Fully C^1-continuous isogeometric analysis of deforming surfaces of arbitrary shape and topology

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ABSTRACT

The high order of continuity achievable using Isogeometric Analysis is key in solving challenging, contemporary scientific problems which are governed by high order differential equations. Although Finite Element Analysis allows for the reduction of the required continuity properties, C^1-continuity is nevertheless needed, for instance, in solving problems related to thin shells and phase field models. In particular, numerical methods to understand the mechanics of lipid bilayers require C^1-continuity since they behave like a fluid in-plane but bend elastically out-of-plane.

When studying deforming surfaces such as lipid bilayers, we require, besides others, two particular geometric requirements: The ability to represent topologies of arbitrary genus and shape, as well as C^1-continuous basis functions. However, when constructing a mesh for IGA using NURBS surfaces, merging of NURBS patches generally results in a C^0-continuous parametrization. Subdivision surfaces, in contrast, generalize the idea of NURBS patches to topologies of arbitrary genus by allowing for extraordinary points. Previous attempts to attain C^1-continuity for the study of thin shell models using Loop and Catmull-Clark subdivision, however, are unable to represent conic sections [1, 2].

We propose a novel method using subdivision surfaces that allow for the construction of arbitrary topologies and shapes, including conic sections. The method maintains C^1-continuity under arbitrary deformations. Since the proposed method relies on subdivision surfaces, the method also allows for local refinement. We present several studies using our proposed method, including examples from shells seen in classical engineering as well as others motivated by the mechanics of lipid bilayers [3].

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