

# Modeling Microarchitectural Plates and Shells via Generalized Continuum Models – Variational Formulations, Isogeometric Analysis and Model Validation

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## ABSTRACT

The plate and shell models of linear strain gradient elasticity theory [1,2,3,5] are studied in the framework of continualization, or homogenization, of microstructural beams, plates and shells such as lattice microarchitecture structures [4,5]. In particular, the roles of length scale parameters and structural dimensions such as thickness are addressed by parameter studies and model validation. Numerical results obtained via isogeometric finite element analysis for the generalized plate and shell models [2,3] and the corresponding three-dimensional solid models are compared to each other for verifying the corresponding dimension reduction procedures. For specific multi-layer honeycomb applications [4,5], the strain gradient plate and shell models are compared, in the sense of model validation, to the corresponding (unhomogenized) fine-scale solid models relying on classical elasticity and standard finite element analysis.

## REFERENCES

- [1] J. Niiranen, A. H. Niemi: Variational formulations and general boundary conditions for sixth-order boundary value problems of gradient-elastic Kirchhoff plates. *European Journal of Mechanics - A/Solids*, 61, 164–179 (2017).
- [2] J. Niiranen, J. Kiendl, A. H. Niemi, A. Reali: Isogeometric analysis for sixth-order boundary value problems of gradient-elastic Kirchhoff plates. *Computer Methods in Applied Mechanics and Engineering*, 316, 328-348 (2017).
- [3] V. Balobanov, J. Kiendl, S. Khakalo, J. Niiranen: Kirchhoff–Love shells within strain gradient elasticity: weak and strong formulations and an  $H^3$ -conforming isogeometric implementation. *Computer Methods in Applied Mechanics and Engineering*, 344, 837-857 (2019).
- [4] S. Khakalo, V. Balobanov and J. Niiranen: Modelling size-dependent bending, buckling and vibrations of 2D triangular lattices by strain gradient elasticity models: Applications to sandwich beams and auxetics. *International Journal of Engineering Science*, 127, 33–52 (2018).
- [5] S. Khakalo and J. Niiranen: Anisotropic strain gradient thermoelasticity for cellular structures: plate models, parameter identification and isogeometric analysis. Submitted for publication (2019).