Construction of C^2 Cubic Splines on Refined Triangulations

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ABSTRACT

The key idea in Isogeometric Analysis is to use spline representations for both the modeling of physical domains and the approximation of the field variables of the governing differential problems. Spline representations with high smoothness proved to be a winning tool in several applications. Smooth tensor-product B-splines are a common choice. However, the tensor-product structure puts severe restrictions on parameterizations and adaptive refinement. This motivates the interest in alternative smooth spline structures, for example, splines on triangulations.

Piecewise polynomials of degree 3 of class C^2 are very appealing because they couple the low degree with a smoothness which allows to efficiently address several problems. When dealing with a general triangulation, to obtain C^2 smoothness in a stable manner, one must use polynomials of degree 9 on each triangle. An alternative is to use lower-degree macro-elements that subdivide each triangle into a number of subtriangles (or more generally subdomains). The most common macro-structures are the Powell-Sabin split and the Clough-Tocher split. The minimum degree to get C^2 smoothness is 5 on the Powell-Sabin split, while piecewise polynomials of degree 6 are needed to achieve C^2 smoothness on the Clough-Tocher split, see [1, 3].

In this talk we consider a family of macro-elements of degree p and maximal smoothness $p-1$ on a triangular region. In particular, we detail the important case of C^2 cubic macro-elements and we discuss the construction of a suitable local B-spline representation for the related spline space. The local B-spline functions are simplex splines [2]. They possess several interesting properties, such as local support, linear independence, and nonnegative partition of unity. The proposed B-spline representation has additional interesting features both from the local and the global point of view:

- on each triangle, it exhibits a natural definition of control points and an intuitive control net which locally mimic the shape of the spline surface;
- it characterizes C^2 joins to neighboring triangles in terms of simple geometric conditions;
- it allows for a local construction for any element of the full space of C^2 cubic splines on the considered refinement for any triangulation.

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