Efficient matrix assembly for hierarchical B-splines

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ABSTRACT

Hierarchical splines are a useful tool in the context of isogeometric analysis, as they allow for local refinement which is not possible in the case of classical tensor-product splines. Unfortunately, similarly as for classical tensor-product splines, the assembly of the Galerkin matrices can be very expensive from the computational point of view, especially for moderate or high spline degree.

To address this issue, we consider the approach presented in [1] for the non-hierarchical case, whose key ingredient is weighted quadrature. The basic idea of weighted quadrature is to approximate the integrals by incorporating the test function in the integration weights. On the other hand, the trial function, the geometry parametrization and the coefficients form the integrand function. Of the main features of this approach is that the number of quadrature points is practically independent of the spline degree p. This is a significant advantage when compared to other more common quadrature approaches, like e.g. Gauss quadrature, where the number of points grows with p.

In this work, we adapt the ideas from [1] to the hierarchical case, by exploiting the local tensor structure of the basis to efficiently compute the quadrature weights. The cost of the resulting assembly algorithm scales favorably with respect to p, as assessed by both complexity analysis and numerical experiments.

REFERENCES

[1] Calbrò, F., Sangalli, G., and Tani, M. Fast formation of isogeometric Galerkin matrices by weighted quadrature. *Comput. Methods Appl. Mech. Engrg.* (2017) **316**:606–622.