IGA-based multi-index stochastic collocation for uncertainty quantification

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ABSTRACT

Partial differential equation (PDE) models are standard tools for solving engineering problems. However, the parameters of PDEs, for example, the coefficients, forcing terms, boundary and initial conditions are usually not known exactly, and thus could be treated as random variables (or random fields).

Uncertainty quantification (UQ) includes estimating how the randomness of these input parameters affects the solution of the PDE or a functional thereof. UQ techniques often rely on repeatedly solving the PDE for different values of the input parameters, which tends to require a substantial computational effort. To reduce such effort multi-level and multi-index methods have recently been proposed. Multilevel methods are based on differences of numerical approximations at different refinement levels controlled by a refinement parameter (for instance the mesh element size, or the number of quadrature points). Multi-index methods are high-dimensional versions of the multilevel technique in the case when multiple refinement parameters are controlling the approximation. These methods are "black-box" techniques in the sense that they allow reuse of legacy PDE solvers, and are suitable for parallelization.

In this talk, we describe Multi-index Stochastic Collocation (MISC), which is closely related to the established sparse-grids stochastic collocation method for the numerical approximation of the solution of PDEs with random data. As show in [1], we join standard MISC together with isogeometric analysis (IGA) solvers to enable solutions of UQ problems on complex geometries. In particular, IGA solvers are suitable in the MISC methodology since they have an intrinsic tensor structure, and this was exploited in [2] to propose a sparse-grid IGA solver.

We show the computational efficiency of the proposed IGA-based MISC that approximates expectations of quantities of interests depending on the solution of elliptic PDEs. Numerical results show MISC is performing better than alternative approaches such as multilevel Monte Carlo and multi-index Monte Carlo.

REFERENCES

[1] Beck, J., Tamellini, L. Tempone, R. IGA-based multi-index stochastic collocation for random PDEs on arbitrary domains. *Comput. Method Appl. M.* (2019) **351**: 330–350.

[2] Beck, J., Sangalli, G., Tamellini, L. A sparse-grid isogeometric solver. *Comput. Method Appl. M.* (2018) **335**: 128–151.