Analysis of BPX preconditioners for (truncated) hierarchical B-splines

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ABSTRACT

Hierarchical B-splines are one of the most used types of splines breaking the tensor-product structure of B-splines allowing for local refinement. They are defined based from a multilevel structure, and their implementation is relatively simple. For this reason, they have been extensively used to construct adaptive schemes for the solution of partial differential equations in the context of IGA, see for instance [1] and references therein.

In this talk we present the construction of additive multilevel preconditioners, also known as BPX, for the solution of the linear system arising in isogeometric adaptive schemes with (truncated) hierarchical B-splines. Our focus is on the construction of a suitable multilevel decomposition. Exploiting the locality of the basis functions, in a similar way to what has been done for adaptive schemes based on T-splines [3], our decomposition provides smaller subspaces than the ones presented in previous works [4], reducing the computational effort on each level.

We prove that, under a suitable grading of the mesh, the condition number of the preconditioned system is bounded independently of the number of levels. The proof is based on the stability of the decomposition, and the strengthened Cauchy-Schwarz inequality [2]. We present several numerical examples to validate the theoretical results, and the performance of the preconditioner.

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