} \}$$
(17)

4 FEM ANALYSIS

FEM analyses are performed for the wrapping fold membrane to evaluate the experiments and the theoretical analysis. The FEM analyses are demonstrated with ABAQUS[4], a commercial software. As the wrapping fold process induces geometrical nonlinearity, the analysis procedure is geometrical nonlinear and static, where the implicit integration scheme is employed. The analysis procedure is carried out by applying General Static and NLgeom option. To stabilize the FEM analyses numerically, the numerical wrapping fold process is significant. We introduce a numerical creasing process proposed in our previous research[5].

5 RESULTS AND DISCUSSION

In this section, we discuss the results obtained by the experiments, by the theoretical analysis, and by the FEM analyses to examine the condition for the local buckling and the interval of the local buckling. In the theoretical analysis and the FEM analyses, Young's modulus and Poisson's ratio are set to be 4.8GPa and 0.30, respectively.

5.1 Condition for local buckling

Figure 6 indicates the condition for the local buckling. The symbol \bigcirc and \times indicate the experimental data where the local buckling occurs and doesn't occur, respectively. The experimental data are plotted for the dominant parameters obtained by the theoretical analysis. The solid and the broken lines show the condition for the local buckling calculated by Eq.(17). As the distribution of the experimental data can be divided into two regions, the effectiveness of the dominant parameters are confirmed. When we apply a proper value to *a*, the condition for the local buckling obtained by the experiments can be qualitatively expressed by the one-dimensional wrapping fold model. On the other hand, to clarify the condition quantitatively, the cross-sectional configuration of the wrapping fold membrane has to be improved.

5.2 Interval of local buckling

As the interval of the local buckling is affected by the experimental condition, the interval is selected as the parameter to examine the mechanics of the local buckling. In this section, we focus on the effects of the tensile force and the diameter of the center hub on the interval.

The effects of the tensile force on the interval of the local buckling obtained by the experiments are indicated in Fig.7. The membrane thickness and the diameter of the center hub are $50\mu m$ and 30mm, respectively. For these data, the solid line and the



Figure 6: Condition for local buckling

broken line represent the results of the crease cl and that of cr, where these results are the average values of the intervals in each round; 1st, 2nd, and 3rd round. When the tensile force is increased from 0.013N/mm to 0.027N/mm, the interval is decreased up to 62% (cr, 3rd). On the other hand, when the tensile force is increased from 0.027N/mmto 0.053N/mm, the interval is decreased up to 81% (cr, 3rd). Thus, although the interval is decreased as the tensile force becomes larger, the rate of the decrement is also reduced. These results are qualitatively expressed by the theoretical analysis.

To examine the effects of the diameter of the center hub on the interval of the local buckling, three kinds of diameters for the center hub are used; 30, 90, and 150mm. The experimental, theoretical, and FEM results are indicated in Fig.8. The tensile force and the membrane thickness are 0.027N/mm and $50\mu m$, respectively. The black dots and the white dots represents the experimental and the FEM results, where the solid line and the broken line are the results of the crease cl and that of cr, respectively. Also, these data are the average value of the intervals in the 1st round. When the diameter of the center hub is increased from 30mm to 90mm, the interval is increased up to 2.8 times (cl), for the experimental data. Also, when the diameter is increased from 90mm to 150mm, the interval is increased up to 1.6 times (cr). These results indicate that the interval is almost proportional to the diameter of the center hub. These experimental results are qualitatively expressed by the FEM results. However, too large interval is obtained by the theoretical analysis. It seems that this result is due to the constant value of a, because in the experiments, the layer pitch becomes large as the diameter of the center hub is increased.

6 CONCLUSIONS

Mechanics of the local buckling was examined to determine the condition for the local buckling. To treat the mechanics, the interval of the local buckling was introduced.



Figure 7: Effects of tensile force

Figure 8: Effects of diameter of center hub

The experimental data indicated that the interval became smaller as the tensile force was increased, and was almost proportional to the diameter of the center hub. In the theoretical analysis, a one-dimensional wrapping model was introduced to examine the mechanics of the local buckling. The dominant parameters for the condition for the local buckling, which were expressed by the tensile force, the membrane thickness, and the radius of the center hub. Also, the experimental data were qualitatively expressed by the one-dimensional wrapping model.

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DETERMINATION OF THE RESPONSE OF COATED FABRICS UNDER BIAXIAL STRESS: COMPARISON BETWEEN DIFFERENT TEST PROCEDURES

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Key words: PVC coated polyester fabric, biaxial test procedures.

Summary. The biaxial response of a PVC-coated polyester fabric is investigated using three different test procedures. The influence of the test procedure on the experimental data is discussed. A new approach based on a response domain is proposed.

1 INTRODUCTION

The estimation of the biaxial mechanical behaviour of coated fabrics is crucial for the design of tensile structures. This design relies on advanced softwares (form-finding, cutting pattern generation, stress determination) that require accurate material data. The material properties obtained from the biaxial tests strongly depend on both the test protocol that generates the stress-strain response and the post-processing of these experimental data. However, there is no standard method for biaxial tests in Europe so far. Therefore most laboratories develop their own protocol based on their experience, resulting in significant differences between the test data to process. Moreover, it has been shown that the post-processing has a major influence on the material parameters¹.

In this paper, the influence of the biaxial test procedure on the stress-strain response of a PVC-coated polyester fabric is investigated by comparing different methods:

- the standard of the membrane structures association of Japan $(MSAJ)^2$;
- the test method proposed by Bridgens *et al.* that tries to reproduce the *in situ* material behaviour with a pre-stressing, a conditioning and a radial load history³; this method is thereafter mentioned as the NCL method, in reference to the Newcastle University;
- the protocols currently used at EMPA.

2 EXPERIMENTAL SETUP

For each test procedure a cruciform specimen made of Valmex 7318 PVC-coated polyester fabric was tested on our biaxial test machine⁴ (Figure 1). The material has a weight of 1000 g/m^2 and a tensile strength of 60 kN/m. The central square of the specimens is 500 mm wide. Each cruciform arm is made of five strips which are independently loaded by an electromechanical drive mounted on linear bearings. Tests are load-controlled by the use of 10 kN load cells fixed between every pair of drive and grip. Strains are measured by the use of two needle-extensometers. Tests are performed in a climatic room ensuring a constant temperature of 22°C.



Figure 2: MSAJ test protocol: load history and corresponding strain measurement

3 TEST PROTOCOLS

3.1 Standard of the Membrane Structures Association of Japan (MSAJ)

The standard of the Membrane Structures Association of $Japan^2$ is the only existing standard for the biaxial testing of coated fabrics so far. This standard allows some flexibility

for the sample geometry and test conditions, so that it is applicable to most biaxial machines. The load profile explores various load ratios with repeated load cycles in order to remove residual strains (Figure 2). The maximum test load is set to 25% of the UTS (Ultimate Tensile Strength). The load is supposed to reach 0 between every load cycle (no pre-stress). However, this condition is not possible to achieve with our biaxial machine. Since the actuators are load-controlled, due to the free movement of all 20 actuators, a pre-stress equal to 0 N can lead to an undesired displacement of the sample in the machine. Therefore a very low pre-stress equal to 0.2 kN/m was applied (0.3% of the UTS). For each load ratio, namely 1:1, 2:1, 1:2, 1:0 and 0:1, three load cycles must be applied. For a material characterization at least 3 specimens must be tested, while for our comparative study only one specimen was used.

