International Conference on Computational Methods in Marine Engineering
MARINE 2011
L.Eça, E. Oñate, J. García, T. Kvamsdal and P. Bergan (Eds)

NUMERICAL SIMULATION OF ENERGY BUOY MOTION IN WAVE MARINE 2011

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Key words: wave energy, energy buoy, offshore hydrodynamics, numerical computations.

Summary. This document provides a description of the numerical method for modelling the energy buoy motion on wave. Computational results were compared with results obtained from model tests carried out on model basin.

1 INTRODUCTION

In recent years, due to the obligations of EU countries, interest in renewable energy resources increases significantly. Most of the current investments in the renewable resources is associated with wind farms. Only a relatively small number of projects deals with the acquiring of energy from sea waves. This article describes the preliminary results of the project, whose subject is to study and optimize the new concept of energy buoy. The principle of the buoy concept is based on the use of wave energy, by inducing the pitching motion of the hull. From the bottom of the buoy extends a long column, at which end a turbine is located. Inside the column is a shaft line. The torque generated on turbine is bring from the turbine to a generator located inside the hull of the buoy.
This paper presents a description of numerical method which was develop for prediction of motion of the buoy on wave. The method allows for modelling of motion in six degree of freedom with taking into account interaction with elements of positioning system (anchor lines).

The concept of the buoy is presented below, Fig. 1.

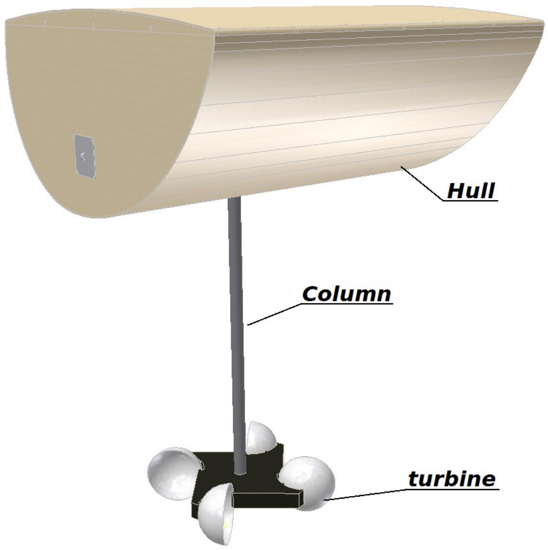


Figure 1: Vizualisation of the energy buoy concept

2 DESCRIPTION OF THE MODEL

2.1 Main Equations

The Equations of rigid body motion in 6 degrees of freedom are as follows:

$$\begin{aligned} m\ddot{x}_G &= \sum_i F_{xi} \quad , \\ m\ddot{y}_G &= \sum_i F_{yi} \quad , \\ m\ddot{z}_G &= \sum_i F_{zi} \quad , \end{aligned} \tag{1}$$

$$\begin{aligned} I_{xx_0}\ddot{\phi}_{x_0} - \left(I_{yy_0} - I_{zz_0}\right)\dot{\phi}_{y_0}\dot{\phi}_{z_0} &= \sum_i M_{x_0 i} \quad , \\ I_{yy_0}\ddot{\phi}_{y_0} - \left(I_{zz_0} - I_{xx_0}\right)\dot{\phi}_{z_0}\dot{\phi}_{x_0} &= \sum_i M_{y_0 i} \quad , \\ I_{zz_0}\ddot{\phi}_{z_0} - \left(I_{xx_0} - I_{yy_0}\right)\dot{\phi}_{x_0}\dot{\phi}_{y_0} &= \sum_i M_{z_0 i} \quad , \end{aligned} \tag{2}$$

where:

m - total mass of the body,

x_0, y_0, z_0 - the main central axes of inertia (a moving coordinate system),

x_G, y_G, z_G - coordinates of the center of the body mass,

$\dot{\phi}_{x_0}, \dot{\phi}_{y_0}, \dot{\phi}_{z_0}$ - instantaneous angular velocities around the main central axis,

$I_{xx_0}, I_{yy_0}, I_{zz_0}$ - main central moments of inertia,

F_{xi}, F_{yi}, F_{zi} - components of the *i*-th force vector (stationary coordinate system),

$M_{x_0 i}, M_{y_0 i}, M_{z_0 i}$ - components of the *i*-th torque vector (a moving coordinate system).

