

# Modeling Microarchitectural Plates and Shells via Generalized Continuum Models – Variational Formulations, Isogeometric Analysis and Model Validation

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## ABSTRACT

The plate and shell models of linear strain gradient elasticity theory [1,2,3,5] are studied in the framework of continualization, or homogenization, of microstructural beams, plates and shells such as lattice microarchitecture structures [4,5]. In particular, the roles of length scale parameters and structural dimensions such as thickness are addressed by parameter studies and model validation. Numerical results obtained via isogeometric finite element analysis for the generalized plate and shell models [2,3] and the corresponding three-dimensional solid models are compared to each other for verifying the corresponding dimension reduction procedures. For specific multi-layer honeycomb applications [4,5], the strain gradient plate and shell models are compared, in the sense of model validation, to the corresponding (unhomogenized) fine-scale solid models relying on classical elasticity and standard finite element analysis.

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