The determination of the material elastic constants from the experimental results is not described in the standard. As a result it is possible to obtain different material properties depending on result interpretation¹.

3.2 Test protocol proposed by Bridgens et al. (NCL)

Bridgens *et al.* recently developed a test protocol with the aim of simulating the normal load condition of an *in situ* fabric³. The test protocol consists of three stages (Figure 3):

- pre-stressing: a pre-stress is applied for approximately 17 hours in order to avoid high initial levels of creep;
- conditioning: the fabric is subjected to loads that are 10% higher than the design load in order to simulate the behaviour of a fabric which has been exposed to environmental loads;
- test: radial load paths are applied that explore the fabric response above and below pre-stress in an order that aims at limiting the influence of the recent load history.

During the test, the sample is loaded up to 20% of the UTS, which is a typical value to avoid tear propagation in the material. The pre-stress is set to 1.3% of the UTS.

It is expected that after the pre-stressing and the conditioning the material has reached the typical behaviour of an *in situ* fabric, with few level of creep. The remaining residual strains are removed from the experimental data under the assumption that the greater the applied load the greater the rate of creep³.

After removal of the residual strains, the *in situ* biaxial stress-strain behaviour of the fabric is obtained for each load cycle A to H.

Based on the test data, Bridgens proposed a new approach that uses 3D response surfaces⁵ in order to allow a better representation of the material non-linear behaviour.

3.3 Test protocol used at EMPA

The protocol used at EMPA is based on the recommendations of the European Design Guide for Tensile Structures⁶, which leads to a similar test to the one described by the Japanese standard. Different load ratios are explored with 5 load cycles each time. The maximum test stress and the pre-stress are set to 20% and 4% of the UTS, respectively. The pre-stress is applied prior to loading and maintained during 2 minutes. The material is also kept at pre-stress level for 2 minutes at every change of load ratio.



Figure 3: Load history and corresponding strain measurement for the two last phases of NCL test protocol



Figure 4: Test protocol used at EMPA: load history and corresponding strain

For the present study, seven load ratios were investigated, namely 1:1, 2:1, 1:2, 5:1, 1:5, 1:0 and 0:1. The corresponding load history and measured strains are presented in Figure 4.

The last loading cycle of each tested load ratio is used for the determination of the material behaviour, which is typically described using a linear elastic orthotropic model or the non-linear model proposed by the authors⁷.

3.4 Comparison

The stress-strain curves measured with the 3 test procedures are compared in Figure 5 for a 1:1 load ratio. The presented results are reduced to the same strain interval from 4% to 20% of the UTS. They show that the test protocols produce quite different results, which emphasizes the influence of the test conditions. There is no unique material behaviour but different possible responses. In the next Section the influence of the test parameters is investigated. The objective is to define a domain that would represent all these possible responses.



Figure 5: Stress-strain behaviour under 1:1 load ratio: comparison between the test procedures

4 PARAMETERS INFLUENCING THE MATERIAL BEHAVIOUR

4.1 Repetition of load cycles

Repetition of load cycles is used to reduce the level of residual strain. After each cycle more residual strain is removed and therefore the material stiffness is changed. The influence of cycle repetition is illustrated in Figure 6 for a 1:1 load ratio. The slope of the stress-strain curves increases at each new cycle in both warp and fill directions until the fourth and fifth cycles which are very similar. It seems therefore that after several cycles a stabilized solution is obtained.

4.2 Past load history

The past load history plays an important role in the material behaviour. The state of the material (crimp in the yarns, level of residual strains) before a new load cycle depends on the previous loadings. In order to investigate this influence a special test protocol is proposed where cycles of 1:1 load ratios are alternated with cycles of other load ratios: 1:1(A), 2:1,

1:1(B), 1:2, 1:1(C), 5:1, 1:1(D), 1:5, 1:1(E). For each load ratio 5 cycles are applied. The stress-strain curves obtained in the warp direction for the first and last cycle of each series of 1:1 load ratio are presented in Figure 8. Similar results are obtained in the fill direction.



Figure 6: Influence of cycle repetition for a 1:1 load ratio (EMPA test)



Figure 7: Test protocol used to investigate the influence of the load history



Figure 8: Influence of the load history in the warp direction for a 1:1 load ratio

Results show that the response of the first 1:1 load cycle of each series is strongly influenced by the loading that has been applied just before. The response is softer if before no load was applied (cycles A) or a very high load was applied in the opposite direction (cycles E for warp). On the contrary after 5 cycles there is a much smaller influence of the load history and similar curves are obtained.

4.3 Pre-stress

The pre-stress is the lowest stress level that is permanently applied on the material. Since the pre-stress is a long-term loading it will result in creep of the material. Therefore, the higher the pre-stress is, the more residual strains are removed. This has an influence on the material behaviour in particular during the first load cycles. After repeated cycles most of the residual strains are removed, so that the response does not depend on the initial pre-stress level. In order to investigate this influence two new samples were initially maintained under pre-stress during 6 hours at 4% of UTS and 1.3% of UTS, respectively, and then loaded under 1:1 load ratio. The stress-strain curves determined for this loading are presented in Figure 9. There is very little influence of the pre-stress in the warp direction which is less affected by large residual strains. In the fill direction however, a higher pre-stress gives a response that is initially stiffer.



Figure 9: Influence of the initial pre-stress level for a 1:1 loading

4.4 Loading rate

Investigating the influence of the loading rate with a single biaxial test protocol is difficult because one cannot separate the contribution of the loading rate to the contribution of the load history on the results. The only solution is to test a new sample for each strain rate. It was therefore chosen to perform uniaxial tests that require less material. Straps (length 500 mm, width 100 mm) were tested under uniaxial tension up to 50% of the UTS in both warp and fill directions. The tensile uniaxial response is presented in Figure 10 for different loading times corresponding to uniaxial loading rate from 25 to 1000 (N/m)/s. Results show that the material only becomes slightly stiffer at higher rates, so its rate-dependency is quite moderate.



Figure 10: Influence of the loading rate measured with uniaxial tensile tests in warp and fill direction

5 DEFINITION OF A RESPONSE DOMAIN

5.1 Proposed test protocol for the definition of the boundaries

It has been shown in Figure 5 that the behaviour of the material depends on the test conditions. Based on the investigations of Section 4 that have emphasize the influence of each test parameter one can define a response domain that would represent all possible responses of the material.

The definition of a domain requires the determination of boundaries. In that case one would need to define an upper "stiff" limit and a lower "soft" limit.