Forces and moments on right hand sides of above equations represents external extractions due to wave, anchoring system and gravity.

Ways of modelling the various excitations will be discussed later in this article.

2.2 Hydrostatic and hydrodynamic reactions

Hydrostatic forces are calculated by integrating the static pressure of the submerged area of hull surface of the buoy. Buoy hull is modelled using quadrilateral panels. During calculated the hydrostatic forces, acting on the panel, it is checked by algorithm whether the panel is fully immersed. If the panel is immersed partly, the force is calculated only for the wetted area. Hydrostatic reaction acting on the panel is described by the following formula:

$$\boldsymbol{R}_{HS} = -\rho_w \, g \, h_C \, S_{wet} \, \boldsymbol{n} \quad , \tag{3}$$

where:

$\boldsymbol{R}_{HS}$ - vector of hydrostatic forces, ρ_w - the density of water, g - acceleration of gravity, h_C , S_{wet} , $\boldsymbol{n}$ - respectively, the immersion of central point of the panel, the area of the panel surface and normal vector directed “outside” the hull.

Hydrodynamic reaction is a sum of two components of frictional resistance and pressure resistance. It is calculated as before, by summing the force at each panel. For a single panel we have:

$$\boldsymbol{R}_{HD} = -\frac{1}{2}\rho_w \left(C_F u_t \boldsymbol{V}_t + C_P u_n \boldsymbol{V}_n \right) S_{wet} \quad , \tag{4}$$

where:

$\boldsymbol{R}_{HD}$ - hydrodynamic resistance, C_F, C_P - drag coefficients of friction and pressure, $\boldsymbol{V}_t, \boldsymbol{V}_n$ - velocity vectors: tangent and normal to the surface of the panel, u_t, u_n - and the modules of the correspondent velocity components.

The resistance of appendages is calculated based on their drag coefficient. For example, the drag of section of cylindrical column is calculated as follows:

$$\Delta \boldsymbol{R}_{HDcyl} = -\frac{1}{2}\rho_w \left(C_{Tcyl} u_o \boldsymbol{V}_o \right) D_{Cyl} \Delta l_{Cyl} \quad , \tag{5}$$

$$\boldsymbol{V}_o = \boldsymbol{V} - (\boldsymbol{V} \cdot \boldsymbol{t})\boldsymbol{t} \quad , \tag{6}$$

Where:

$\boldsymbol{V}$ is a vector of velocity of the concerned section (relative to the water), $\boldsymbol{t}$ - tangent vector to the axis of the cylinder, C_{Tcyl} - total resistance coefficient of the cylinder's section [4], D_{Cyl} - diameter of the cylinder, Δl_{Cyl} - length of the cylinder's section

2.3 Hydrodynamic forces due to wave

In the presented method water wave motion is modelled using a linear theory of waves. In addition, corrections have been introduced taking into account the impact of the free surface deformation on the value of hydrostatic pressure.
The velocity potential for a deep water regular wave is determined by the formula [1], [2]:

$$\Phi(x, z, t) = \frac{ag}{\omega} e^{kz} \sin(kx - \omega t) \quad , \tag{7}$$

where:

- a - wave amplitude,
- g - gravity,
- ω - wave circular frequency,
- k - wavenumber, $k = \omega^2 / g$ for deep water
- x, z - global coordinates,
- t - time.

The condition on the free surface is defined as follows

$$\zeta = -\frac{1}{g}\frac{\partial \Phi}{\partial t} \tag{8}$$

By substituting the velocity potential expression to the above condition we obtain the wave profile ordinate:

$$\zeta(x, t) = a \cos(kx - \omega t) \tag{9}$$

The components of the velocity vector, obtained by differentiated the potential in terms of coordinate *x* and *y*, are:

$$u_x(x, z, t) = \frac{akg}{\omega} e^{kz} \cos(kx - \omega t) \quad , \qquad u_z(x, z, t) = \frac{akg}{\omega} e^{kz} \sin(kx - \omega t) \quad . \tag{10}$$

Hydrodynamic pressure is calculated from the Bernoulli equation:

$$-\frac{\partial \Phi}{\partial t}+\frac{1}{2}u^2+\frac{p}{\rho}+g(z+\zeta)=0 \quad , \tag{11}$$

Hence, after substitution of variables and conversion, we obtain:

$$p_{wave}(x,z,t)=\rho\, a\, g\, e^{kz}\cos(kx-\omega t)-g(z+\zeta)-\frac{1}{2}\rho\left(\frac{akg}{\omega}e^{kz}\right)^2 \tag{12}$$

2.4 Added masses

Impacts, related to the presence of water added masses, is modelled using a simplified method based on the strip theory [2],[3]. Buoy hull is divided into strips with a specified thickness.

