Solving the Laplace-Beltrami Equation on Curved Two-dimensional Manifolds using the Finite Element Method with Catmull-Clark Subdivision Surfaces

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The early works on isogeometric analysis focused on geometries modelled through Non-Uniform Rational B-Splines (NURBS). However, a NURBS surface is a tensor product surface generated by two NURBS curves. This has limitations in modelling geometries with complex topologies. Catmull-Clark subdivision surfaces are derived as tensor-products of Catmull-Clark subdivision curves, but allowing "extraordinary vertices". This ensure the Catmull-Clark subdivision surfaces can be used for modelling complex geometries with arbitrary topologies.

The present work proposes and analyses an isogeometric approach for solving the Laplace-Beltrami equation on a two-dimensional manifold embedded in three-dimensional space using the finite element method with Catmull-Clark subdivision surfaces. The Catmull-Clark subdivision bases are used to discretise both the geometry and physical fields. A fitting method is also proposed to generate control meshes to approximate any given geometries with Catmull-Clark subdivision surfaces. Sample points are chosen from the given surface and least square fitting is used to obtain an optimal control mesh for the surface. The Catmull-Clark subdivision method is also compared to the conventional finite element method. Subdivision surfaces with no extraordinary vertices have the optimal $p + 1$ convergence rate. However the extraordinary vertices introduce relatively large errors in the analysis which decreases the convergence rate. A comparative study shows the effects of the number and valences of the extraordinary vertices. An adaptive quadrature scheme and other remedies are also implemented to reduce the error.