Smooth Splines on Meshes with Polar Singularities

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ABSTRACT

Representing arbitrary surfaces with a finite number of polynomial patches requires the introduction of polar points for high-valence neighborhoods in quadrilateral meshes. Such holes can be filled by means of polar spline surfaces, where the basic idea is to use periodic spline patches with one collapsed boundary. Building splines over such singularities requires special rules to ensure smoothness; ensuring suitability for design and analysis imposes further constraints.

A general framework for building C^k polar spline parametric patches of arbitrary degree and with arbitrary number of elements at the hole boundary was presented in [1]. Apart from a simple, geometric construction of smooth basis functions, it was shown that it is possible to endow upon the spline basis interesting properties such as non-negativity and partition of unity. Numerical experiments indicating optimal approximation behavior, even at the singular point, were presented in [1]. In this talk, we will present construction of smooth parameterizations of conic sections and solutions of high-order PDEs on surfaces (e.g., Cahn-Hilliard equations); the smoothness afforded by the smooth spline basis allows straightforward numerical discretization and implementation. Moreover, we will present a pointwise divergence free discretization for Stokes flow on arbitrarily curved surfaces.

REFERENCES

[1] D. Toshniwal, H. Speleers, R.R. Hiemstra, and T.J.R. Hughes. *Multi-degree smooth polar splines: A framework for geometric modeling and isogeometric analysis*, Comput. Methods Appl. Mech. Engrg. 316, 1005-1061, 2017.