It has been shown in Figure 6 that after 5 cycles a stabilized response is obtained. Moreover it has been shown in Figure 8 that after 5 cycles the load history has no more influence. Therefore the EMPA test procedure that includes 5 cycles per load ratio can be used for the determination of this upper limit.

The definition of a lower limit is not as straightforward since the material behaviour is affected by the load history. It can be seen from Figure 6 that the first cycle usually exhibits a much softer behaviour compared to the following cycles. This first cycle represents in that case the very first response of the material to a 1:1 load cycle after a short initial pre-stressing. Such initial response emphasizes the initial large strains that occur in a new material. These permanent strains are usually taken into account by means of compensation tests to estimate the final shape of the structure. In order to determine the material initial behaviour it is necessary to test a new sample for each load ratio, which is material- and time-consuming. Moreover, this behaviour only happens once at the very first loading. The following loadings will all show a stiffer behaviour as most of the initial residual strains will have been removed. This definition of an initial response under operation conditions. The objective of defining such a response domain is indeed to represent the limit of the material behaviour on a real structure. A more appropriate solution would be to measure the response of a sample after it has been

initially loaded removing the large initial residual strains. The measurement would then be done based on a unique loading after a period of rest.

A test protocol is proposed that aims at representing the material response on a structure after a period of rest (loaded under pre-stress only). The load history is presented in Figure 11. A new sample is first maintained under pre-stress during 6 hours. Then load cycles are alternatively performed under 1:1, 2:1 and 1:2 load ratios. After each cycle the sample is kept under pre-stress for 1 hour before the next loading. Each series of three load ratios is performed three times. At the end of the protocol additional load cycles can be integrated for the estimation of the upper limit.



Figure 11: Load history for the determination of the domain boundaries

Results show that the second and third series of loadings give very similar results. The first series appears to be much softer and therefore affected by initial residual strains. As an example the stress-strain curves obtained for a 1:1 load ratio are presented in Figure 12. The lower limit is finally defined as the average of the second and third curves.



Figure 12: Results obtained on the 3 first cycles of 1:1 load ratio based on the proposed protocol

The influence of the pre-stress and of the loading rate on the proposed test protocol was

investigated. Three tests were performed on three new samples using two different pre-stress and different loading times for the measurement of the lower and upper limits. Results of the investigation are presented in Figure 13. As it could have been expected from the observations of Section 4, the pre-stress level does affect the material behaviour while the loading rate has no influence. The upper limit is also not very sensitive to the test parameters.



Figure 13: Influence of test parameters on the lower and upper limit in the fill direction

5.2 Comparison between the response domain and the MSAJ and NCL tests

It has been shown that the pre-stress has a significant influence on the material behaviour. In the previous comparison (Figure 5) the pre-stress used for the EMPA procedure was significantly higher than for the two other tests. For this new comparison a pre-stress of 1.3% of the UTS is chosen similarly to the NCL protocol. The parameters of each test are summarized in Table 1.

	MSAJ	NCL	EMPA upper	EMPA lower	
Load cycles	3	1 (with steps)	5	1	
Past load history	none (new sample)	pre-stressing and conditioning	pre-stressing and 9 load cycles	pre-stressing and 3 load cycles	
Pre-stress	0.3% UTS	1.3% UTS	1.3% UTS	1.3% UTS	
Loading rate	123.3 (N/m)/s	14 (N/m)/s	93.5 (N/m)/s	93.5 (N/m)/s	

Table 1 : Comparison between the test parameters

The final results are presented in Figure 14. Overall, there is a reasonably good match between the NCL test procedure, the MSAJ standard and the response domain defined by EMPA's test protocol if similar pre-stress levels are used. The results obtained with the Japanese standard are indeed always included in the response domain in green. This is presumably due to the similarities between both test procedures. However, it can be emphasized that the behaviour derived from the Japanese standard test is much softer than the upper limit of the domain. This proves the strong influence of the amount of cycles on the material response. It seems that three cycles are not sufficient to completely remove the residual strains and therefore to obtain a converged solution.



Figure 14: Comparison between the MSAJ standard, the NCL protocol and EMPA's response domain

The lower limit however is not always the softest measured response. In particular in case of a 2:1 load ratio the response measured with the NCL method is much softer. The cause of this difference cannot be related to the pre-stress or to the cycle repetitions that are very
similar. The loading rates are different but it has shown very little influence on the results. It must therefore be due to the load history. In fact it can be seen in Figure 3 that prior to the 2:1 load ratio (cycle E) a 0:1 load ratio is applied (cycle A). This can have a significant influence on the stiffness measured in the warp direction under the 2:1 load ratio as it has been explained in Section 4.

6 CONCLUSION

Three test protocols for the investigation of the biaxial response of PVC-coated polyester fabrics have been compared. Results emphasize the significant influence of the test conditions on the material response, in particular the level of pre-stress, the amount of repeated load cycles and the initial material conditioning. It is therefore impossible to assess which method is more appropriate to the investigation of the biaxial mechanical behaviour of coated-fabrics.

A new approach has been proposed that could give a representation of the material behaviour variability and therefore help to define the limits of the possible behaviour of tensile structures. A response domain for the material has been presented whose limits can be experimentally determined. The upper limit would represent the material stiffest solution obtained after several load repetitions. The lower limit would represent the softest material response on a real structure, defined as the response of an initially loaded sample after a period of rest. Any further loading of the fabric is then expected to produce a response that is included within the previously described boundaries if the pre-stress is adjusted in the test procedure to match the design requirements. It has been observed for the studied material that the upper limit could be up to 60% stiffer than the lower limit. Those two limits might be used to calculate two extreme cases of the structure behaviour. If the variability is not so pronounced then an average of both limits might also be used for the material model.

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EFFECTS ON ELASTIC CONSTANTS OF TECHNICAL MEMBRANES APPLYING THE EVALUATION METHODS OF MSAJ/M-02-1995

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Summary. The non-linear load-deform ation behaviour of textile membranes highly depends on the ratio of the applied membrane forces in warp and weft direct ion (called load ratio hereafter). In practice, usually for each membrane structure the biaxial material behaviour is determined experimentally. The Japane se Standard MSAJ/M-02-1995 describes a standardized biaxial testing procedure. To achie ve input parameters for the structural design process, the commentary to this standard expl ains some methods how to evaluate one set of fictitious elastic constants based on the expe rimental results which, sim ultaneously, envelop different load ratios and do not reflect the non-linear material behaviour anymore. Different approaches of determining such simplified, fictitious elastic constants have been investigated in the present contribution, with m ainly two conclusions: firstly, to have one set of elastic constants by means of which all types of structures under all types of loading can be treated is a highly disputable objective a nd secondly, the values of the determined elastic constants react quite sensitively on the underlying determination option, which should be defined by the users themselves.

1 INTRODUCTION

Typical coated woven f abrics used in m embrane structures are m ade of Glass/PTFE or Polyester/PVC. Both fabrics show an extremely nonlinear load-deformation behaviour under biaxial tension, which is the common loading condition of textile membranes.