Each section is associated a mass of water which causes hydrodynamic reaction during accelerated motion. Mass of water around frame depends on its shape and direction of motion. For linear motion, the value of the hydrodynamic reactions acting on the i-th slice is calculated with the formulas:

$$\begin{aligned} \Delta R^i_{AMy} &= -m^i_{22}a^i_y\,\Delta l^i \quad , \\ \Delta R^i_{AMz} &= -m^i_{33}\,a^i_z\,\Delta l^i \quad . \end{aligned} \tag{13}$$

The torque around the axis Ox, due to presence of added masses, is calculated as follows:

$$\Delta M^i_{AMx}=-m^i_{44}\,\epsilon^i_x\,\Delta l^i \quad . \tag{14}$$

where:

$\Delta R^i_{AMy}, \Delta R^i_{AMz}$	- hydrodynamic reaction due to added masses on y and z direction,
ΔM^i_{AMz}	- hydrodynamic torque due to added masses on x direction,
m^i_{22}, m^i_{33}	- added masses of *i*-th frame for horizontal (*y*-direction) and vertical (*z*-direction) motions, respectively,
m^i_{44}	- added masses in motion around the *x*-axis,
a^i_y, a^i_z	- linear acceleration of *i*-th frame in the *y*- and *z*-direction, respectively,
ϵ^i_x	- angular acceleration of *i*-th frame,
Δl^i	- the length (thickness) of *i*-th frame section.

Coefficients m_{ii} were taken from the literature [2], [3]. Coefficients depend on the frame's immersion and the circular wave frequency.

Vector of frame acceleration is calculated as:

$$\boldsymbol{a}_i = \boldsymbol{a}_G + \boldsymbol{\epsilon} \times \boldsymbol{r}_{G P_{Fr}} + \boldsymbol{\omega} \times \boldsymbol{\omega} \times \boldsymbol{r}_{G P_{Fr}} \quad , \tag{15}$$

where:

$\boldsymbol{a}_G$ - acceleration of gravity center of the buoy hull,

$\boldsymbol{\epsilon}$ - angular acceleration of buoy hull,

$\boldsymbol{\omega}$ - rotational speed of buoy hull,

$\boldsymbol{r}_{G P_{Fr}}$ - $G P_{Fr}$ vector, where G – gravity centre, P_{Fr} – point at frames keel

Total hydrodynamic reaction due to added masses is calculated as follows:

$$\begin{gathered} \boldsymbol{R}_{AM} = \sum_i \boldsymbol{R}_{AM}{}^i \quad , \\ \boldsymbol{M}_{AM} = \sum_i \left(\boldsymbol{r}^i_{G P_{Fr}} \times \boldsymbol{R}_{AM}{}^i + \boldsymbol{M}^i{}_{AM} \right) \quad , \end{gathered} \tag{16}$$

where:

$$\begin{gathered} \boldsymbol{R}_{AM}{}^i = R^i_{AMy} \boldsymbol{e}_{Hy} + R^i_{AMz} \boldsymbol{e}_{Hz} \quad , \\ \boldsymbol{M}_{AM}{}^i = M^i_{AMx} \boldsymbol{e}_{Hx} \quad , \end{gathered} \tag{17}$$

$\boldsymbol{e}_{Hx}, \boldsymbol{e}_{Hy}, \boldsymbol{e}_{Hz}$ - versors of hull local coordinate system

In order to calculate the above reactions, the buoy acceleration ought to be known. The acceleration, in turn, depends on the sum of the forces acting on the hull. Therefore, calculation of the forces due to added masses shoud be done by successive approximations.

2.5 Reaction due to anchoring system

The anchoring system of energy buoy consist of two main ropes and two auxiliary lines. Main ropes are (almost) vertical. The ropes are strained to increase draft of the boy. Auxiliary lines are horizontal, they role is to prevent yawing. The stiffness of vertical ropes is high to prevent heave motions. However horizontal lines are flabby, in order to minimize yawing without reducing pitching, which is the desired motion. The sketch of anchoring system is presented at Fig. 2.

