$2k$-order generalized-α methods for parabolic and hyperbolic equations

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ABSTRACT

The generalized-α method was introduced by Chung and Hulbert in [3] for solving (hyperbolic) structural dynamics problems. The method provides second-order accuracy in time and has a feature of user-control on the numerical dissipation in the higher frequencies of the discrete spectrum. This method includes a wide range of time integrators such as the Newmark method, the HHT-α method by Hilber, Hughes and Taylor, and the WBZ-α method by Wood, Bossak and Zienkiewicz; see [3]. The generalized-α method was then extended to computational fluid dynamics governed by the parabolic differential equations such as the Navier-Stokes equations [4].

We propose a new class of high-order generalized-α methods that maintain all the attractive features. In particular, we extend the method to $2k$ order of accuracy in time, where $k > 1$. For hyperbolic problems, the accuracy is obtained by solving k matrix systems implicitly and updating the other $2k$ variables explicitly at each time-step. For parabolic equation, we solve k matrix systems and update other k variables. The dissipation control is also provided by introducing parameters corresponding to each set of equations. Hence, our high-order schemes require simple modifications of the available implementations of the generalized-α method.

We consider finite elements and isogeometric analysis for the spatial discretization. The overall method maintains the optimal rates (h^{p+1} in L^2 norm and h^p in H^1 semi-norm of solution) in space. We present the spectrum analysis on the amplification matrix by following the analysis in [1, 2] and establish that the method is unconditionally stable. Various numerical examples illustrate the performance of the overall methodology and show the optimal approximation accuracy.

REFERENCES

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