The structural design of membrane structures depends on this load-deformation behaviour, which can vary even for one membrane type of one fabricator from batch to batch. Due to this

fact biaxial tensile tests are usually performed for each membrane structure to determine its specific load-deformation behaviour as source for r realistic input parameters for the design calculation.

From the engineering point of view an intern ational standardized testing and evaluation procedure is desirable for the determination of the load-deformation behaviour of membrane materials. A standard ized procedure should allow the comparison of different m embrane materials on an objective base. The Mem brane Structures Association of Japan developed such a standardized biaxial testing proced ure, which was publishe d 1995 in the standard MSAJ/M-02-1995 "Testing Method for Elasti c Constants of Me mbrane Materials"¹. This excellent standard has been more and more internationally accepted during the last 15 years and has been used increasing ly as a basis for r contractual arrangements between design engineers, contractors, manufacturers and/or fabricators.

The main characteristic of the MSAJ/M-02-1995 testing proce dure is that five different load ratios for the m embrane forces in warp and weft di rection have to be applied in a precisely defined sequence. Herewith, different non-linear load-strain-paths are m easured depending on the applied load ratios.

Usually, the design ca lculation of a m embrane structure is performed using m odern software packages which are based on finite elements and which are able to handle globa 1 geometric non-linearity as well as m aterial non-linearity, although the latter only in terms of the membrane's inability to carry in-plane compression. For simplicity, the load-deformation-behaviour of the m embrane in tension is usually treated linear-elastically, which means that the non-linear load-deform ation-behaviour is not considered in the design process. There seems to exist a great lack of knowledge how to simulate and herewith how to include the non-linearity of the membrane material in the design process.

The main topic of the MSAJ/M-02-1995 is the standardized biaxial testing procedure in order to deliver realistic information on the load-strain behaviour. Optimally, for each loading condition the specifically m easured non-linear load-strain-charac teristics would directly be introduced into the design calc ulation. However, up to now this is not feasible. The commentary to MSAJ/M-02-1995 therefore expl ains exemplarily so me methods how to simplify the non-linear load-strain behaviour in order to achiev e certain fictitious elastic constants which shall approximately describe the membrane material.

The simplified evaluation of the experim ental load-strain-paths according to the commentary of MSAJ/M-02-1995 has already led to intensive discussions, e. g. by Bridgens & Gosling³. In addition to their investigations, the aim of this contribution is firstly, to discuss the application of the simplified m ethods on principle and secondly, to present and discuss results of different options for the simplified determination of such fictitious elastic constants. The quantitative effects of these different determination options on the resulting sets of elastic constants will be investigated by means of exemplary test data.

2 BIAXIAL TESTING APPLYING MSAJ/M-02-1995

As already m entioned, MSAJ/M-02-1995 princi pally describes a standardized biaxial testing procedure for woven m embrane materials. The scope is to obta in the non-linear load-strain relationship. The biaxial tests are performed applying tensile loads in the warp and weft

direction on a cross-shaped specim en for five different defined load ratios of the mem brane forces in warp and weft direction, see figure 1 and table 1. Figure 1 shows one of the biaxial testing machines of the Essener Labor für Le ichte Flächentragwerke of the University of Duisburg-Essen, which has maximal loads of 50 kN in each direction.

Direction of yarn	Load ratio				
Warp direction	1	2	1	1	0
Weft direction	1	1	2	0	1

Table 1: Load ratios applied during biaxial tensile testing starting with 1:1 and ending with 0:1

Figure 2 (a) shows a typical load-straindiagram as a result of a biax ial test according to MSAJ/M-02-1995 of a Glass/PTFE m aterial, type G6 according to the European Design Guide for Tensile Surface Structures⁴. This load-straindiagram consists of ten load-strain-paths: one load-strain-path each for the warp and weft direction at five load ratios, see figure 2 (b). The two zero-load-paths for the load ratios 1:0 and 0:1 appear as horizontal straight lines in this particular way of plotting.



Figure 1: Biaxial testing machine of the Essener Labor für Leichte Flächentragwerke at the University of Duisburg-Essen with a maximum load of 50 kN in each direction



Figure 2: (a) Load-strain-diagram as a result of a MSAJ biaxial test on Glass/PTFE material Type G6, (b) ten load-strain-paths (warp/weft at five load ratios), as extracted from the diagram

Architectural fabrics are wove n from single yarns and coated afterwards. The yarns lay crimped in the f abric matrix. The crimp value depends on the stress in the warp and weft direction that is app lied during the weaving process ⁴. As the stresses in warp and weft

direction usually do not have the same values, due to the crimp interchange the fabric shrinks differently in both directions . Herewith, woven m embranes behave orthogonal anisotropic, see figure 2.

3 DETERMINATION OF ELASTIC CONSTANTS APPLYING THE COMMENTARY OF MSAJ/M-02-1995

In design practice, the m embrane material is considered as a linear-elastic orthogonal anisotropic two dim ensional plane-stress stru cture. For this reason, the comm entary of MSAJ/M-02-1995 describes several possibilities how to determine a set of fictitious elastic constants for the use in practical de sign, which consist of the stiffness , E_x and E_y , and Poisson's ratio, v_{xy} and v_{yx} , each in warp and weft directi on. The defined set of constant ts meets the requirements of constitutive equations for linear-elastic, orthotropic materials used for numerical simulations, see exemplary Münsch & Reinhardt². This set of constants describes an optimized approximation while using specified load-strain-paths considering the full range of experimental load values for the evaluation. The sets of elastic constants have to be treated as "fictitious" elastic constants because firstly, they shall es timate the non-linear load-deformation behaviour of the m aterial and secondly, they shall envelop all load combinations in warp and weft direction.

On the basis of the afore described simplifications, the commentary of MSAJ/M-02-1995 proposes to express the relationship between load and strain with the following equations

$$\varepsilon_x = \frac{n_x}{E_x \cdot t} - v_{xy} \cdot \frac{n_y}{E_y \cdot t},\tag{1}$$

$$\varepsilon_y = \frac{n_y}{E_y \cdot t} - v_{yx} \cdot \frac{n_x}{E_x \cdot t}.$$
 (2)

Hereby, ε describes the strain, *n* is the load, *E* is the stiffness and *v* is the Poisson's ratio with v_{xy} is the transverse strain in x-d irection caused by a load in y-direction and v_{yx} is the transverse strain in y-direction caused by a load in x-direction. The x-direction corresponds to the warp direction of the fabric, the y-dire ction to the weft dire ction. The number of unknowns in these equations is four: the two s tiffnesses and the two Poisson's ratios. The further idealisation of the membrane material to a linear-elastic orthogonal anisotropic plane stress plate with a symmetric stiffness matrix leads to the constraint

$$\frac{E_x \cdot t}{E_y \cdot t} = \frac{v_{yx}}{v_{xy}},\tag{3}$$

which is referred to as the "reciprocal rela tionship" in the commentary of MSAJ/M-02-1995. This additional constraint reduces the num ber of unknowns to three, but it does not necessarily correspond to the beh aviour of woven membrane materials. Modelling of the membrane by assuming a linear-elastic orthotropic plane stress is well k nown to be a rather rough structural model for a coated woven fa bric with its above-m entioned nonlinear load-strain behaviour. Over all, it must be aware that this way of modelling of the load-strain behaviour is just a vague approximation.

