Applications of Smooth Functional Surfaces over an Unstructured Quadrilateral Mesh

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ABSTRACT

Extending IgA analysis to unstructured multipatches allows the solution of PDEs over complex engineering domains. One recent breakthrough is the construction a basis for C^1-smooth isogeometric function spaces over unstructured C^0 plane meshes: given a function defined on such a mesh, whose graph surface also has a B-spline representation, the resulting C^1 basis functions can be constructed along the edges while using higher order B-splines inside each patch. This has been the subject of recent publications, see [1], for the references.

A natural application of this paradigm is the numerical solution of fourth-order PDE problems such as the biharmonic equation, using the isogeometric method. A natural question is how can this construction be extended to real life 3D problems.

We propose to review several extensions based on the ideas of Babuska and Vogelius [3] on Geometric Domain Reduction. By considering (quasi)-cylindrical domains (in CAD often referred to 2.5 dimensional domains), we can reduce the study of such 3D solids to a collection of 2-D problems by the use of a projection onto a space of polynomials defined over the "thickness". Thus one can build a hierarchy of plate models based on C^1 2D B-splines and extend this to plates with variable thickness, described by unstructured meshes.. The same method, with different polynomials, can be used to generalize "thin" flows such as Helle-Shaw equations [2], (injection between two plates), and lubrification, while keeping the structure nearly $2D$. A nice collection of problems arises then such as locking and boundary layers (thick plates) or stability (flows). But the formulation being 3D in nature there is no need to use complex mixed methods such as the $\overline{B}bar$ one. Hence the IgA method can be extended to simulations that are defined on 2.5 domains and still retain their $3D$ details.

REFERENCES

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