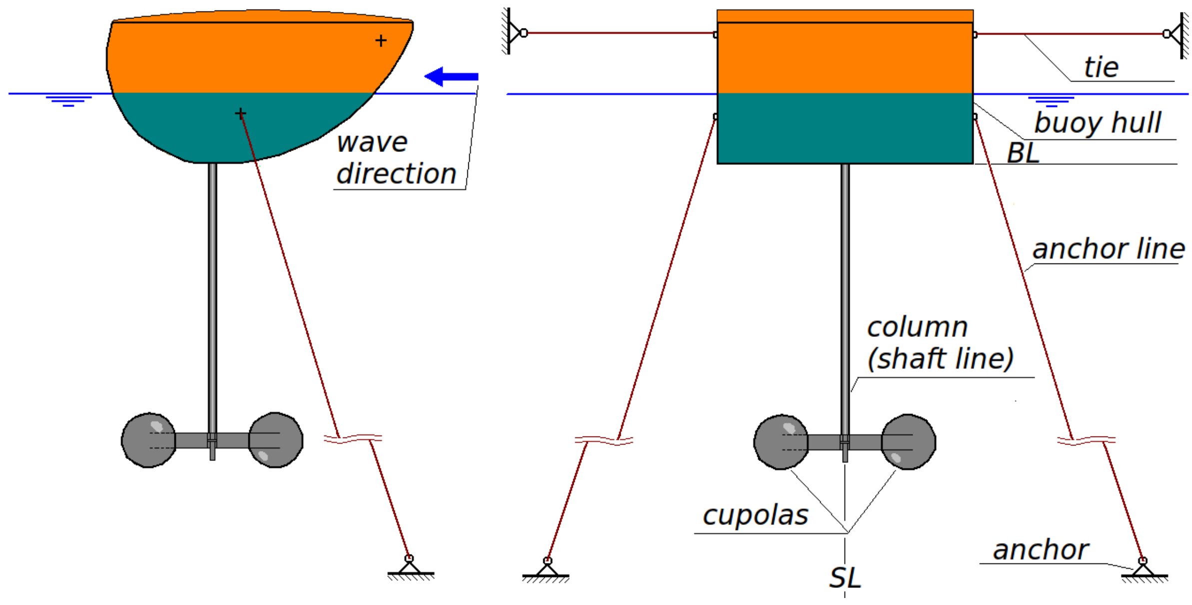


Figure 2: a) Sketch of anchoring system

Rope stiffness characteristics are described by the following formula:

$$\boldsymbol{R}_R = \begin{cases} s_R \dfrac{\Delta l_R}{l_{R0}} \boldsymbol{e}_R, & \Delta l_R > 0; \\ \boldsymbol{0}, & \Delta l_R \leq 0; \end{cases} \quad , \tag{18}$$

where:

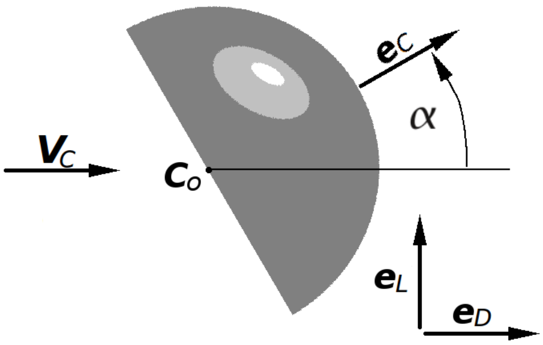
- $\boldsymbol{R}_R$ - rope pull force,
- s_R - rope (linear) stiffness $s_R = EA$, where E is a Young's modulus, A is a ropes cross-section area,
- Δl_R - elongation of the rope, $\Delta l_R = l_{R1} - l_{R0}$,
- l_{R0} - initial length of the rope (without tension),
- $\boldsymbol{e}_R$ - ropes directional versor.

