Explicit estimates for spline approximation of arbitrary smoothness in isogeometric analysis

Espen Sande*, **Carla Manni and Hendrik Speleers**

Department of Mathematics, University of Rome Tor Vergata
Via della Ricerca Scientifica 1, 00133 Rome, Italy

ABSTRACT

Classical error estimates in spline approximation read as follow [1]: for any $u \in H^r(0,1)$, and any knot vector τ, there exists a spline $s \in \mathcal{S}^k_{p,\tau}$ such that

$$\|u - s\| \leq C(p,k,r)h^r \|u^{(r)}\|, \quad p \geq r-1,$$

where h denotes the maximal knot distance of τ and $\|\cdot\|$ is the L^2 norm. Here the "constant" $C(p,k,r)$ is independent of h, but depends on the degree p, the smoothness of the spline k, and the Sobolev regularity r. In [2] a representation in terms of Legendre polynomials was exploited to provide a constant $C(p,k,r)$ of the form $C/(p-k)^r$ for spline spaces of degree $p \geq 2k+1$ and smoothness C^k.

In this talk we extend the result of [2] in two ways: (i) we prove that the constant $C(p,k,r)$ is of the form $C/(p-k)^r$ for any $-1 \leq k \leq p-1$ and (ii) we provide an explicit upper bound of the unknown constant C. We further discuss the extension of these results to the case of tensor product spline approximation and to the case of mapped geometries, both for a single patch and for multiple patches.

REFERENCES

[1] L. L. Schumaker. *Spline Functions: Basic Theory, 3rd edn*. Cambridge University Press (2007).

[2] Beiro da Veiga, L., Buffa, A., Rivas, J. and Sangalli, G. *Some estimates for $h-p-k$-refinement in Isogeometric Analysis* Numer. Math. (2011) 118: 271.