The determination of the fictitious elastic constants from the load-strain-paths has to be

performed stepwise in a double step correlation analysis. In the first step each curved loading path has to be substituted by a st raight line. In the second step the slopes of the straight lines obtained in the first step have to be modified in such a way that they satisfy the equations of the assumed linear elastic plane stress behaviour and describe all experimental loading paths for all five load ratios optimally by just one set of four fictitious c onstants. The commentary of the MSAJ-Standard recommends to use eight of the ten measured loading paths omitting the two zero load paths, although, four paths woul d be sufficient to determ ine a set of four fictitious elastic constants. Bridgens & Gosling ³ already have discussed the significantly different results in the determination of the elastic constants when using all ten pa ths instead of the eight paths as recommended in the MSAJ-Standard.

To determine the optimum set of elastic constants the commentary of MSAJ/M-02-1995 proposes the "least squares m ethod", the "best approximation method" and other sim plified methods. The "best approximation method" and the other methods are not presented here. The "least squares m ethod" is known from the determination of regression lines in statistic calculations and has been used in the present investigations. The scope is to minimize the sum of squares of errors in a certain subject interval [a, b] between a continuous function f(x) and an approximation equation y(x):

$$S = \int_{a}^{b} [f(x) - y(x)]^2 dx \to \min .$$
⁽⁴⁾

The errors can either be defined as the vertical differences (load errors S_{σ}) or the horizontal differences (strain errors S_{ε}). For the determ ination of the elastic constants this m eans that either the load term or the strain term can be m inimized: $S_{\sigma} \rightarrow \min$ or $S_{\varepsilon} \rightarrow \min$. For clarification see figure 3 (a) a nd (b), each showing three exemplary errors - load and strain, respectively - between an experim ental load-strain-path and an arbitrary line. The commentary of MSAJ/ M-02-1995 recommends the application of various methods to determine the elastic constants and to use the most satisfactory combination of constants. It has not to be noted here, that this procedure does not fit with a "standardized procedure" and will lead to variable values depending on the chosen procedure of the user, too.



Figure 3: (a) Vertical errors are calculated in order to minimize the load term, (b) horizontal errors are calculated in order to minimize the strain term

In the design process for a membrane structure the residual strains are taken into account in the process of com pensation. This means that the membrane material is shortened by the value of the residual strains before installati on. Usually, the residual strains are not included in the static calculation of a membrane structure. Therefore, it is reasonable to remove the residual strains from the test data for the determination of the elastic constants.

The commentary of MSAJ/M-02-1995 recomm ends to use straight lines connecting the point of 2 kN/ m (for Glass/PTFE mem branes) and the point of the m aximum experimental load for the determination of the constants. He rewith, the fictitious elastic constants and the corresponding lines are determ ined with the aim to reflect the strain at the m aximum experimental load in the best way. Although this procedure satisf ies the desire for standardization, the service loads of the most membrane structures do not reach the maximum experimental loads during their life time. Herewith, this procedure might not be sufficient for practical design efforts.

4 ROUTINE FOR THE DETERMINATION OF ELASTIC CONSTANTS

For the determination of the fictitious elastic constants from test data a correlation analysis routine was programmed by using the commercial software MATLAB⁵. The basis of the routine is the calculation of regression lines using the least squares method as proposed in the commentary of MSAJ/M-02-1995. A regression line e in a load-strain-diagram follows the linear equation (5), in which n is the load, m is the slop e, ε is the stra in and b is the eintersection point of the regression line with the load-axis at zero strain:

$$i = m \cdot \varepsilon + b . \tag{5}$$

In a f irst step, the r outine evaluates the regression lines for all experim ental load-strainpaths. Herewith, ten regression lines and their values for *m* and *b* are determined so that each of the ten loa d-strain-paths is fitted optimally. A regression line for an arbitrary experimental load-strain-path is show n in figure 4. It is the nature of a regression line to reflect the slope of the path in a good m anner. Usually, the regression line has another intersection point b with the load-axis at zero strain than the test data path itself. To describe the stiffness of a linearelastic material in a load-strain-diagram the intersection point of the regression line is n ot important but the slope. To provide the typical illustration of a linear load-strain behaviour, the



Figure 4: Regression and switched lines for a test data path

intersection point of the regression line may be switched into the intersection point of the test data path for the plots, see figure 4.

In order to set up fictitious straight load-strain-lines the programmed routine generates in a second step all possible combinations of the four fictitious elastic constants within lim it

values and increments established by the user. The increm ents may be quite rough in a first step of the analysis. They can be set to smaller values in an adjacent fine analysis, which will be conducted in the periphery of the best fit result of the rough analysis. In case that the reciprocal relationship, see eq. (3), is applied, only those comb inations are taken into account that satisfy this constraint within arbitrary limits. In the investigations for this contribution the limits are set to

$$\frac{v_{yx}}{v_{xy}} - 0.005 < \frac{E_x}{E_y} < \frac{v_{yx}}{v_{xy}} + 0.005, \qquad (6)$$

which seems to be precise enough.

In a third step, the strain values of the fict itious load-strain-lines are calculated for one arbitrary load level at each load ratio according to equations (1) and (2) inserting the generated constants. Knowing the strain values enables the evaluation of the slope of the fictitious load-strain-lines. Each fictitious load-strain-line j is related to the load -strain-path j of the test data. The slopes of the fictitious load-strain-lines j are calculated with equation (7) at the various load ratios using arbitrary values for n_x and n_y . The only constraint is that the ratios of n_x and n_y satisfy the respective load ratio.

$$m_j = \frac{n_j}{\varepsilon_j} \tag{7}$$

For the further procedure the intersection point of each load-strain-line at the load-axis at zero strain is set to the respective value b of the related regression line. This ensures that those load-strain-lines with a slope that ap proaches the slope of the respective regression lines lead to the "least squares". In order to calculate the strain values for a fict itious load-strain-line j for each existent test data po int i of the related load-strain-path j, equation (5) has to be transformed into equation (8)

$$\varepsilon_i = \frac{n_i - b_j}{m_j} \,. \tag{8}$$

Finally, the sum of squared strain errors over all n test data points and m load-strain-paths considered in a determination of constants can be calculated using the following equation

$$S_{\varepsilon} = \sum_{j=1}^{m} \sum_{i=1}^{n} \left(\varepsilon_{i} - \overline{\varepsilon_{i}} \right)^{2}, \qquad (9)$$

in which ε_i is the result of equation (8) and $\overline{\varepsilon_i}$ is the value of the related test data point, respectively. The value S_{ε} is the sum of all squared horizontal differences explained in figure 3 (b). The optimum set of constants in the m eaning of the commentary of MSAJ/M-02-1995 is the one combination of elastic constants with the minimum value S_{ε} .