2.6 Hydrodynamic reaction acting on the turbine canopies

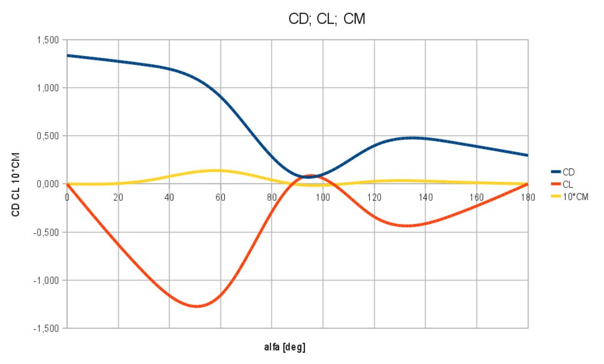
The buoy turbine is moving progressive (due to pitch motion) and rotating Hydrodynamic reaction on rotor is calculated as the sum forces induced on canopies and additional elements to which the canopies are attached. Hydrodynamic reaction induced on singular (isolated) canopy can be written as:

$$\boldsymbol{R}_{HDc}=\frac{1}{2}\rho u_c^2 S_c\left(\boldsymbol{e}_D C_{Dc}(\alpha)+\boldsymbol{e}_L C_{Lc}(\alpha)\right) \quad , \tag{19}$$

where:
u_c - module of velocity of canopy (relative to the water), S_c - frontal canopy area, $\boldsymbol{e}_D, \boldsymbol{e}_L$ - directional versors for drag and lift force, $C_{Dc}(\alpha), C_{Lc}(\alpha)$ - characteristics of drag coefficient and lift force coefficient in a function of angle of attack
To use the above formula, one must define the characteristics of drag and lift coefficients. Within the presented project $C_{Dc}(\alpha), C_{Lc}(\alpha)$ characteristics were calculated using RANSE-CFD, Fig. 3.



a)



b)

Figure 3: a) Sketch of cupola, assumed convention; b) Lift and drag force coefficents (from RANSE-CFD)

The total hydrodynamic force and moment reaction induced on the turbine is calculated from the below formula:

$$\boldsymbol{R}_{HDT}=\sum_i \boldsymbol{R}_{\boldsymbol{HDci}}+\Delta \boldsymbol{R}_{HDT} \quad , \tag{20}$$

$$\boldsymbol{M}_{HDT}=\sum_i \boldsymbol{r}_i \times \boldsymbol{R}_{HDTi} \quad , \tag{21}$$

where:
$\boldsymbol{r}_i$ – vector perpendicular to the axis connecting the point on the axis of rotation of the turbine and the center of the i-th canopy.
Inflow velocity of water on the canopy is calculated from the formula:

$$\boldsymbol{V}_i=\boldsymbol{V}_{\boldsymbol{inf}}-\left(\boldsymbol{V}_{G_H}+\boldsymbol{\omega}_H \times \boldsymbol{r}_{GO}+\omega_T \boldsymbol{e}_z \times \boldsymbol{r}_{OCo}\right) \quad , \tag{22}$$

where:
$\boldsymbol{V}_{inf}$ - inflow velocity,
$\boldsymbol{V}_{G_H}$ - velocity of center of gravity of buoy,

$\boldsymbol{\omega}_H$ - rotational speed of buoy hull,

$\boldsymbol{r}_{GO}$ - vector with beginning at a central point and end point at the axis of rotation.

The total torque of the turbine is the difference between hydrodynamic interactions and torque response to the generator:

$$\boldsymbol{M}_T = \boldsymbol{M}_{HDT} + \boldsymbol{M}_{Gen} \quad . \tag{23}$$

Torque on the generator is a function of turbine rotational speed. In this model, we have assumed that this is a linear function:

$$\boldsymbol{M}_{Gen} = a_{Gen} \boldsymbol{\omega}_T \tag{24}$$

Where a_{Gen} ia a torque growth rate.
Hence, the equation of rotation of the turbine becomes:

$$I_0 \frac{d\,\boldsymbol{\omega}_T}{dt} = \boldsymbol{M}_{\boldsymbol{HDT}} + a_{Gen} \boldsymbol{\omega}_T \tag{25}$$

I_0 - moment of inertia of the turbine relative to its axis (including the moments of inertia of the mechanisms, which are rotating together with the turbine)

3 NUMERICAL SIMULATION AND MODEL TESTS

3.1 Description of the configuration

Calculations were made for the buoy model of the geometry and mass shown in the table below:

(Assumed) scale factor	λ	5
Mass of the buoy (model)	m	671 kg
Draught of the boy before anchoring	T_0	0.320 m
Draught of the anchored buoy	T_k	0.370 m
Moment of inertia related to axis Ox_0	I_{xx_0}	372 kg m^2
Moment of inertia related to axis Oy_0	I_{yy_0}	372 kg m^2
Moment of inertia related to axis Oz_0	I_{zz_0}	311 kg m^2
Longitudinal centre of gravity	x_G	0.793 m
Transverse centre of gravity	y_G	0.000 m
Height of centre of gravity	z_G	0.408 m

