A constraint based local refinement methodology for isogeometric analysis (IGA) of the weighted least-squares (WLS) form of the neutron transport equation with application to radiation shielding analyses

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ABSTRACT

In this paper a constraint based local refinement algorithm is developed for the weighted least-squares (WLS) form of the neutron transport equation discretised using isogeometric analysis (IGA). The WLS equation is an elliptic partial integral-differential equation (PIDE). When this PIDE is discretised using a Bubnov-Galerkin (BG) discretisation of the weak form of the equation it forms symmetric positive definite (SPD) matrices which are amenable to solution by computational efficient linear iterative solution algorithms. The WLS equation may be used to model the transport of neutral particles through a host medium such as a nuclear reactor core or a radiation shield within a nuclear power plant (NPP). Furthermore, the inverse of the macroscopic total cross section is not present in the formulation meaning that the behaviour of neutral particles in voided regions can be modelled. The weak form of the WLS equation can be discretised using a wide variety of spatial basis functions and discretisation techniques such as the finite element (FE), spectral element (SE) or isogeometric analysis (IGA) methods. However, the focus of this paper is the development of locally refined IGA methods for continuous BG discretisations of the WLS equation.

IGA is a spatial discretisation methodology with two key benefits over traditional FE methods. Firstly, the NURBS basis functions used to approximate the solution in IGA allow for the exact representation of geometries created using computer aided design (CAD) method. This includes conic sections in two dimensions and quadric surfaces in three dimensions as well as more complex shapes to be modelled. Secondly, the approximation space used in NURBS based IGA enables high-order continuity basis functions to be used within a NURBS patch. One potential deficiency of the IGA method is that, for continuous spatial discretisations, refinement propagates throughout the spatial domain which is caused by the tensor product structure of the basis functions in multi-dimensions.

In the research presented in this paper local constraints are applied to the boundaries between NURBS patches. This is used in order to enable patch specific local refinement of the underlying NURBS mesh for continuous spatial discretisations of the neutron transport equation. This constraint based local refinement methodology is applied to the WLS-S_N equation. Several radiation shielding verification benchmark test cases are presented. These test cases are used to determine the magnitude of the discretisation error incurred by this constraint based refinement approach. The method of manufactured solutions (MMS) is used to illustrate the order of spatial convergence of the constraint based approach. Furthermore, heuristic error estimators and goal based error indicators are compared in order to ascertain an optimal self-adaptive mesh refinement strategy.