The programmed routine was validated with the exemplary test data presented in the commentary of MSAJ/M-02-1995. Hereby, very similar results were achieved by using the least squares method minimizing the strain term compared to the presented ones in the commentary of MSAJ/M-02-1995.

5 EXEMPLARY EVALUATION OF TEST DATA

Based on the afore m entioned evaluation pr ocedure, the influence of different "determination options" on the resulting elastic constants has been investigated. For this purpose, test data of 70 biaxial tests on the Verseidag-Indutex membrane material B 18089 were considered. This is an often used a nd well-proved Glass/PTFE material type $G6^4$ with nominal tensile strength values of 140/120 kN/m in warp/weft direction.

All mentioned tests had been conducted in the context of real projects in the las t three years at the Essener Labor für Leichte Flächentragwerke of the University of Duisburg-Essen. In order to get an insight in to how much even for one t ype of material produced by a manufacturer with high quality m anagement level the calculated values of fictitious elastic constants might inevitably vary, three tests were systematically selected out of the 70 biaxial tests – in the following referred to as T1, T2 and T3 – with the aim to cover approximately the whole realistic spectrum. Within the re latively narrow range of observed behaviour characteristics, Test T2 represents the aver age, while Te st T1 shows a som ewhat stiffer behaviour in warp direction combined with a somewhat softer one in we ft direction, and Test T3 behaves the other way around (som ewhat stiffer in weft and softer in warp direction). The maximum test load was max. n = 30 kN/m.

Table 2 s hows the calcu lated elastic constants using eight di fferently defined "determination options". All results were ca lculated using the least squares m ethod minimizing the strain term as described in ch apter 4. The first four determination options make use of all f ive load ratio s applied in the standardized MS AJ test, see table 1. Calculations have been perfor med either using eight load-strain-paths as proposed in the commentary of the MSAJ-Standard (i.e. omitting the zero-load-paths), see options 1 and 2, or using all ten load-strain-paths as proposed by Bridgens & Gosling ³, see options 3 and 4. Additionally, a differentiation was made with regard to applyi ng the recipro cal relationship (yes or no), see options 1, 3 versus options 2 and 4.

The last four determ ination options 5 to 8 in table 2 have been de fined by the authors to simulate reasonable decisions of rationally thinking structural design engineers with regard to their specific membrane structure. For a synclastic structure with almost identical membrane forces in warp and weft direction under design loading, the determination might reasonably be conducted using the load ratio 1:1, com bined with either 2:1 or 1:2 (at least four load-strain-paths are needed for the determ ination of the unknowns). For an anticla stic structure with predominant warp stressing under the critical de sign load case, the load ratios 2:1 and 1:0 might be reasonable (option 7), and for the opposite type of stre ssing the load ratios 1:2 and 0:1 (option 8). For all determ ination options 5 to 8, the reciproc al relationship is applied as proposed in the commentary of the MSAJ-Stand ard. Furthermore, in determination options 7 and 8 three load-strain-paths are used omitting the zero-load-paths.

Figure 5 exemplarily shows the experimental load-strain-paths of Test T2 together with the theoretical straight lines obtained with the f ictitious elastic constants from determination option 1 in table 2, i.e. using eight load-strain-paths in compliance with the commentary of MSAJ/M-02-1995 and applying the reciprocal relationship. Figure 6 shows the corresponding results for determination option 3 in table 2, i.e. using all ten load-strain-paths and also the reciprocal relationship applied. In figures 5 and 6 the strains are plotted against the "leading

Determination options		Test	Stiffness [kN/m]		Poisson's ratio [-]		c	Note
Determination options	data	E _x t	E _y t	ν _{xy}	v _{yx}	S_{ϵ}	NOLE	
1	All load ratios - 8 load-strain-paths - reciprocal relationship applied	T1	1322	718	0,55	1,01	41,06	MGAL
		T2	1292	816	0,57	0,90	32,72	original
		T3	1122	884	0,68	0,86	24,64	onginar
2	All load ratios - 8 load-strain-paths - reciprocal relationship not applied	T1	1212	766	0,71	0,74	27,94	MSAL
		T2	1188	864	0,73	0,69	23,09	modified
		T3	1066	924	0,79	0,71	20,96	moullieu
3	All load ratios - 10 load-strain-paths - reciprocal relationship applied	T1	918	544	0,82	1,38	696,66	MSAJ
		T2	914	610	0,83	1,24	637,19	modified as
		T3	756	660	1,00	1,15	520,31	in [3]
4	All load ratios - 10 load-strain-paths - reciprocal relationship not applied	T1	888	558	0,89	1,28	690,59	MSAI
		T2	860	634	0,94	1,08	625,49	modified
		Т3	766	658	0,99	1,19	519,63	modified
	Two load ratios: 1:1 / 2:1 - reciprocal relationship applied	T1	1580	798	0,45	0,89	4,89	practical
5		T2	1600	924	0,48	0,83	2,14	approach
		Т3	1566	1074	0,55	0,80	0,45	(synclastic)
6	Two load ratios: 1:1 / 1:2 - reciprocal relationship applied	T1	1600	738	0,55	1,19	7,96	practical
		T2	1336	824	0,60	0,97	4,42	approach
		Т3	1250	920	0,65	0,88	3,05	(synclastic)
7	Two load ratios: 2:1 / 1:0 - 3 load-strain-paths - reciprocal relationship applied	T1	520	770	0,88	0,59	0,67	practical
		T2	520	894	1,33	0,78	0,62	approach
		Т3	500	1083	1,43	0,69	0,35	(anticlastic)
	Two load ratios: 1:2 / 0:1 - 3 load-strain-paths - reciprocal relationship applied	T1	522	372	0,75	1,05	0,23	practical
8		T2	500	732	1,40	0,96	0,98	approach
		T3	540	500	0,74	0,80	0,29	(anticlastic)

membrane force ", which is meant to be the larger one at each load ratio. This form of plotting was chosen to avoid meaningless horizontal lines for the zero-load-paths.

 Table 2: Elastic constants from three biaxial tests of the same Glass/PTFE material Type G6 obtained using different determination options

Using the determination options based on the commentary of MSAJ/M-02-1995 – fully original or modified, options 1 to 4 – results in an "alarmingly" great variety of values for the calculated elastic constants: $E_x \cdot t$ varies between 756 kN/ m and 1322 kN/m, $E_y \cdot t$ between 544 kN/m and 924 kN/m, v_{xy} between 0.55 and 1.00, and v_{yx} between 0.69 and 1.38.

