Space-time isogeometric efficient solver for parabolic problems

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ABSTRACT

In this talk, we focus on the weak formulation of the heat equation presented in [2]. We consider a space-time isogeometric discretization of the problem, adopting smooth splines both in space and in time. This choice makes the numerical method inherently global. In this context, the main challenge is the design of a computationally efficient solver.

Exploiting the tensor structure of the basis functions, we propose a preconditioner that is based on the fast diagonalization (FD) method, a direct solver that has recently been employed in a preconditioning strategy devised for a least-squares formulation of the heat equation in [1]. In order to apply the FD method, the preconditioning matrix is required to be the sum of Kronecker products of matrices that admit a stable generalized eigendecomposition. While for the least-squares formulation these requests are easy to achieve, in our context we have to dealt with time matrices that does not admit a stable generalized eigendecomposition. We circumvent this difficulty in [3] by introducing an ad-hoc factorization of the time-dependent matrices involved which allows to design a solver conceptually similar the fast diagonalization, with similar computational cost and robust with respect to the spline degree.

The computational times required for the application of the preconditioning strategy, for a serial execution, are almost proportional to the number of degrees-of-freedom and independent of the polynomial degree. Furthermore, this approach is also suitable to parallelization.

REFERENCES

[1] Montardini, M., Negri, M., Sangalli, G., and Tani, M., Space-Time Least-Squares Isogoemetric Method for Parabolic Problems, *arXiv:1809.10026v4 [math.NA]*

[2] Steinbach, O., Space-time finite element methods for parabolic problems. *Comp. Met. Appl. Math.*, (2015) **15(4)**: 551–566.

[3] Loli, G., Montardini, M., Sangalli, G., and Tani, M., Space-time isogeometric preconditioners for parabolic problems, *in preparation*.