

# **B-spline approximation of anisotropic diffusion problems: spectral symbols and anisotropic multi-iterative solvers**

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## **ABSTRACT**

In plasma physics, tokamak reactors are magnetic toroidal confinement devices that aim to harvest energy from the fusion of small atomic nuclei heated to the plasma state. The heat diffusion in a tokamak shows an intrinsic anisotropy that generates ill-conditioning and consequently various numerical difficulties.

In this work, we focus on the discretization of anisotropic diffusion operators with B-splines. In order to overcome the difficulties caused by the ill-conditioning, we conduct a detailed spectral study of the resulting coefficient matrices, by using the theory of generalized locally Toeplitz (GLT) sequences [1]. We exploit then the obtained spectral information to design fast anisotropic multi-iterative solvers for the corresponding linear systems combining anisotropic multigrid techniques [2] with GLT-based preconditioners for Krylov methods.

## **REFERENCES**

- [1] Garoni C., and Serra-Capizzano S. *Generalized Locally Toeplitz Sequences: Theory and Applications - Vol I*. Springer Monographs, (2017).
- [2] Fischer R., and Huckle T. Multigrid methods for anisotropic BTTB systems. *Linear Algebra Appl.* (2006) **417**: 314-334.