It can be seen from figure 5 that for determination option 1 the calculated load-strain-lines match the experimental load-strain-paths, in particular the points of maximum experimental load quite well – of course exce pt for the ze ro-load-paths of the load ratios 1:0 and 0:1, because they were omitted from the correlation process. Bridgens & Gosling³ propose to take into account these zero-load-paths, too, because they contain relevant mechanical information regarding the load bearing behaviour of anticlastic structur es. However, it may be concluded by plausibility from figure 5 th at, in ord er to achieve an improved matching of the two calculated zero-load-lines with their experimental counterparts, smaller theoretical values for ε_y at 1:0 and ε_x at 0:1 would be necessary. This would imply smaller values for the stiffnesses and higher values for the Poisson's ratios, as becomes obvious from eqns. (1) and (2). For example: at 1:0, with $n_y = 0$, the s train ε_y in eq. (2) decreases if v_{yx} increases and $E_x \cdot t$ decreases.



Figure 5: Results for Test T2, 8 load-strain-paths, reciprocal relationship applied (det. opt. 1)



Figure 6: Results for Test T2, 10 load-strain-paths, reciprocal relationship applied (det. opt. 3)

Figure 6 shows that – using determ ination option 3 – the two calculated zero-load-lines fit indeed somewhat better with their experim ental counterparts, but for the "price" of greater disagreement for all other calculated load-strain-lines. This effect is reflected by the results in table 2, comparing options 1 and 2 with options 3 and 4. For example, the calculated stiffness $E_x \cdot t$ decreases dramatically from values greater than 1000 kN/m to values smaller than 1000 kN/m when the zero-load-paths are taken into account. Attention should be paid to the m uch worse correlation measure S_{ε} for the determination options 3 and 4 (column 8 in table 2).

A comparison of the determ ination options 1 and 2 shows, that applying the reciprocal relationship has a significant infl uence on the calculated constants if only eight load-strainpaths are evaluated, especially on the Poisson's ratios. Applying the reciprocal relationship increases the values of v_{yx} and decreases those of v_{xy} , e.g. for Test T2 f rom 0.90 to 0.69 and from 0.57 to 0.73, respectively. The in fluence of the reciprocal relationship is smaller if ten load-strain-paths are evaluated, as can be seen f rom the results f or determination options 3 and 4: For Test T2, v_{yx} decreases from 1.24 to 1.08 and v_{xy} increases from 0.83 to 0.94.

If a practical, i.e. a stru ctural design engineer's approach is used for the determ ination of the fictitious constants, see determination options 5 to 8 in table 2, the results vary even more. Especially, the s tiffness values reach extrem e values: $E_x \cdot t$ varies from 500 kN/m up to 1600 kN/m and $E_y \cdot t$ varies from 372 kN/m up to 1083 kN/m.

It can be summarized, that the values of fictitious elastic constants evaluated from one and the same biaxial MSAJ-test depend extrem ely on the underlying determination option – even if, as perform ed in the present investigations, only one nu merical correlation m ethod is applied (here: the least squares method minimizing the strain term), and if the calculated lines are optimized only for one load range (here: between minimum and maximum experimental test load).

6 CONCLUSIONS

The Japanese Standard MSAJ/M-02-1995 describes first and forem ost a standardized experimental biaxial testing procedure. It is the main feature of the procedure, that the specimens are loaded in warp an d weft direction with a precisely defined consecutive sequence of five different load ratios. In the authors' opinion this is the prim ary merit of MSAJ/M-02-1995.

The secondary (and highly ambitious) scope of the MSAJ-Standard is to provide the design engineer with information how to transform the observed biaxial load-s train-behaviour into ready-to-use stiffness parameters for his design calculations. The commentary of the MSAJ-Standard idealizes the m embrane material for this purpose as a linear-elastic orthotrophic plane stress material, which may be described by only three fictitious elastic constants, but which is known to be a rather rough structural model for woven membranes with their highly nonlinear load-strain-behaviour. On this basis, the commenta ry gives recommendations how to extract an optimum set of these elastic constants from the biaxial test data.

Disregarding the roughness of the model, the e ffects of different determination options on the resulting sets of fi ctitious elastic constants were investigated in this contribution. To determine the optimum sets of elastic constants, a MATLAB correlation analysis routine was programmed using the least squares m ethod minimizing the strain term, which is one of the proposed methods in the MSAJ/M-02-1995 comment ary. Three real test data sets were investigated with this tool us ing several determination options. They represent, on the one hand, the evaluation proposals of the commentary of MSAJ/M-02-1995, both in their original version and in the modified version according to Bridgens & Gosling, and, on the other hand, thinkable design engineer's approaches aim ing at covering the actual load bearing behaviour of typical membrane structures.

It could be dem onstrated that a great variety of values for r the elastic constants can be obtained for one and the sam e material, only depending on the different determination options. Having the roughness of the underlying structural model in mind, the question arises, if it is not a disputable objective of the commentary of MSAJ/M-02-1995 to determ ine only one single set of fictitious constants by means of which all types of membrane structures under all types of load cases shall be treated. In the design practice it m ight be more reasonable to use constants which are determ ined for specific load ranges and load ratios depending on the project's needs. Further, concerning the design practice, it m ight be recommendable in the light of the great variety of the constants' values to calculate membrane structures with two limitative sets of elastic constants instead of using only one single set.

Nonetheless, from an engineering point of view an international standardized procedure for testing and evaluating the biaxial load-strain-behaviour is desirable to enable the comparison of materials on an objective base. However, it is not reasonable to evaluate values for fictitious elastic constants with an ostensibly high accuracy considering the rough character of a linear approximation of the material behaviour and the variety of possible determination options and evaluation methods.

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ISSUES WITH MANAGEMENT, MAINTENANCE AND UPKEEP IN ETFE ENCLOSURES

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1 INTRODUCTION

Ethylene tetra-fluoro-ethylene (ETFE) foil, as a single layer or as multi-layer inflated cushions, has been in used in the building industry for nearly 30 years as a medium to cover and clad both façades and atria. Its longevity has been well publicised and proven with many projects showing little or no signs of degradation.

The number of ETFE foil structures has been steadily rising in recent years, and with this the inevitable need for maintenance has also risen. The anticipated life of ETFE foil is now suggested to be as long as 50 years [1], and as with any other building material, regular inspections are necessary to ensure the continued optimal operation of the enclosure.

2 CASE STUDY 1 – HREOD DOME



Figure 1. ETFE cushion roof covering atrium of Nova Hreod College courtyard

An installation by Architen Landrell Associates in the UK, the Hreod Dome is an open structure in the South of England which covers the courtyard of Nova Hreod College (A new development in 2005 to relocate Hreod Parkway School). The dome comprises a radial array of fritted (printed) 2-layer ETFE cushions with a centre 'oculus' cushion. The ETFE cushion roof, located 15m above ground level, was chosen to cover the courtyard and provide a sheltered communal space for students as well as allow trees and plants to grow in the covered area.

The school is located within a kilometre of a decommissioned waste recycling site. The proximity to this site has resulted in a large number of birds in the vicinity; consequentially there have been occasions where maintenance is required due to damage to the cushions by the birds pecking at the surface. Often the necessity for remedial work is not immediately noticed and reported; small holes in the foil are easily offset by the capacity of the air handling units and although the internal pressure may be maintained, the pressurising fan will need to operate for longer and/or more frequently to replace the leaking air. Regular monitoring may potentially reveal such defects through the detection of abnormal fan usage.

