The surrogate matrix methodology: Low-cost assembly for isogeometric analysis

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ABSTRACT

This talk will introduce two new assembly strategies based on the surrogate matrix methodology [1,3–5] which lower the cost of matrix assembly in IGA applications *without sacrificing accuracy*. It is well-known that isogeometric methods face a great computational burden at the point of matrix assembly. This is due, in large part, to the quadrature involved in directly computing B-spline or NURBS basis function interactions; see, e.g., [2,6]. Nevertheless, we will show that significant classes of B-splines and NURBS bases have an intrinsic structure which can be easily exploited to avoid most of this quadrature.

The new assembly strategies we will present involve performing quadrature for only a small fraction of the IGA basis function interactions and then *approximating the rest* through, for example, interpolation. Therefore, most of the quadrature involved in standard IGA assembly is not performed at all and the predominant expense in most IGA assembly algorithms is avoided. Our strategies, which may be viewed as constructing variable-scale approximations (i.e., surrogates) for each system matrix, retain the accuracy of the standard methods because the structure of our B-spline/NURBS bases allow for a simple correspondence between matrix entries and smooth functions [5].

In this talk, we will summarize the theoretical aspects of the surrogate matrix methodology which, in turn, certify the convergence of new surrogate IGA methods for Poisson's equation, membrane vibration, plate bending, Stokes' flow, and other problems. We will also focus on the implementation of the methodology in existing IGA code, using the open-source GeoPDEs library [7] as an example. For the sake of demonstration, we will show assembly speed-ups of up to fifty times, after only a few small modifications to this software. The capacity for even further speed-ups is clearly possible and similar modifications could be made to other contemporary software libraries.

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