Table 1 : Model of the buoy - test configuration

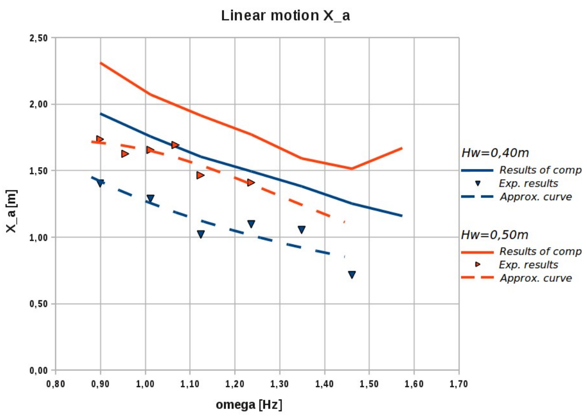
The regular wave parameters (for model) were as follows:

Wave height:	H_{WM} [m]	0.4; 0.5
Circular frequency:	ω_{WM} [Hz]	1.8 – 3.2

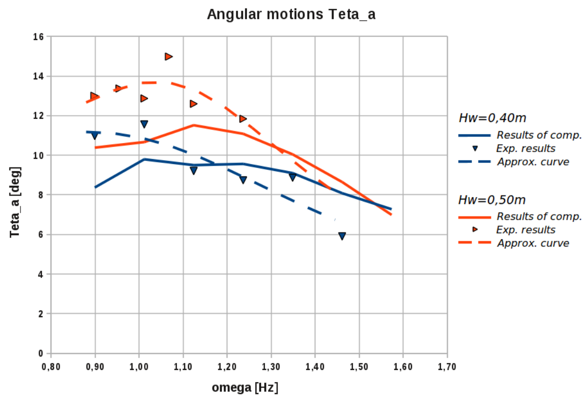
Table 2 : Wave parameters (for the model)

3.2 Results

The figure below contains results of numerical modelling of pitch motion extrapolated to full scale and comparison with model tests results. Model tests has been carried out in large model basin of Ship Design and research Centre (CTO S.A.) in Gdansk. The size of model basin is 260m x 12m x 6m, Fig. 5.



a)



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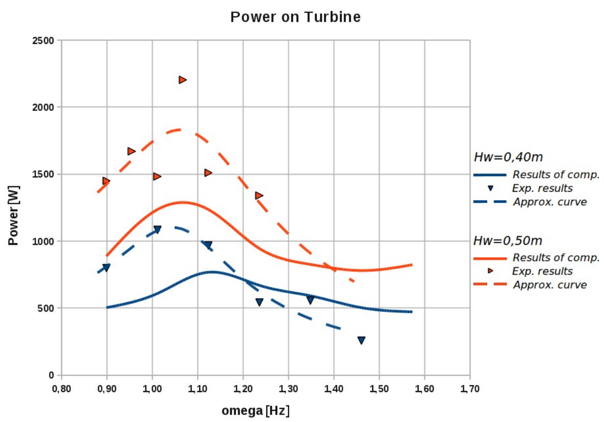

c)

Figure 4: Comparison of the results of calculations with experimental data: a) linear motion along the x-axis – surge; b) angular motions – pitch , c) power generated on the turbine

Pictures below shows the buoy during model test in CTO's towing tank:

a) b)

Figure 5: a) Preparations for model test; b) Enwrgy buoy during tests on regular wave

4 CONCLUSIONS

- The computational results of buoy pitch motions were close to results obtained from model tests, however surge motions were overestimated (amount of error is about 30-50%),
- Numerical results of power generated on turbine are (mainly) below the curve obtained from model tests. The average difference between computational predictions and experimental results is about 30% .
- The parameters of numerical model should be corrected after carefully analysis of model tests results. The authors believe, that correction of added masses and dumping coefficients should improve the accuracy of the model results,
- Although the computational model parameters still require adjustment, the accuracy of presented results provides, that the model can be used for preliminary evaluation of tested configuration.

ACKNOWLEDGEMENT

The research presented in this paper has been conducted with the support of the Research Grant no. N N511 367 537 of the Polish Ministry of Science and Higher Education

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