Following the unusually cold winter with a large amount of snow, emergency work was necessary due to water ingress through a puncture in the cushion. In this case a site inspection revealed that the oculus cushion had deflated due to the size of the puncture. This resulted in the cushion inverting under the weight of snow and allowing water from the melted snow to enter the cushion. After the cushion was drained of water the hole could be patched using adhesive ETFE tape. The root of the problem was the unusually large volume of snow over the early months of 2010. This had caused the bird wire (used to prevent birds from landing

on the aluminium extrusion) to be pulled from the holders, enabling birds to then land on the perimeter of the cushions. This also provoked a redesign of the bird wire holders to ensure they were more resilient to the load imposed by sliding snow.



Figure 2. Water pooling on and inside centre oculus cushion of ETFE roof

The perimeter of the dome allows full access for maintenance personnel however due to the circumferential design of the dome; there is no walkway for access to the top of the structure. This was taken into account during the design process of both the steelwork that supports the roof as well as for the extrusion that holds the perimeter of the cushions.



Figure 3. Steelwork with hanger plate anchor (indicated) designed for installation and maintenance access



Figure 4. Aluminium extrusion used for securing edges of ETFE cushions

A channel in the steelwork allows safe contractor access to the oculus (Figure 3). Hanger plate anchors were applied at regular intervals on the steelwork channel to provide access to

the top of the structure by qualified personnel. Additionally the extrusion is wide enough to allow trained maintenance crew to gain access to interim cushions that are not located directly next to the steelwork (Figure 4).

In this instance the access and repair of the ETFE cushion took no more than 2 hours; also including a routine inspection of the air handling unit (AHU) to ensure that the system continued to perform optimally.



3 CASE STUDY 2 – HERTFORDSHIRE UNIVERSITY

Figure 5. ETFE cushion entrance walkway at Hertfordshire University's Art & Design department

Approximately 10 years old and having had few inspections and very little maintenance in that time, the structure consists of a series of nine ETFE inflated cushions (with an area of nominally 225m²) covering the general public access space to the Art and Design building. The walkway comprises three ETFE cushions forming the North face of the structure, and six inflatable cushions forming the South face; all cushions are constructed from translucent white ETFE.

Architen Landrell Associates was called to site to perform general maintenance on the structure due to a steady decline in pressure in the system due to apparent damage to the cushions. The maintenance brief involved repairing any damage to the cushions, an inspection of the air supply, cleaning of the top side of the ETFE cushions and cleaning the aluminium extrusion at the perimeter of the cushions.

Upon closer inspection of the cushions, it was found that some remedial work had previously been undertaken for small punctures that had been caused by birds pecking at the surface (Figure 6 & Figure 7) however since the system was last inspected there was a

significant amount of additional damage to the surface. This was potentially the cause of the pressure drop in the cushions (the symptom being the fans running more than normally) and additionally allowing dirt deposits in the cushions through water ingress (Figure 8).



Figure 6. Existing repairs to ETFE surface



Figure 7. Additional damage as a result of birds pecking at surface



Figure 8. Dirt deposits as a result of water ingress

The air handling unit was serviced in order to ensure the cushions could achieve their full inflation pressure (nominally around 300 Pa) and to reduce the necessity of having the fans running constantly. This process involves ensuring the air supply manifolds have sufficient sealing, a check on the electronics and replacement of filters (if any) to ensure satisfactory continued operation. In this case a suspected cause of inadequate pressure in the cushions was that the filters did not appear to have been changed for a considerable time.



Figure 9. Clogged filter contributing to restricted air flow



Figure 10. New filter to replace old

Dust and particles in the air drawn in to the air handling unit and caught in the filter (in place to prevent these particles from entering the cushion) had clogged up the air intake, reducing the amount of air that was able to be pumped into the cushions and contributing to the drop in pressure. Figure 9 shows an image of one of the filters before it was replaced with a new one (Figure 10). This not only highlights the need for regular maintenance inspections by the installer to ensure continual satisfactory operation of the system, but also the need for repair kits to be located on s ite to enable building management services to diagnose and remedy simple maintenance issues with the ETFE systems. Good practice at hand-over of the structure would be to include information on regular maintenance of the roof within the Operation and Maintenance (O&M) manual (including topics such as frequency of filter changes) that can be performed by building services. This will aid in diagnosing and remedying simple matters that arise over the life of the structure.

4 CASE STUDY 3 – BARNSLEY INTERCHANGE



Figure 11. Entrance to Barnsley Interchange Station showing externally visible array of ETFE cushions

In December 2010 a problem was reported with an ETFE structure at Barnsley interchange railway station; a cushion was not inflating and there was concern over snow and ice build-up. It was also reported on site that following a recent air inflation pump change full inflation had not been achieved in the two cushions which had suffered ponding due to the weight of snow.

A site-visit showed that a combination of sunlight and warm air in the cushion had melted the snow and with subsequent sub-zero ambient temperatures the water had frozen in the inverted cushion and become sheets of solid ice (Figure 12 & Figure 13).



Figure 12. Image of ice build-up on cushions as a result of ponding



Figure 13. Image showing ice build-up of approx. 3"

The resulting weight of the ice that had pooled on one of these cushions also appeared to damage a previous repair, detaching a seam that was held with ETFE repair tape. Opening of this seam seemed to be the cause of the pressure loss in the cushions, and not (as was initially thought) the service that was performed previously on the air handling unit.



Figure 14. Ice sheet being removed from ETFE cushion



Figure 15. Previous repair of split weld possibly due to weight of ice

These symptoms might possibly be attributed to the initial design of the structure in particular the patterning of the cushions, the design of which builds in sufficient cushion rise to reduce the lateral forces on the supporting steelwork. Comparing a cushion with a higher radius of curvature than the cushions shown here (under equal pressure) the lateral loading on the steelwork will be higher. This is often visually desirable as it can produce a pleasing aesthetic to the building; however this could result in the issue discussed above in addition to increasing the amount of supporting steelwork which will have to be more substantial to allow for the higher loads.

5 CONCLUSION

As with any structure, for ETFE systems it is necessary for building management to actively participate in the maintenance of the structure. This can be achieved by performing regular checks and preventative maintenance on the foil, the perimeter and the air handling units. By carrying out these checks it is possible to avoid instances where more substantial remedial works may be necessary and to ensure continuous effective operation of the structure.

Likewise it is also the duty of the installer to provide information to the building management team on how best to look after the ETFE system. This should involve sections in O&M manuals that cover what checks should be made, how often, and what actions they can perform to keep the system at optimum health. This can also include information on how often user-serviceable parts, such as filters, should be maintained. To ease these processes it must be considered in the design phase of the structure to allow safe access to the structure to perform visual checks to both the ETFE and the air handling units.

Additionally, as with any low-profile roof, the risks of ponding or snow build-up (potentially leading to issues were cushions could be damaged) should be taken into account at design stage to minimise those risks.

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