Multiparametric Shell Eigenvalue Problems

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ABSTRACT

Real-life engineering problems are often subject to significant parameter uncertainties due to e.g. manu-facturing imperfections or uncertain material properties. Including such uncertainties in the underlying mathematical model results in a multiparametric problem, in which the stochastic dimension of the sy-stem is approximated by using a finite, but possibly large, number of parameters.

In this talk we consider multiparametric shell eigenvalue problems that arise from introducing random material properties to the shell model. A notable feature of such problems is that, whereas the smallest eigenmode of a second order elliptic problem is always simple, for a shell eigenvalue problem this need not be the case. In fact we demonstrate that the eigenmodes of shells of revolution appear to be clustered due to periodic nature of the angular components and in special cases also due to physical properties of the problem.

Sparse stochastic collocation [1] and spectral inverse iteration algorithms [2] have recently been proposed for resolving simple eigenmodes of multiparametric eigenvalue problems. A certain class of axisymme-tric shells can indeed be reduced to one-dimensional problems in which eigenvalues are separated by wavenumber, and therefore the aforementioned algorithms may be applied directly. In order to deal with more general cases, however, we extend the algorithms in a way that subspaces associated to clustered eigenvalues may also be computed [3].

Numerical experiments suggest that the proposed subspace algorithms are able to resolve the eigen-clusters effectively. The observed convergence rates for the stochastic collocation and spectral inverse iteration algorithms agree well with the theoretical estimates known for elliptic second order problems.

REFERENCES

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